European option pricing under cumulative prospect theory with alternative probability weighting functions

Martina Nardon and Paolo Pianca

Department of Economics, Ca' Foscari University of Venice
mnardon@unive.it, pianca@unive.it

Abstract. In this contribution, we evaluate European financial options under continuous cumulative prospect theory. We focus on investors' probability risk attitudes and consider alternative probability weighting functions. In particular, the *constant relative sensitivity* weighting function is the only one, amongst those proposed in the literature, which is able to model separately curvature and elevation. Curvature models optimism and pessimism when one moves from extreme probabilities, whereas elevation can be interpreted as a measure of relative optimism. We performed a variety of numerical experiments and studied the effects of both these features on options prices.

Keywords: European option pricing; cumulative prospect theory; probability weighting function; curvature; elevation.

JEL Classification: C63, D81, G13.

1 INTRODUCTION

Prospect theory has recently begun to attract attention in the literature on financial options valuation; when applied to option pricing in its continuous cumulative version (Tversky and Kahneman [1992]; Davis and Satchell [2007]), it seems a promising alternative to other models, for its potential to explain option mispricing. Empirical studies on quoted options highlight systematic differences between the market prices and the Black and Scholes model; this may be due to different causes, such as assumptions regarding the price dynamics (volatility, in particular), markets frictions, information imperfections, and investors' attitude toward risk. Normally one tries to improve the performance of models assuming alternative dynamics for the prices of the underlying assets, but leaving unchanged decision maker's preferences. A different approach is to price options considering behavioral aspects of the operators.

The literature on behavioral finance (see e.g. Barberis and Thaler [2003] and Subrahmanyam [2007] for a survey) and prospect theory is ample, whereas a few studies in this field focus on financial options. A first contribution which applies prospect theory to options valuation is the work of Shefrin and Statman [1993], who consider covered call options in a one period binomial model. A list of paper in this field should include: Poteshman and Serbin [2003], Abbink and Rockenbach [2006], Breuer and Perst [2007]. Versluis *et al* [2010] apply the cumulative prospect theory in the continuous case in order to evaluate European call options. Nardon and Pianca [2014] extend the model of Versluis *et al* [2010] to the European put option; the authors also consider both the positions of the writer and the holder.

According to prospect theory, individuals do not always take their decisions consistently with the maximization of expected utility. Decision makers are risk averse when considering gains and risk-seeking with respect to losses. They are loss averse: people are much more sensitive to losses than they are to gains of comparable magnitude. Gambles are evaluated based on potential gains and losses relative to a *reference point*, rather than in terms of final wealth. Decision makers tend to underweight high probabilities and overweight low probabilities; empirical evidence of such behaviors is reported in Kahneman and Tversky [1979]. Risk attitude, loss aversion and subjective probabilities are described by two functions: a value function and a weighting function, which models probability perception.

Shiller [1999] argues that the weighting function may be one of the possible causes of overpricing of out-of-the-money and in-the-money options, thus it may explain the options smile. This phenomenon could be justified in terms of the distortion in probabilities represented by the weighting function, due to the overestimation of small probabilities and underestimation of medium and large probabilities. The weighting function could also explain the down-turned corners that some smiles exhibit if for extreme strike prices, the discontinuities at the extremes of the weighting function become relevant (Shiller [1999]).

In this contribution, we focus on the effects on European option prices of the shape of the probability weighting function. Such a function models probabilistic risk behavior; its *curvature* is related to the risk attitude towards probabilities. Empirical evidence suggests a particular shape of probability weighting functions which turns out in a typical *inverse-S*: the function is initially concave (probabilistic risk seeking or *optimism*) for small probabilities and convex (probabilistic risk aversion or *pessimism*) for medium and large probabilities. A linear weighting function describes probabilistic risk neutrality or objective sensitivity towards probabilities, which characterizes expected utility. Empirical findings indicate that the intersection between the weighting function and the linear function (*elevation*) is for probability around 0.33. Curvature of the weighting function models optimism and pessimism when one moves from extreme probabilities, whereas elevation can be interpreted as a measure of relative optimism. Alternative weighting functions have been used in many theoretical and experimental studies. The *constant relative sensitivity* weighting function proposed by Abdellaoui *et al* [2010] is the only one, amongst those in the literature, which is able to model separately curvature and elevation. We are interested in studying the effects of both these features on options prices.

The rest of the paper is organized as follows. Section 2 synthesizes the main features of prospect theory. Section 3 focuses on the probability weighting function. Section 4 presents the option pricing models under continuous cumulative prospect theory. In Section 5 numerical results are provided and discussed. Section 6 concludes.

2 PROSPECT THEORY

Prospect theory,[1] in its formulation proposed by Kahneman and Tversky [1979], is based on the subjective evaluation of *prospects*. A preference relation is introduced over the set of all prospects; originally prospect theory deals only with a limited set of prospects. With n potential future outcomes $\{x_1, x_2, \dots, x_n\}$, a prospect is a vector of pairs $(\Delta x_i, p_i)$, for $i = 1, 2, \dots, n$. A probability p_i is assigned to any possible outcome. Assume $\Delta x_i \leq \Delta x_j$, for $i < j$, with $\Delta x_i \leq 0$ for $i \leq k$ and $\Delta x_i > 0$ for $i > k$. Infinitely many outcomes may also be considered (Schmeidler [1989]).

It is worth noting that outcome Δx_i is defined relative to a certain *reference point* x^*; being x_i the absolute outcome, we have $\Delta x_i = x_i - x^*$. An important difference between expected utility and prospect theory is that in the former results are evaluated considering the final wealth, whereas in the latter results are evaluated through a value function v which considers only outcomes. Zero is usually taken as a reference point (the status quo).

The degree of risk aversion or risk seeking appears to depend not only on the values, but also on the probability and ranking of the outcomes. Subjective values $v(\Delta x_i)$ are not multiplied by objective probabilities p_i, but using *decision weights* $\pi_i = w(p_i)$, computed via a weighting function.

The shape of the value function and the weighting function becomes significant in capturing the full complexity of actual choice patterns. It is also relevant to separate gains from losses, as negative and positive outcomes may be evaluated differently by the investors: the function v is typically convex in the range of losses and concave and steeper in the range of gains; whereas subjective probabilities may be evaluated through a weighting function w^- for losses and w^+ for gains, respectively.

Let us now denote with Δx_i, for $-m \leq i < 0$ negative outcomes and for $0 < i \leq n$ positive outcomes, with $\Delta x_i \leq \Delta x_j$ for $i < j$. Subjective value of a prospect is displayed as follows:

$$V = \sum_{i=-m}^{n} \pi_i \cdot v(\Delta x_i)\,, \tag{1}$$

with decision weights π_i and values $v(\Delta x_i)$ based on relative outcomes. In the case of expected utility, the weights are $\pi_i = p_i$ and a utility function is considered. In the following, in order to simplify the notation, it will be convenient to write x_i instead of Δx_i for the net outcomes, but still considering outcomes interpreted as deviations from a reference point.

Cumulative prospect theory developed by Tversky and Kahnemann [1992] overcomes some drawbacks (such as violation of stochastic dominance) of the original prospect theory. In cumulative prospect theory decision weights π_i are differences in transformed (through a weighting function w) cumulative probabilities of gains or losses. Formally:

$$\pi_i = \begin{cases} w^-(p_{-m}) & i = -m \\ w^-\left(\sum_{j=-m}^{i} p_j\right) - w^-\left(\sum_{j=-m}^{i-1} p_j\right) & i = -m+1, \ldots, -1 \\ w^+\left(\sum_{j=i}^{n} p_j\right) - w^+\left(\sum_{j=i+1}^{n} p_j\right) & i = 0, \ldots, n-1 \\ w^+(p_n) & i = n. \end{cases} \tag{2}$$

Specific parametric forms have been suggested in the literature for the value function. Let x be an outcome, a function which is used in many empirical studies is

$$\begin{cases} v^- = -\lambda(-x)^b & x < 0 \\ v^+ = x^a & x \geq 0, \end{cases} \tag{3}$$

with positive parameters that control risk attitude ($0 < a \leq 1$ and $0 < b \leq 1$) and loss aversion ($\lambda \geq 1$); v^- and v^+ denote the value function for losses and gains, respectively. Function (3) has zero as reference point; it is concave for positive outcomes and convex for negative ones, it is steeper for losses. Parameters values equal to one imply risk and loss neutrality.

