# Is the output growth rate in NIPA a welfare measure?[*]

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**Abstract**

National Income and Product Accounts (NIPA) measure real output growth by means of a Fisher ideal chain index. Bridging modern macroeconomics and the economic theory of index numbers, this paper shows that output growth as measured by NIPA is welfare based. In a dynamic general equilibrium model with general recursive preferences and technology, welfare depends on present and future consumption. Indeed, the associated Bellman equation provides a representation of preferences in the domain of current consumption and current investment. Applying standard index number theory to this representation of preferences shows that the Fisher-Shell true quantity index is equal to the Divisia index, in turn well approximated by the Fisher ideal index used in NIPA.

KEYWORDS: Growth measurement, Quantity indexes, NIPA, Fisher-Shell index, Embodied technical change.

JEL CLASSIFICATION NUMBERS: C43, D91, O41, O47.

# 1   Introduction

The present paper bridges modern macroeconomics and the economic theory of index numbers[1] to show that the class of chain indexes used by National Income Product Accounts (NIPA) properly reflect changes in welfare when applied to a dynamic general equilibrium economy with recursive preferences and quasi-concave technology. In doing so, it evaluates the suitability of NIPA methodology for measuring output growth in a general model economy with explicit preferences and technology. In this framework, preferences are defined over consumption streams, present and future, but NIPA is constrained to use observable information by aggregating the main components of current final demand, encompassing consumption and investment. To overcome this problem, this paper notes that the Bellman equation provides with a representation of preferences over current consumption and investment. Index number theory is applied to this representation of preferences to show that a Fisher-Shell true quantity index is equal to the Divisia index, which is known to be well approximated by the Fisher ideal chain index used by NIPA in the US.[2] This means that the output growth rate in National Accounts is a welfare based measure in the very precise sense of compensating variation.

Until the 90's the Bureau of Economic Analysis (BEA) featured in its National Income and Product Accounts a Laspeyres fixed-base quantity index to measure real output growth. The traditional fixed-base quantity index yields a reasonable good measurement of real growth provided that relative prices remain stable. Indeed, since the mid-80's, following the seminal contribution of Gordon (1990), the BEA publishes a constant quality price index for equipment investment. After controlling for quality improvements, the price of durable goods, notably computers and peripheral equipment, permanently declines relative to the price of non-durable consumption goods and services. It was then realized that when the relative price of equipment permanently declines, the weight of investment with respect to consumption in the Laspeyres fixed-base index becomes

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[1]For economic index number theory, see Diewert (1993), Triplett (1992), Fisher and Shell (1998) and IMF (2004, chapter 17), among many others. A renewal of interest in using money metric utility for price measurement is in Redding and Weinstein (2018).

[2]See Fisher and Shell (1971) for a definition of a Fisher-Shell index and for a discussion about the conditions of its applicability. See Triplett (1992) for a discussion on the properties of the Fisher ideal index. National Accounts in Europe measure real growth by the mean of chained-type Laspeyres index following the Commission Decision 98/715/EC.

obsolete quickly enough to have a relevant impact on growth measurement due to the well-known substitution bias problem.[3] The observed substitution bias effect in the Laspeyres fixed-base quantity index lead then the BEA to consider alternative measures of output growth. As a reaction, since the early 1990's, NIPA moved to a chained-type index built on the Fisher ideal index. However, the theoretical legitimation of this change has not yet been explored. This paper provides a rational for it based on the idea that index numbers reflect the underlying preferences of households in a well-defined technological environment, meaning that measuring output growth by the mean of a Fisher ideal chined index is a welfare based measure.

The interest of the exercise also stems from understanding better the notion of real growth and its connection with welfare in models with more than one sector.[4] Growth theory has been reformulated in the late nineties in order to replicate the observed permanent decline in the relative price of durable to nondurable goods. Based on the hypothesis first formulated by Solow (1960) that technical progress is embodied in capital goods, Greenwood et al (1997) propose a simple two-sector optimal growth model with investment specific technical change where productivity grows faster in the investment than in the consumption sector.[5] In this family of models, as in the data, investment grows faster than consumption, which raises the fundamental problem of measuring output growth. The general methodology suggested in this paper is then applied to the two-sector AK model proposed by Rebelo (1991), which replicates the empirical regularities referred above –see Felbermayr and Licandro (2005). Index number theory identifies then the growth rate of output with the Divisia index, meaning that in this context the output growth rate as measured by NIPA is welfare based.

This theoretical framework sheds light on an old debate in the growth and growth accounting literature. The so-called Solow-Jorgenson controversy was revived by the differing interpretations found in Hulten (1992) and Greenwood et al (1997). The con-

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[3] Appendix A2 illustrates how the substitution bias implicit in a Lasyperes fixed-base index overestimates output growth when the price of investment goods declines relative to the price of non-durable consumption. Appendix A3 illustrates how a Fisher ideal chained index solves the problem.

[4] If all components of final demand grow at the same rate, aggregation is not an issue: the growth rate of the economy is the common growth rate of all final demand components.