In financial applications, and in particular when dealing with options, prospects may involve a continuum of values; hence, prospect theory cannot be applied directly in its original or cumulative versions. Davis and Satchell [2007] provide the continuous cumulative prospect value:[2]

$$V = \int_{-\infty}^{0} \psi^{-}[F(x)]\, f(x)\, v^{-}(x)\, dx + \int_{0}^{+\infty} \psi^{+}[1-F(x)]\, f(x)\, v^{+}(x)\, dx, \tag{4}$$

where $\psi = \frac{dw(p)}{dp}$ is the derivative of the weighting function w with respect to the probability variable, F is the cumulative distribution function and f is the probability density function of the outcomes.

3 THE WEIGHTING FUNCTION

Prosect theory involves a probability weighting function which models probabilistic risk behavior. A weighting function w is uniquely determined, it maps the probability interval $[0,1]$ into $[0,1]$, and is strictly increasing, with $w(0) = 0$ and $w(1) = 1$. In this work we will assume continuity of w on $[0,1]$, even thought in the literature discontinuous weighting functions are also considered.

The *curvature* of the weighting function is related to the risk attitude towards probabilities. Empirical evidence suggests a particular shape of probability weighting functions: small probabilities are overweighted $w(p) > p$, whereas individuals tend to underestimate large probabilities $w(p) < p$. This turns out in a typical *inverse-S shaped* weighting function: the function is initially concave (probabilistic risk seeking or *optimism*) for probabilities in the interval $(0, p^*)$, and convex (probabilistic risk aversion or *pessimism*) in the interval $(p^*, 1)$, for a certain value of p^*. A linear weighting function describes probabilistic risk neutrality or objective sensitivity towards probabilities, which characterizes expected utility. Empirical findings indicate that the intersection (*elevation*) between the weighting function and the 45$^{\text{o}}$ line, $w(p) = p$, is for p^* in the interval $(0.3, 0.4)$.

The sensitivity towards probability is increased if (Abdellaoui *et al* [2010])

$$\frac{w(p)}{p} > 1, \qquad p \in (0, \delta) \qquad \text{and} \qquad \frac{1-w(p)}{1-p} > 1, \qquad p \in (1-\varepsilon, 1),$$

whereas a weighting function exhibits decreased sensitivity if

$$\frac{w(p)}{p} < 1, \qquad p \in (0, \delta) \qquad \text{and} \qquad \frac{1-w(p)}{1-p} < 1, \qquad p \in (1-\varepsilon, 1),$$

for some arbitrary small $\delta > 0$ and $\varepsilon > 0$.

Some weighting functions (e.g. the functions suggested by Goldstein and Einhorn [1987], Tversky and Kahneman [1992] and Prelec [1998]) display *extreme sensitivity*, in the sense that $w(p)/p$ and $(1-w(p))/(1-p)$ are unbounded as p tends to 0 and 1, respectively.

As already noticed, empirical studies on probability perception suggest the typical inverse-S shaped form for w, which combines the increased sensitivity with concavity for small probabilities and convexity for medium and large probabilities. In particular, such a function captures the fact that individuals are extremely sensitive to changes in (cumulative) probabilities which approach to 0 and 1. Abdellaoui *et al* [2010] discuss how optimism and pessimism are possible sources of increased sensitivity.

Different parametric forms for the weighting function with the above mentioned features have been proposed in the literature, and their parameters have been estimated in many empirical studies. Single parameter probability weighting functions are those proposed by Karmarkar [1978, 1979], Röell [1987], Currim and Sarin [1989], Tversky and Kahneman [1992], Luce *et al* [1993], Hey and Orme [1994], Prelec [1998], Safra and Segal [1998], and Luce [2000]. Two (or more) parameters probability weighting functions have been proposed by Bell [1985], Goldstein and Einhorn [1987], Currim and Sarin [1989], Lattimore *et al* [1992], Wu and Gonzales [1996], Prelec [1998], Diecidue *et al* [2009], and Abdellaoui *et al* [2010].

Karmarkar [1978] considers the following function

$$w(p) = \frac{p^{\gamma}}{p^{\gamma} + (1-p)^{\gamma}}, \tag{5}$$

with $\gamma > 0$. Function (5) is a special case (when $\delta = 1$) of the two parameters family proposed by Wu and Gonzales [1996],

$$w(p) = \frac{p^{\gamma}}{\left(p^{\gamma} + (1-p)^{\gamma}\right)^{\delta}}, \tag{6}$$

with δ and γ positive.

Tversky and Kahneman [1992] use the Quiggin's [1982] functional of the form

$$w(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}}, \tag{7}$$

where γ is a positive constant (with some constraint in order to have an increasing function). The parameter γ captures the degree of sensitivity toward changes in probabilities from impossibility (zero probability) to certainty (Tversky and Kahneman [1992]). When $\gamma < 1$, one obtains the typical inverse-S shaped form; the lower the parameter, the higher is the curvature of the function. Note that $w(0) = 0$ and $w(1) = 1$ for the above defined functions.

Prelec [1998] suggests a two parameter function of the form

$$w(p) = e^{-\delta(-\ln p)^{\gamma}}, \qquad p \in (0,1), \tag{8}$$

with $w(0) = 0$ and $w(1) = 1$. The parameter δ (with $0 < \delta < 1$) governs elevation of the weighting function relative to the 45^{o} line, while γ (with $\gamma > 0$) governs curvature and the degree of sensitivity to extreme results relative to medium probability outcomes. When $\gamma < 1$, one obtains the inverse-S shaped function. In this model, the parameter δ influences the tendency of over- or under-weighting the probabilities, but it has no direct meaning.

As an alternative, one can also consider the more parsimonious single parameter Prelec's weighting function

$$w(p) = e^{-(-\ln p)^{\gamma}}, \qquad p \in (0,1), \tag{9}$$

which only allows for curvature to be varied. Note that in this case, the unique solution of equation $w(p) = p$ for $p \in (0,1)$ is $p = 1/e \simeq 0.367879$ and does not depend on the parameter γ.

In an empirical study, Wu and Gonzales [1999] consider both the Prelec [1998] weighting function and the *linear in log odds* function proposed by Goldstein and Einhorn [1987],

$$w(p) = \frac{\delta\, p^{\gamma}}{\delta\, p^{\gamma} + (1-p)^{\gamma}}. \tag{10}$$

Function (10) has also been used by Lattimore *et al* [1992] in a variant functional form, Tversky and Fox [1995], Birnbaum and McIntosh [1996], and Kilka and Weber [2001]. The weighting function proposed by Karmarkar [1978, 1979] is a special case of (10) with $\delta = 1$.

An interesting parametric function is the *switch-power weighting function* proposed by Diecidue *et al* [2009], which consists in a power function for probabilities below a certain value $\hat{p} \in (0,1)$ and a dual power function for probabilities above $\hat{p}$; formally w is defined as follows:

$$w(p) = \begin{cases} cp^a & \text{if } 0 \leq p \leq \hat{p}, \\ 1 - d(1-p)^b & \text{if } \hat{p} < p \leq 1, \end{cases} \tag{11}$$

with five parameters a, b, c, d, and $\hat{p}$. All the parameters are strictly positive, assuming continuity and monotonicity of w. When $\hat{p}$ approaches 1 or 0, w reduces to a power or a dual power probability weighting function, respectively. Diecidue *et al* [2009] provide preference foundation for such a family of parametric weighting functions and inverse-S shape under rank dependent utility based on testable preference conditions.

Parameters in (11) reduce to three (a, b, and $\hat{p}$) by assuming continuity of $w(p)$ at $\hat{p}$ and differentiability. Hence one obtains

$$c = \hat{p}^{1-a} \left(\frac{b\hat{p}}{b\hat{p} + a(1-\hat{p})} \right), \tag{12}$$

and

$$d = (1-\hat{p})^{-b} \left(\frac{a(1-\hat{p})}{b\hat{p} + a(1-\hat{p})} \right). \tag{13}$$

For $a,b \leq 1$, the function w is concave on $(0,\hat{p})$ and convex on $(\hat{p},1)$ (it has an inverse-S shape), while for $a,b \geq 1$ the weighting function is convex for $p < \hat{p}$ and concave for $p > \hat{p}$ (it has an S-shape). Both parameters a and b govern the curvature of w when $a \neq b$. In particular, parameter a describes probabilistic risk attitude for small probabilities; whereas parameter b describes probabilistic risk attitude for medium and large probabilities. In the case with $a \neq b$, parameter $\hat{p}$, which signals the point where probabilistic risk attitudes change from risk aversion to risk seeking (for an inverse-S shaped function), may not lie on the 45$^{\text{o}}$ line, hence it has not the meaning of dividing the region of over- and under-weighting of the probability.

When $a = b$, then w intersects the 45^o line at $\hat{p}$. In such a case, one obtains the following two parameter probability weighting function

$$w(p) = \begin{cases} \hat{p}^{1-a} p^a & \text{if } 0 \leq p \leq \hat{p}, \\ 1 - (1-\hat{p})^{1-a}(1-p)^a & \text{if } \hat{p} < p \leq 1. \end{cases} \tag{14}$$

Parameter $\hat{p}$ separates the regions of over- and under-weighting of probabilities. This is the same form as the *constant relative sensitivity* weighting function considered by

Abdellaoui *et al* [2010]:

$$w(p) = \begin{cases} \delta^{1-\gamma} p^{\gamma} & \text{if } 0 \le p \le \delta, \\ 1-(1-\delta)^{1-\gamma}(1-p)^{\gamma} & \text{if } \delta < p \le 1, \end{cases} \tag{15}$$

with $\gamma > 0$ and $\delta \in [0,1]$. For $\gamma < 1$ and $0 < \delta < 1$ it has an inverse-S shape. The derivative of w at δ equals γ; this parameter controls for the curvature of the weighting function. The parameter δ indicates whether the interval for overweighting probabilities is larger than the interval for underweighting, and therefore controls for the elevation. Hence, such a family of weighting functions allows for a separate modeling of these two features.

Remember that a convex weighting function characterizes probabilistic risk aversion and a concave weighting function characterizes probabilistic risk proneness, whereas a linear weighting function characterizes probabilistic risk neutrality. Then the role of δ is to demarcate the interval of probability risk seeking from the interval of probability risk aversion.[3] In such a case, overweighting corresponds to risk seeking (or optimism) and underweighting corresponds to risk proneness (or pessimism). Elevation represents the relative strength of optimism versus pessimism, hence it is a measure of relative optimism, and δ may be interpreted as an index of relative optimism.

The intersection between the weighting function and the 45^{o} line is for p in the interval $(0.3, 0.4)$. Gonzales and Wu [1999] and Abdellaoui *et al* [2010] find that the weighting function is more elevated for losses than for gains. In Abdellaoui *et al* [2010] the relative index of optimism for gains δ^+ is lower than the relative index of pessimism for losses δ^-.

Curvature is a measure of the degree of sensitivity to changes from impossibility to possibility (Tversky and Kahneman [1992]), it represents the diminishing effect of optimism and pessimism when moving away from extreme probabilities 0 and 1. Hence parameter γ, controlling for curvature, measures the relative sensitivity of the weighting function. This suggests an interpretation for such a parameter as a measure of relative risk aversion. The index of relative sensitivity (Abellaoui *et al* [2010]) of

w as defined in (15) is

$$
\begin{aligned}
RS(w,p) &= -\frac{p\frac{\partial^2 w(p)}{\partial p^2}}{\frac{\partial w(p)}{\partial p}} && \text{for} \quad p \in (0,\delta], \\
RS(w,p) &= -\frac{(1-p)\frac{\partial^2(1-w(p))}{\partial(1-p)^2}}{\frac{\partial(1-w(p))}{\partial(1-p)}} && \text{for} \quad p \in (\delta,1),
\end{aligned}
\tag{16}
$$

which is constant on the interval $(0, 1)$ and equals $1-\gamma$. For this reason, probability functions of the form (15) are called *constant relative sensitivity* weighting functions.

Gonzales and Wu [1999] discuss the importance of modeling curvature and elevation independently, providing psychological interpretation. To our knowledge, the functional form in (15) is the only one, amongst those used in the literature, which is able to capture separately the effects of curvature and elevation.

4 EUROPEAN OPTIONS VALUATION

We evaluate European financial options within continuous cumulative prospect theory; in particular, in the applications we use the constant relative sensitivity weighting function defined in the previous section.

Versluis *et al* [2010] provide the prospect value of writing call options, considering different time aggregation of the results. Nardon and Pianca [2014] extend the model of Versluis *et al* [2010] to the case of put options; the authors also consider the problem both from the writer's and holder's perspective, and use alternative weighting functions.

Let S_t be the price at time t, with $t \in [0,T]$, of the underlying asset of a European option with maturity T; in a Black-Scholes setting, the underlying price dynamics is driven by a geometric Brownian motion. The probability density function of the underlying price at maturity S_T is

$$
f(x) = \frac{1}{x\sigma\sqrt{2\pi T}} \exp\left(\frac{-\left[\ln(x/S_0)-(\mu-\sigma^2/2)T\right]^2}{2\sigma^2 T} \right), \tag{17}
$$

where μ and $\sigma > 0$ are constants, and the cumulative distribution function is

$$
F(x) = \Phi\left(\frac{\ln(x/S_0)-(\mu-\sigma^2/2)T}{\sigma\sqrt{T}} \right), \tag{18}
$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard Gaussian random variable.

Let c be the call option premium with strike price X. At time $t=0$, the option's writer receives c and can invest the premium at the risk-free rate r, obtaining $c\,e^{rT}$. At maturity, the amount $S_T - X$ is paid to the holder if the option expires in-the-money.

The option premium represents a certain gain for the writer, while the negative payoff is a possible loss; the writer may aggregate or separate results in different ways, depending on what leads to the highest possible prospect value (*hedonic framing*). Multiple gains are preferred to be segregated, which is not the case of multiple losses which are usually integrated to alleviate the loss, mixed outcomes may be integrated in order to cancel out losses when there is a net gain or a small loss, in case of large losses with a small gain (e.g. the option premium), they are segregated to preserve the *silver lining*.

Results can also be integrated or separated over time. In the *time segregated* case, the option premium is evaluated separately through a value function from the option payoff. Considering zero as a reference point (*status quo*), the prospect value of the writer's position is

$$V_s = v^+\left(c\,e^{rT}\right) + \int_X^{+\infty} \psi^-\left(1-F(x)\right) f(x) v^-\left(X-x\right) dx, \tag{19}$$

with f and F being the probability density function and the cumulative distribution function of the future underlying price S_T, and v is defined as in (3).

The investor is indifferent between the prospect value V and the status quo when $V=0$. Equating V_s at zero and solving for the price c, yields

$$c = e^{-rT}\left(\lambda \int_X^{+\infty} \psi^-\left(1-F(x)\right) f(x)\left(x-X\right)^b dx\right)^{1/a}, \tag{20}$$

which requires numerical approximation of the integral.