[5] Many other papers have followed. See, for example, Krusell (1998), Gort et al (1999), Greenwood et al (2000), Cummins and Violante (2002), Whelan (2003), Boucekkine et al (2003,2005), Felbermayr and Licandro (2005) and Fisher (2006).

troversy can be shown to boil down to the issue of the aggregation of consumption and investment when these are measured in different units and, more importantly, when its relative price has a trend. In our conceptual framework, it becomes clear that Greenwood et al (1997) take a path that is more consistent with the theory. However, implicitly, these authors –and others following like Oulton (2007) –develop a modern version of the paradigm that consumption, and consequently its growth rate, is the relevant measure of real growth.[6] In this paper, we claim that investment growth, as reflected in the Divisia index, also matters for output growth. Notice that NIPA methodology stresses the fact that the growth rate of investment does contain information relevant to the welfare of the representative household since it reflects utility gains associated with postponed consumption. This is particularly relevant in a world where technical change is embodied in durable goods, and hence where technical progress only materialize through the incorporation of new physical capital.

The issue of trends in relative prices and different growth rates is also critical in the recent literature of structural transformation, since agriculture, manufacturing and services grow at different rates during the development process.[7] In line with our findings, Duernecker et al (2017) claim that using chain indexes more accurately reflects the effects of secular changes in relative prices.

Interestingly, the main result on this paper that output growth in NIPA is a welfare measure does not require a representative household. The proof that a Fisher-Shell index is equal to the Divisia index holds true even when agents have different preferences and income, and consequently equilibrium may differ from the equilibrium of an equivalent representative agent economy. When a Fisher-Shell index is applied to a dynamic general equilibrium economy with heterogenous households, money is used as a common norm to evaluate welfare changes across individuals; money metric utility implicitly adopts an utilitarian approach weighting each households proportional to its own

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[6]Greenwood et al (1997), in fact, is not a normative paper. It does perform the positive exercise of measuring the contribution of embodied technical change to US growth. However, in doing so, they measure output and its growth rate in units of consumption, de facto identifying real output growth with consumption growth. Cummins and Violante (2002) generalize the exercise and use standard NIPA methodology to the same objective, finding similar quantitative results. See also Greenwood and Jovanovic (2001). Sections 3 and 4 further discus these issues.

[7]For structural transformation, see Acemoglu and Guerrieri (2008), Duarte and Restuccia (2010), Herrendorf et al (2013) and Ngai and Pissarides (2007), among many others.

income. The approach leads to apply the Fisher-Shell index to individual households separately, and then aggregate their individual growth rates weighted by their shares on total income. Starting from the usual assumption that income levels may be used to make inter-households comparisons, inter-households differences in income growth aggregate on the average growth rate of the economy. Such an approach reduces to the analysis of the income side of NIPA and, if data were available, it may be used to study the social welfare implications of observed phenomena like job polarization, where different occupations face different growth rates in earnings depending on their position in the income distribution.[8]

But, what do we mean by output growth in National Accounts is a welfare measure? Or, in other terms, what does National Accounts measure? In order to answer these questions, let us better explain the implications of applying money metric utility in this context. Firstly, the welfare of a country is nothing else than the discounted flow of residents' utility, which critically depends on total assets in the country, and output is just the return to these assets. The growth rate of output in NIPA measures then the growth rate of the return to total assets in a country. Second, a utility function is a particular representation of households preferences: monotonic transformations of it change the level of utility leaving the preference map intact; then, the growth rate of a particular representations is meaningles. To overcome this problem, index number theory adopts current income as a sensible norm to measure changes in welfare; this is the sense of money metric utility. Finally, since income as measured by National Accounts represents the return to assets, the Fisher-Shell quantity index and then the Divisia index are income compensating measures quantifying changes in the return to assets. Interestingly, when the subjective discount rate is time independent, the growth rate in NIPA also measures changes in welfare.

In Section 3.3, we formally analyze this issue for the two-sector AK model and show that the growth rate of output as measured by the Fisher-Shell index is equal to the growth rate of welfare, explicitly writing the particular representation of preferences that grows at this rate. At any moment in time, an economy posses a set of assets including among other things a geographical environment, a myriad of different types of physical, human and intangible capital, as well as different forms of political, social and cultural institutions, all of them resulting from a long history of human investments

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[8]For job polarization, see Autor and Dorn (2013) and Goos et al (2014), among others.

and achievements of many different nature. These assets allow individuals living in the economy to produce different types of goods and services, including political, social and cultural activities, that will contribute to the current wellbeing of people, as well as to increase, replace and maintain the set of assets.[9] National accounts classify all these different type of goods and services in the broad categories of consumption and investment, respectively. Consequently, not all current production generates current wellbeing, since net investment in new assets will produce wellbeing in the future. For this reason, as pointed out by Weitzman (1976), when measuring human wellbeing, we should consider both consumption and net investment.