When considering the *time aggregated* prospect value, gains and losses are integrated in a unique prospect, then one obtains

$$\begin{aligned} V_a = {} & w^+\left(F(X)\right) v^+\left(c\,e^{rT}\right) + \\ & + \int_X^{X+c\,e^{rT}} \psi^+\left(F(x)\right) f(x) v^+\left(c\,e^{rT} - (x-X)\right) dx + \\ & + \int_{X+c\,e^{rT}}^{+\infty} \psi^-\left(1-F(x)\right) f(x) v^-\left(c\,e^{rT} - (x-X)\right) dx. \end{aligned} \tag{21}$$

In this latter case, the option price is implicitly defined by the equation $V_a = 0$.

In order to obtain the value of a European put option, we can no longer use put-call parity arguments. Let p be the put option premium at time $t = 0$; the prospect value of the writer's position in the time segregated case is

$$V_s = v^+ \left(p\,e^{rT}\right) + \int_0^X \psi^- \left(F(x)\right) f(x) v^- \left(x - X\right) dx\,, \tag{22}$$

and one obtains

$$p = e^{-rT} \left(\lambda \int_0^X \psi^- \left(F(x)\right) f(x) \left(X - x\right)^b dx\right)^{1/a}. \tag{23}$$

In the time aggregated case the put option value is implicitly defined equating at zero the following expression

$$\begin{aligned} V_a = &\int_0^{X - p\,e^{rT}} \psi^- \left(F(x)\right) f(x)\, v^- \left(p\,e^{rT} - (X - x)\right) dx + \\ &+ \int_{X - p\,e^{rT}}^X \psi^+ \left(1 - F(x)\right) f(x)\, v^+ \left(p\,e^{rT} - (X - x)\right) dx + \\ &+ w^+ \left(1 - F(X)\right) v^+ \left(p\,e^{rT}\right), \end{aligned} \tag{24}$$

which has to be solved numerically for p.

4.1 Option valuation from holder's perspective

When one considers the problem from the holder's viewpoint, the prospect values both in the time segregated and aggregated cases change. Holding zero as reference point, the prospect value of the holder's position for a call option in the time segregated case is

$$V_s^h = v^- \left(-c\,e^{rT}\right) + \int_X^{+\infty} \psi^+ \left(1 - F(x)\right) f(x) v^+ (x - X)\, dx\,, \tag{25}$$

with f and F being the probability density function and the cumulative distribution function of the future underlying price S_T.

Consider v as defined in (3). We equate V_s^h at zero and solve for the price c, obtaining

$$c_s^h = e^{-rT} \left(\frac{1}{\lambda} \int_X^{+\infty} \psi^+ \left(1 - F(x)\right) f(x) \left(x - X\right)^a dx\right)^{1/b}. \tag{26}$$

In the time aggregated case, the prospect value has the following integral representation

$$
\begin{aligned}
V_a^h = {} & w^-\left(F(X)\right)v^-\left(-c\,e^{rT}\right)+\\
& +\int_X^{X+c\,e^{rT}} \psi^-\left(F(x)\right)f(x)v^-\left((x-X)-c\,e^{rT}\right)dx+\\
& +\int_{X+c\,e^{rT}}^{+\infty} \psi^+\left(1-F(x)\right)f(x)v^+\left((x-X)-c\,e^{rT}\right)dx\,.
\end{aligned}
\tag{27}
$$

In order to obtain the call option price in equilibrium, one has to solve numerically for c.

In an analogous way one can derive the put option prospect values for the holder's position. In the segregated case the prospect value is

$$
V_s^h = v^-\left(-p\,e^{rT}\right)+\int_0^X \psi^+\left(F(x)\right)f(x)v^+(X-x)\,dx\,. \tag{28}
$$

Equating at zero and solving for the price p, one obtains

$$
p_s^h = e^{-rT}\left(\frac{1}{\lambda}\int_0^X \psi^+\left(F(x)\right)f(x)\,(X-x)^a\,dx\right)^{1/b}. \tag{29}
$$

Finally, in the time aggregated setting, the prospect value from holder's viewpoint is

$$
\begin{aligned}
V_a^h = {} & w^-\left(1-F(X)\right)v^-\left(-p\,e^{rT}\right)+\\
& +\int_{X-p\,e^{rT}}^{X} \psi^-\left(1-F(x)\right)f(x)v^-\left((X-x)-p\,e^{rT}\right)dx+\\
& +\int_0^{X-p\,e^{rT}} \psi^+\left(F(x)\right)f(x)v^+\left((X-x)-p\,e^{rT}\right)dx\,.
\end{aligned}
\tag{30}
$$

The put option value is implicitly defined by the equation $V_a^h = 0$.

5 NUMERICAL RESULTS

We performed a wide sensitivity analysis on call and put options values considered both from writer's and holder's perspective, computed with the models presented in the previous section. We have calculated the options prices both in the time segregated and aggregated case, and applied alternative weighting functions. In this contribution, we report only the results for the constant relative sensitivity weighting function (15) proposed by Abdellaoui *et al* [2010], as we are interested in separating the effects of curvature governed by the parameter γ and elevation controlled by δ.

We computed the option prices for several values of the volatility and the strike price X. We let vary the parameters $\gamma \in [0.7, 1.0]$ and $\delta \in [0.3, 0.4]$. For the value function, we compared different parameters sets, ranging from *TK sentiment* (Tversky and Kahnemann [1992]) to more *moderate sentiment*; a linear function (with $a = b = 1$ and $\lambda = 1$) is considered as a limiting case (no sentiment). Tversky and Kahnemann [1992] used the following values for the parameters: $a = b = 0.88$, $\lambda = 2.25$, $\gamma^+ = 0.61$, and $\gamma^- = 0.69$. Versluis *et al* [2010] adopt 10 % of TK sentiment; in their study the parameters are: $a = b = 0.988$, $\lambda = 1.125$, $\gamma^+ = 0.961$, and $\gamma^- = 0.969$.

The choice of moderate sentiment for the values of the parameters is motivated in order to obtain realistic option prices. TK sentiment parameters yield too high options prices to be used in practice, in particular in the segregated case (Versluis *et al* [2010]; Nardon and Pianca [2014]); 10 % of the TK sentiment provides results more in line with market prices. A value for γ in the interval $[0.95, 1]$ seems more realistic. We also considered different sensitivity to probability risk for positive and negative outcomes ($\gamma^+ \neq \gamma^-$ and $\delta^+ \neq \delta^-$). The choice of δ is suggested by empirical evidence, as discussed above.

It is also worth noting that, when we set $\mu = r$, $a = b = 1$, $\lambda = 1$, and $\gamma = 1$, we obtain the same results as in the Black-Scholes (BS) model.

Numerical results suggest that option prices are increasing with δ (elevation) within the interval $[0.3, 0.4]$; prices increase at a decreasing rate, and the effect is more important the lower is γ (the higher the curvature). Note that this is true with some rare exceptions, as it is the case of deep-in-the-money puts from holder's perspective, which may be due to possible round-off errors in the numerical procedure applied in order to approximate the integrals and to numerically solve the equations presented in the previous section.

The effect of γ (curvature) is non-trivial, depending on the moneyness and the model (time-aggregated or segregated) which is used. In particular, in the time-aggregated model (writer's perspective), option prices are decreasing with respect to γ; in the time-segregated model (writer's perspective), option prices are decreasing with respect to γ, with the exception of deep-in-the-money calls and puts. A possible explanation may be the framing effect (together with the specific shape of the the value function and the weighting function) in the segregated model. In such cases, it is more likely that the option is exercised at maturity, hence the result for the writer is the sum of a highly negative outcome and a relative small positive gain. In the segregated case, the option premium is evaluated separately, hence preserving the silver lining.

Tables 1 and 2 report the results for the European calls and puts in the time-aggregated model, from writer's perspective, for different strikes and elevation. Similar results are obtained in the time-segregated model (which are not reported in the paper for the sake of brevity). Here we focus on the effect of elevation. In these examples, the parameters of the value function a, b and λ are fixed. We assume moderate sensitivity of the value function; in particular, the tables show the results for parameters corresponding to 10% of the TK sentiment.[4] Here we assume $\delta^+ = \delta^-$ and $\gamma^+ = \gamma^-$. The parameter δ is letting vary in the interval $[0.3, 0.4]$. As regards the parameter γ, we calculate option prices for a wide interval ranging from $\gamma = 0.7$ (which is closer to the value used by Tversky and Kahnemann [1992]) to $\gamma = 1$ (in this latter case the only effect of the value function applies and of course the prices are constant with respect to γ). We observe that for lower values of γ, options prices deviates sensitively from Black-Scholes prices.

Tables 3 and 4 report the results for the European calls and puts in the time-aggregated models from holder's perspective, for different strikes and elevation. Also in these cases, options prices are increasing with δ.

The effect of the curvature on option prices is more important. As γ increases the option value decreases; this happens both when we evaluate options from the writer's and holder's viewpoint. Figure 1 shows some results for the call and put options in the time-aggregated model, when $\delta = 0.325$, $a = b = 0.976$, and $\lambda = 1.125$; in these cases, option premia are decreasing with curvature of the weighting function, approaching the BS price when γ tends to 1.

Note that writer's prices are always above BS prices. This is not the case when we consider holder's perspective. If one considers the pricing problem both from the writer's and holder's perspective, it is possible to obtain an interval for the prices of call and put options for certain values of the sentiment parameters which are of practical interest. Balck-Scholes price lies in the interval bounded by the holder's price from below and the writer's price from above. The range of such an interval depends on the value of the parameters which govern investor's sentiment (attitude toward risk, loss aversion, and probability bias). More moderate sentiment implies smaller price intervals.

It is also interesting to study the effects of γ and δ on the implied volatility. As already observed (see also Shiller [1999]), the under- and over-weighting of probabilities might be a possible cause of deviations of options prices from the theoretical Black-Scholes model. We consider options values obtained with the models described above, and compute implied volatilities in the BS formula. As an example, figure 2 shows implied volatilities for the call and put options in the time-aggregated model, when $\delta = 0.325$, $a = b = 0.988$, and $\lambda = 1.125$. Implied volatility is increasing with the sentiment (lower values of γ); higher values of δ yield higher volatility, even though the effect of elevation is less pronounced than the effect of curvature. Implied volatilities exhibit a different effect for call and put options: in both cases there is a smile (or skewed) effect, but volatility is decreasing with moneyness for the call option, whereas it is increasing for the put options.

6 CONCLUSIONS

Prospect theory applied to option valuation seems a promising approach in order to explain options mispricing. In this contribution, we used the constant relative sensitivity weighting function, within the framework of cumulative prospect theory in its continuous version. Such a weighting function allows for separate modeling of curvature and elevation, which has an interesting interpretation in terms of probabilistic optimism and pessimism. Option prices are sensitive to the choice of the values of the sentiment parameters. We performed a number of numerical experiments in order to study the effects of curvature and elevation on option premia and on implied volatility.

NOTES

1. The book of Wakker [2010] provides a thorough treatment on prospect theory.
2. See also Rieger and Wang [2008], Wakker [2010], and Kothiyal *et al* [2011].

3. This is not the case for weighting function (11); when $a \neq b$, both parameters controls for curvature and all parameters may influence elevation.
4. In other numerical trials we used alternative values of the parameters a, b and λ; in particular 20% of the TK sentiment.

REFERENCES

Abbink, K. and B. Rockenbach. "Option pricing by students and professional traders: A behavioral investigation." *Managerial and Decision Economics*, Vol. 27, No. 6, (2006), pp. 497–510.

Abdellaoui, M., O. L'Haridon and H. Zank. "Separating curvature and elevation: A parametric probability weighting function." *Journal of Risk and Uncertainty*, Vol. 41, (2010), pp. 39–65.

Barberis, N. and R. Thaler. "A survey of behavioral finance." In G.M. Constantinides, M. Harris and R. Stulz, eds, *Handbook of the Economics of Finance*, Elsevier Science, (2003), pp. 1051–1121.

Bell, D.E. "Disappointment in decision making under uncertainty." *Operations Research*, Vol. 33, (1985), pp. 1–27.

Birnbaum, M.H. and W.R. McIntosh. "Violations of branch independence in choices between gambles." *Organizational Behavior and Human Decision Processes*, Vol. 67, (1996), pp. 91–110.

Black, F. and M. Scholes. "The pricing of options and corporate liabilities." *Journal of Political Economy*, Vol. 81, No. 3, (1973), pp. 637–654.

Breuer, W. and A. Perst. "Retail banking and behavioral financial engineering: The case of structured products." *Journal of Banking and Finance*, Vol. 31, No. 3, (2007), pp. 827–844.

Currim, I.S. and R.K. Sarin. "Prospect versus utility." *Management Science*, Vol. 35, No. 1, (1989), pp. 22–41.

Davies, G.B. and S.E. Satchell. "The behavioural components of risk aversion." *Journal of Mathematical Psychology*, Vol. 51, No. 1, (2007), pp. 1–13.

Diecidue, E., U. Schmidt, and H. Zank. "Parametric weighting functions." *Journal of Economic Theory*, Vol. 144, No. 3, (2009), pp. 1102–1118.

Goldstein, W.M. and H.J. Einhorn. "Expression theory and the preference reversal phenomena." *Psychological Review*, Vol. 94, No. 2, (1987), pp. 236–254.

Gonzalez, R. and G. Wu. "On the shape of the probability weighting function." *Cognitive Psychology*, Vol. 38, (1999), pp. 129–166.

Hey, J.D. and C. Orme. "Investigating generalizations of expected utility theory using experimental data." *Econometrica*, Vol. 62, No. 6, (1994), pp. 1291–1326.

Kahneman, D. and A. Tversky. "Prospect theory: An analysis of decision under risk." *Econometrica*, Vol. 47, No. 2, (1979), pp. 263–292.

Karmarkar, U.S. "Subjectively weighted utility: A descriptive extension of the expected utility model." *Organizational Behavior and Human Performance*, Vol. 21, (1978), pp. 61–72.

Karmarkar, U.S. "Subjectively weighted utility and the Allais Paradox." *Organizational Behavior and Human Performance*, Vol. 24, (1979), pp. 67–72.

Kilka, M. and M. Weber. "What determines the shape of the probability weighting function under uncertainty?" *Management Science*, Vol. 47, No. 12, (2001), pp. 1712–1726.

Kothiyal, A., V. Spinu and P.P. Wakker. "Prospect theory for continuous distributions: A preference foundation." *Journal of Risk and Uncertainty*, Vol. 42, (2011), pp. 195–210.

Lattimore, P.K., J.R. Baker and A.D. Witte. "The influence of probability on risky choice: A parametric examination." *Journal of Economic Behavior and Organization*, Vol. 17, No. 3, (1992), pp. 377–400.

Luce, D.R. *Utility of Gains and Losses: Measurement-Theoretical and Experimental Approaches*. Lawrence Erlbaum Publishers, London, (2000).

Luce, D.R., B.A. Mellers and S-J. Chang. "Is choice the correct primitive? On using certainty equivalents and reference levels to predict choices among gambles." *Journal of Risk and Uncertainty*, Vol. 6, (1993), pp. 115–143.

Nardon, M. and P. Pianca. "A behavioural approach to the pricing of European options." In M. Corazza and C. Pizzi, eds, *Mathematical and Statistical Methods for Actuarial Sciences and Finance*. Springer, (2014), pp. 217–228.

Poteshman, A.M. and V. Serbin. "Clearly irrational financial market behavior: Evidence from the early exercise of exchange traded stock options." *Journal of Finance*, Vol. 58, No. 1, (2003), pp. 37–70.

Prelec, D. "The probability weighting function." *Econometrica*, Vol. 66, No. 3, (1998), pp. 497–527.

Quiggin, J. "A theory of anticipated utility." *Journal of Economic Behavior and Organization*, Vol. 3, (1982), pp. 323–343.

Rieger, M.O. and M. Wang. "Prospect theory for continuous distributions." *Journal of Risk and Uncertainty*, Vol. 36, (2008), pp. 83–102.

Röell, A. "Risk aversion in Quiggin and Yaaris rank-order model of choice under uncertainty." *The Economic Journal*, Vol. 97, (1987), pp. 143–159.

Safra, Z. and U. Segal. "Constant risk aversion." *Journal of Economic Theory*, Vol. 83, (1998), pp. 19–42.

Schmeidler, D. "Subjective probability and expected utility without additivity." *Econometrica*, Vol. 57, (1989), pp. 571–587.

Shefrin, H. and M. Statman. "Behavioral aspects of the design and marketing of financial products." *Financial Management*, Vol. 22, No. 2, (1993), pp. 123–134.

Shiller, R.J. "Human behavior and the efficiency of the financial system." In J.B. Taylor and M. Woodford, eds, *Handbook of Macroeconomics*, Vol. 1C, Elsevier, (1999), pp. 1305–1340.

Subrahmanyam, A. "Behavioural finance: A review and synthesis." *European Financial Management*, Vol. 14, No. 1, (2007), pp. 12–29.

Tversky, A. and C.R. Fox. "Weighing risk and uncertainty." *Psychological Review*, Vol. 102, (1995), pp. 269–283.

Tversky, A. and D. Kahneman. "Advances in prospect theory: Cumulative representation of the uncertainty." *Journal of Risk and Uncertainty*, Vol. 5, (1992), pp. 297–323.

Versluis, C., T. Lehnert and C.C.P. Wolff. "A cumulative prospect theory approach to option pricing." Working paper, LSF Research Working Paper Series 09–03, Luxembourg School of Finance, (2010).

Wakker, P.P. *Prospect Theory: For Risk and Ambiguity*. Cambridge University Press, Cambridge, (2010).

Wu, G., and R. Gonzalez. "Curvature of the probability weighting function." *Management Science*, Vol. 42, No. 12, (1996), pp. 1676–1690.

Wu, G., and R. Gonzalez. "Nonlinear decision weights in choice under uncertainty." *Management Science*, Vol. 45, No. 1, (1999), pp. 74–85.

Table 1. Sensitivity of the call option prices (writer's position in the aggregated model) to the elevation of the weighting function, for different values of γ (curvature), with $\gamma^+ = \gamma^-$. Parameters of the value function: $a = b = 0.988$, and $\lambda = 1.125$. Option parameters: $S_0 = 100$, $X \in [80, 120]$, $r = 0.01$, $\sigma = 0.2$, $T = 1$. BS is the Black-Scholes price with $\gamma = 1$, $a = b = 1$, and $\lambda = 1$

γ	X	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
	80	21.8633	24.9153	24.9766	25.0279	25.0699	25.1119
	90	14.1929	17.5572	17.6425	17.7162	17.7773	17.8267
0.7	100	8.4333	11.6587	11.7635	11.8558	11.9371	12.0084
	110	4.6101	7.3239	7.4192	7.5057	7.5843	7.6559
	120	2.3406	4.3881	4.4611	4.5284	4.5907	4.6485
	80	21.8633	24.4130	24.4616	24.5024	24.5358	24.5625
	90	14.1929	16.9713	17.0428	17.1023	17.1514	17.1911
0.75	100	8.4333	11.0690	11.1536	11.2282	11.2938	11.3513
	110	4.6101	6.7960	6.8715	6.9400	7.0023	7.0590
	120	2.3406	3.9610	4.0172	4.0690	4.1168	4.1613
	80	21.8633	23.9789	24.0159	24.0468	24.0726	24.0931
	90	14.1929	16.4604	16.5157	16.5620	16.5999	16.6379
0.8	100	8.4333	10.5501	10.6159	10.6739	10.7248	10.7694
	110	4.6101	6.3347	6.3923	6.4446	6.4920	6.5352
	120	2.3406	3.5931	3.6347	3.6731	3.7085	3.7413
	80	21.8633	23.6009	23.6273	23.6497	23.6680	23.6828
	90	14.1929	16.0090	16.0492	16.0829	16.1106	16.1329
0.85	100	8.4333	10.0903	10.1384	10.1807	10.2179	10.2504
	110	4.6101	5.9286	5.9699	6.0072	6.0412	6.0721
	120	2.3406	3.2737	3.3026	3.3293	3.3539	3.3767
	80	21.8633	23.2696	23.2864	23.3006	23.3124	23.3218
	90	14.1929	15.6077	15.6336	15.6555	15.6735	15.6879
0.9	100	8.4333	9.6803	9.7116	9.7390	9.7632	9.7843
	110	4.6101	5.5685	5.5948	5.6186	5.6403	5.6599
	120	2.3406	2.9943	3.0122	3.0287	3.0439	3.0580
	80	21.8633	22.9775	22.9854	22.9925	22.9980	23.0025
	90	14.1929	15.2488	15.2614	15.2721	15.2808	15.2878
0.95	100	8.4333	9.3126	9.3278	9.3413	9.3530	9.3633
	110	4.6101	5.2472	5.2598	5.2712	5.2816	5.2910
	120	2.3406	2.7483	2.7566	2.7643	2.7714	2.7779
	80	21.8633	22.7186	22.7186	22.7186	22.7186	22.7186
	90	14.1929	14.9263	14.9263	14.9263	14.9263	14.9263
1	100	8.4333	8.9811	8.9811	8.9811	8.9811	8.9811
	110	4.6101	4.9590	4.9590	4.9590	4.9590	4.9590
	120	2.3406	2.5305	2.5305	2.5305	2.5305	2.5305

Table 2. Sensitivity of the put option prices (writer's position in the aggregated model) to the elevation of the weighting function, for different values of γ (curvature), with $\gamma^+ = \gamma^-$. Parameters of the value function: $a = b = 0.988$, and $\lambda = 1.125$. Option parameters: $S_0 = 100$, $X \in [80, 120]$, $r = 0.01$, $\sigma = 0.2$, $T = 1$. BS is the Black-Scholes price with $\gamma = 1$, $a = b = 1$, and $\lambda = 1$

γ	X	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
	80	1.0673	1.9858	2.0202	2.0520	2.0815	2.1089
	90	3.2974	4.8642	4.9236	4.9772	5.0257	5.0696
0.7	100	7.4383	9.4446	9.5145	9.5750	9.6270	9.6713
	110	13.5156	15.5706	15.6199	15.6605	15.6934	15.7193
	120	21.1466	22.8941	22.9157	22.9334	22.9481	22.9593
	80	1.0673	1.7992	1.8258	1.8503	1.8731	1.8942
	90	3.2974	4.5811	4.6291	4.6723	4.7115	4.7469
0.75	100	7.4383	9.1167	9.1742	9.2241	9.2671	9.3037
	110	13.5156	15.2698	15.3102	15.3428	15.3696	15.3908
	120	21.1466	22.6780	22.6946	22.7084	22.7196	22.7285
	80	1.0673	1.6364	1.6561	1.6743	1.6912	1.7069
	90	3.2974	4.3288	4.3659	4.3994	4.4297	4.4572
0.8	100	7.4383	8.8224	8.8678	8.9074	8.9415	8.9705
	110	13.5156	15.0015	15.0326	15.0583	15.0793	15.0959
	120	21.1466	22.4898	22.5022	22.5124	22.5208	22.5273
	80	1.0673	1.4934	1.5072	1.5199	1.5316	1.5425
	90	3.2974	4.1024	4.1294	4.1538	4.1758	4.1957
0.85	100	7.4383	8.5570	8.5906	8.6201	8.6454	8.6670
	110	13.5156	14.7609	14.7836	14.8024	14.8177	14.8299
	120	21.1466	22.3253	22.3338	22.3410	22.3468	22.3515
	80	1.0673	1.3672	1.3757	1.3836	1.3909	1.3976
	90	3.2974	3.8983	3.9158	3.9315	3.9458	3.9587
0.9	100	7.4383	8.3166	8.3387	8.3581	8.3749	8.3891
	110	13.5156	14.5441	14.5589	14.5710	14.5810	14.5889
	120	21.1466	22.1806	22.1859	22.1903	22.1939	22.1968
	80	1.0673	1.2551	1.2591	1.2628	1.2661	1.2693
	90	3.2974	3.7135	3.7220	3.7296	3.7365	3.7428
0.95	100	7.4383	8.0979	8.1088	8.1184	8.1267	8.1337
	110	13.5156	14.3480	14.3551	14.3611	14.3659	14.3698
	120	21.1466	22.0529	22.0553	22.0574	22.0590	22.0604
	80	1.0673	1.1552	1.1552	1.1552	1.1552	1.1552
	90	3.2974	3.5454	3.5454	3.5454	3.5454	3.5454
1	100	7.4383	7.8981	7.8981	7.8981	7.8981	7.8981
	110	13.5156	14.1697	14.1697	14.1697	14.1697	14.1697
	120	21.1466	21.9396	21.9396	21.9396	21.9396	21.9396

Table 3. Sensitivity of the call option prices (holder's position in the aggregated model) to the elevation of the weighting function, for different values of γ (curvature), with $\gamma^+ = \gamma^-$. Parameters of the value function: $a = b = 0.988$, and $\lambda = 1.125$. Option parameters: $S_0 = 100$, $X \in [80, 120]$, $r = 0.01$, $\sigma = 0.2$, $T = 1$. BS is the Black-Scholes price with $\gamma = 1$, $a = b = 1$, and $\lambda = 1$

γ	X	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
	80	21.8633	22.7435	22.7879	22.8247	22.8547	22.8783
	90	14.1929	15.6447	15.7202	15.7824	15.8283	15.8742
0.7	100	8.4333	10.1142	10.2080	10.2903	10.3624	10.4254
	110	4.6101	6.1865	6.2704	6.3464	6.4155	6.4783
	120	2.3406	3.6227	3.6851	3.7427	3.7960	3.8454
	80	21.8633	22.3281	22.3627	22.3914	22.4148	22.4333
	90	14.1929	15.1405	15.2010	15.2511	15.2921	15.3250
0.75	100	8.4333	9.6001	9.6760	9.7426	9.8009	9.8521
	110	4.6101	5.7330	5.7994	5.8596	5.9143	5.9640
	120	2.3406	3.2638	3.3117	3.3559	3.3967	3.4346
	80	21.8633	21.9711	21.9969	22.0184	22.0360	22.0499
	90	14.1929	14.6988	14.7454	14.7844	14.8159	14.8414
0.8	100	8.4333	9.1482	9.2072	9.2590	9.3044	9.3439
	110	4.6101	5.3374	5.3879	5.4336	5.4754	5.5132
	120	2.3406	2.9554	2.9908	3.0234	3.0524	3.0814
	80	21.8633	21.6620	21.6801	21.6952	21.7076	21.7174
	90	14.1929	14.3091	14.3428	14.3710	14.3940	14.4126
0.85	100	8.4333	8.7480	8.7912	8.8290	8.8622	8.8910
	110	4.6101	4.9894	5.0256	5.0584	5.0881	5.1152
	120	2.3406	2.6881	2.7127	2.7353	2.7554	2.7755
	80	21.8633	21.3926	21.4039	21.4134	21.4211	21.4273
	90	14.1929	13.9630	13.9847	14.0028	14.0177	14.0296
0.9	100	8.4333	8.3915	8.4195	8.4442	8.4657	8.4845
	110	4.6101	4.6813	4.7044	4.7252	4.7442	4.7614
	120	2.3406	2.4548	2.4700	2.4840	2.4969	2.5088
	80	21.8633	21.1579	21.1626	21.1662	21.1698	21.1727
	90	14.1929	13.6537	13.6642	13.6730	13.6802	13.6860
0.95	100	8.4333	8.0719	8.0856	8.0977	8.1082	8.1173
	110	4.6101	4.4068	4.4178	4.4278	4.4369	4.4451
	120	2.3406	2.2499	2.2569	2.2634	2.2694	2.2749
	80	21.8633	20.9483	20.9483	20.9483	20.9483	20.9483
	90	14.1929	13.3760	13.3760	13.3760	13.3760	13.3760
1	100	8.4333	7.7841	7.7841	7.7841	7.7841	7.7841
	110	4.6101	4.1608	4.1608	4.1608	4.1608	4.1608
	120	2.3406	2.0687	2.0687	2.0687	2.0687	2.0687

Table 4. Sensitivity of the put option prices (holder's position in the aggregated model) to the elevation of the weighting function, for different values of γ (curvature), with $\gamma^+ = \gamma^-$. Parameters of the value function: $a = b = 0.988$, and $\lambda = 1.125$. Option parameters: $S_0 = 100$, $X \in [80, 120]$, $r = 0.01$, $\sigma = 0.2$, $T = 1$. BS is the Black-Scholes price with $\gamma = 1$, $a = b = 1$, and $\lambda = 1$

γ	X	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
	80	1.0673	1.6322	1.6614	1.6885	1.7136	1.7370
	90	3.2974	4.1278	4.1807	4.2284	4.2715	4.3104
0.7	100	7.4383	8.2730	8.3359	8.3906	8.4374	8.4771
	110	13.5156	14.0124	14.0510	14.0824	14.1075	14.1270
	120	21.1466	21.0363	21.0436	21.0482	21.0528	21.0555
	80	1.0673	1.4758	1.4983	1.5191	1.5385	1.5564
	90	3.2974	3.8825	3.9252	3.9636	3.9983	4.0297
0.75	100	7.4383	7.9852	8.0369	8.0820	8.1208	8.1535
	110	13.5156	13.7557	13.7868	13.8121	13.8323	13.8479
	120	21.1466	20.8735	20.8783	20.8816	20.8841	20.8856
	80	1.0673	1.3397	1.3564	1.3718	1.3861	1.3993
	90	3.2974	3.6642	3.6972	3.7269	3.7538	3.7781
0.8	100	7.4383	7.7272	7.7681	7.8037	7.8345	7.8605
	110	13.5156	13.5272	13.5512	13.5707	13.5863	13.5984
	120	21.1466	20.7339	20.7368	20.7387	20.7399	20.7406
	80	1.0673	1.2205	1.2321	1.2428	1.2527	1.2619
	90	3.2974	3.4687	3.4926	3.5142	3.5337	3.5513
0.85	100	7.4383	7.4948	7.5249	7.5514	7.5743	7.5936
	110	13.5156	13.3226	13.3400	13.3542	13.3653	13.3743
	120	21.1466	20.6135	20.6150	20.6159	20.6164	20.6165
	80	1.0673	1.1155	1.1227	1.1293	1.1354	1.1411
	90	3.2974	3.2927	3.3081	3.3221	3.3347	3.3460
0.9	100	7.4383	7.2842	7.3041	7.3215	7.3367	7.3495
	110	13.5156	13.1386	13.1498	13.1589	13.1661	13.1718
	120	21.1466	20.5093	20.5099	20.5101	20.5102	20.5100
	80	1.0673	1.0225	1.0259	1.0289	1.0317	1.0344
	90	3.2974	3.1335	3.1410	3.1477	3.1538	3.1594
0.95	100	7.4383	7.0930	7.1027	7.1113	7.1188	7.1252
	110	13.5156	12.9723	12.9777	12.9821	12.9856	12.9883
	120	21.1466	20.4187	20.4188	20.4188	20.4187	20.4186
	80	1.0673	0.9397	0.9397	0.9397	0.9397	0.9397
	90	3.2974	2.9889	2.9889	2.9889	2.9889	2.9889
1	100	7.4383	6.9184	6.9184	6.9184	6.9184	6.9184
	110	13.5156	12.8214	12.8214	12.8214	12.8214	12.8214
	120	21.1466	20.3398	20.3398	20.3398	20.3398	20.3398

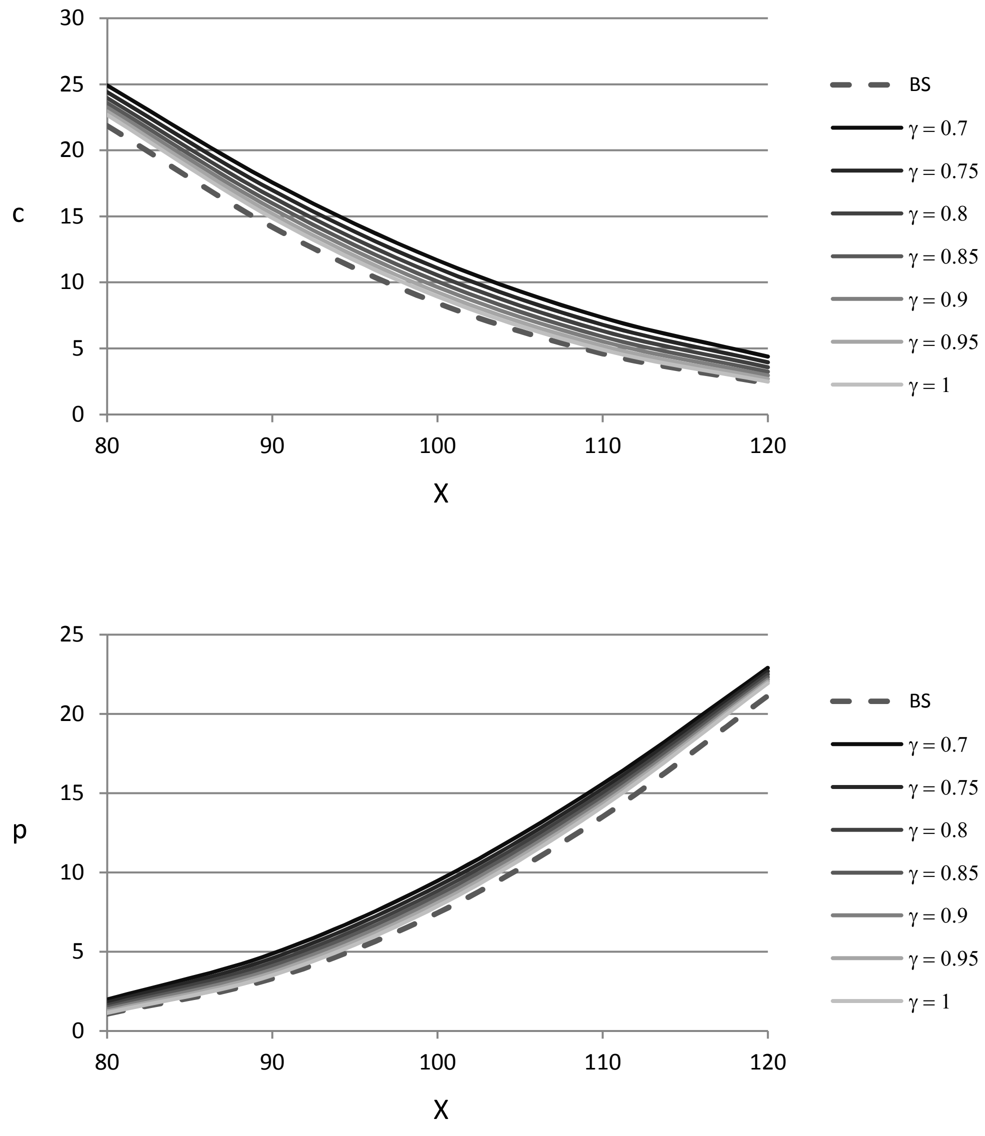
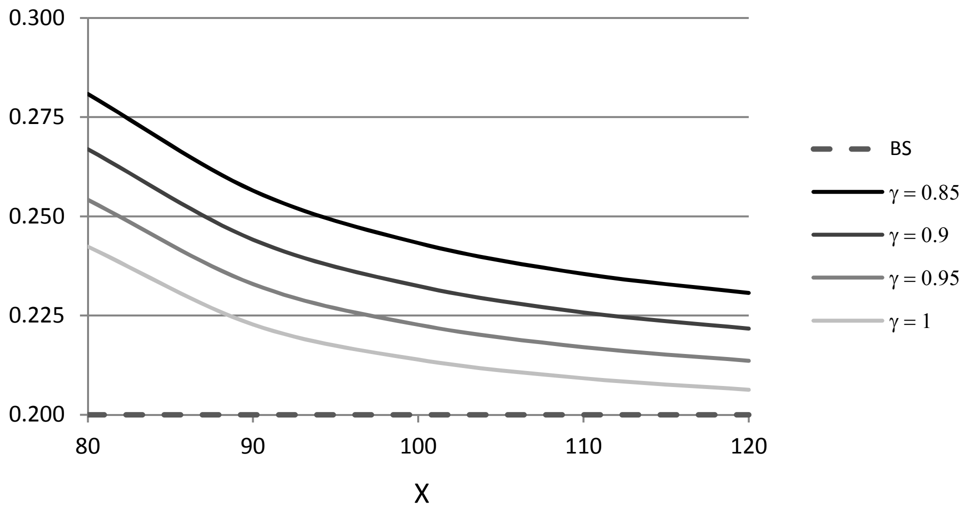


Fig. 1. Sensitivity of the call (above) and put (below) option prices (writer's position in the time-aggregated model) to the curvature of the probability weighting function, $\gamma \in [0.7, 1.0]$, with $\delta = 0.325$. BS is the Black-Scholes price (with $\gamma = 1$, $a = b = 1$, and $\lambda = 1$). The option parameters are: $S_0 = 100$, $X \in [80, 120]$, $r = 0.01$, $\sigma = 0.2$, $T = 1$; the parameters of the value function are: $a = b = 0.976$, and $\lambda = 1.125$



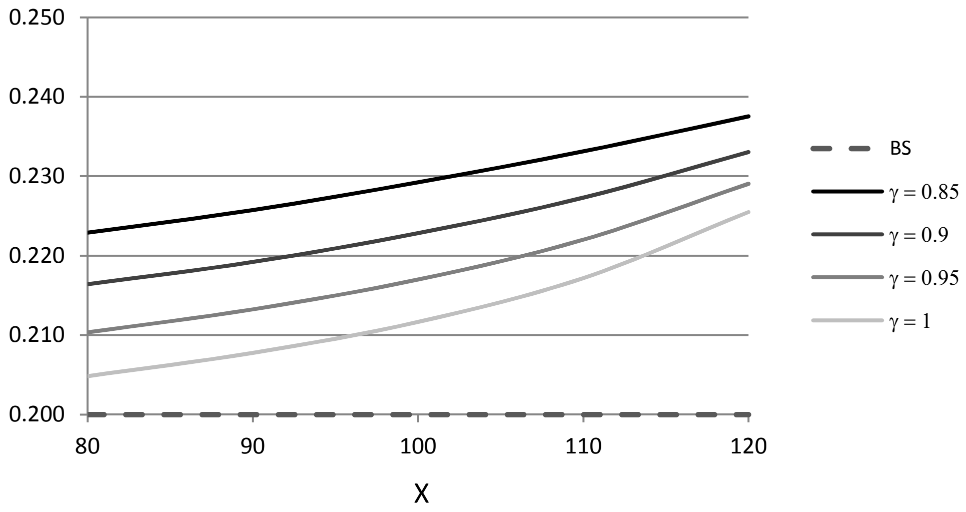


Fig. 2. Implied volatility of the call (above) and put (below) options (writer's position in the time-aggregated model) as the curvature of the probability weighting function varies, $\delta = 0.325$. Other parameters are: $S_0 = 100$, $X \in [80, 120]$, $r = 0.01$, $\sigma = 0.2$ for the BS prices, $T = 1$; the parameters of the value function are: $a = b = 0.976$, and $\lambda = 1.125$