This paper is organized as follows. Section 2 describes the economy with general recursive preferences and general technology. It applies index number theory to it and proves that the Fisher-Shell true quantity index is equal to the Divisia index. Section 3 illustrates it in the interesting case of the two-sector AK model economy replicating the permanent decline in the relative price of investment. This section provides a rational for the BEA movement from fixed-base to chained-type indexes. Finally, Section 4 discusses the main implications of our results and Section 5 concludes and suggests some possible extensions.

## 2  Measuring output growth

Consider a two-sector non-stochastic perfectly competitive dynamic general equilibrium economy with two goods, consumption and investment, and a general technology transforming capital and labor into these two goods. Firms hire capital and labor to produce them, and under the usual intertemporal budget constraint, a representative household chooses continuously consumption and investment in order to maximize intertemporal utility. All along this paper, we assume that preferences and technology are such that an equilibrium path exists and is unique.

In this paper, we try to understand the problem faced by a National Statistical Office (NSO) operating in this economy, that only observes current nominal consumption and investment, and the corresponding prices, but has no information about individual

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[9]Even if this paper restricts the analysis to those type of good and services registered in National Accounts, we want to point out that many other dimensions of human wellbeing could be added if we were able to collect the relevant information.

preferences, technology and future consumption. Let us finally assume that the NSO uses this information to measure the growth rate of both real consumption and real investment and, then, computes a chained Fisher ideal index of real output growth. As it is well know, in continuous time, this is equivalent to compute a Divisia index.

The general problem in National Accounts is to find an index built out of observables at $t$, current consumption and investment, and the corresponding prices, that measures changes in real output. For our fictitious economy, we aggregate equilibrium consumption and investment by the mean of a Fisher-Shell true quantity index –controlling for changes in equilibrium prices. Section 2.2 shows that in this context the Fisher-Shell index is equal to the Divisia index, which in continuous time is equal to the Fisher ideal chain index –the one used in NIPA to measure GDP growth.[10] The resulting rate of output growth is then welfare based. Section 2.3 generalizes the result to heterogeneous households and Section 3.3 discuss the meaning of the statement that the growth rate of output is welfare based.

## 2.1 Bellman equation under recursive preferences

The economy evolves in continuous time. For any date $t \geq 0$ and any consumption path $C : [0, \infty) \to \mathbb{R}_+$ let $_tC$ denote the restriction of $C$ to the interval $[t, \infty)$, preferences of the representative household are represented by some recursive utility function $U$ generated by the differential equation

$$\frac{d}{dt} U(_tC) = -f(c_t, U(_tC)). \tag{1}$$

The generating function $f$ is assumed to be differentiable with $f_1 > 0$ and $f_2 < 0$. Note that $f_1$ is marginal utility from current consumption, lost when we move an infinitesimal period of time ahead, and so the negative sign in (1). In turn, $f_2 < 0$ is related to the implicit subjective discount rate.[11] For instance, the classical additively separable utility function is an important particular case of the general specification above in which

$$U(_tC) = \int_t^\infty e^{-\rho(s-t)} u(c_s) ds$$

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[10] The property that in continuous time the Fisher ideal chain index is equal to the Divisia index is shown in Appendix A3.

[11] Epstein (1987) explores conditions under which a generating function $f$ represents a recursive utility function $U$. Becker and Boyd (1997, chapter 1) motivates the study of general recursive preferences.

with $u'(c) > 0$, $u''(c) < 0$ and $\rho > 0$. Differentiate with respect to time $t$ to write

$$\frac{d}{dt}U(_tC) = -u(c_t) + \rho U(_tC).$$

Hence, in this case, $f(c,u) = u(c) - \rho u$ and indeed $f_1(c,u) = u'(c) > 0$ while $f_2(c,u) = -\rho < 0$. Indeed, this clarify the interpretation given above that $f_1$ is the marginal utility from current consumption, lost when we move an infinitesimal period of time ahead, and $f_2$ is the return to household assets, which value is represented by $U(_tC)$ and the discount rate is $\rho$.

Each instant $t$, a social planer chooses individual consumption $c_t$ and per capita net investment $\dot{k}_t$ such that $(c_t, \dot{k}_t) \in \Gamma(k_t, \Theta_t)$ is feasible, where $k_t$ is capital and $\Theta_t$ represents a vector of exogenous non-stochastic states. In the following, we assume that, for a given $k_t > 0$, there exists a unique consumption and investment path equilibrium $(c_s, \dot{k}_s)_{s \geq t}$ that maximizes $U(_tC)$ subject to the technological constraint. Then, total utility is $U(_tC)$ and the current change in welfare as measured by $U(_tC)$ is simply given by (1).

In addition to the well known problem that preferences are not univocally represented by a utility function, we face here the additional problem, from an accounting point of view, that neither preferences nor foreseen consumption are observable by National Statistical Offices. In this context, we wish to build a quantity index that reflects changes in welfare using only current consumption $c_t$ and current net investment $x_t = \dot{k}_t$, both observables at instant $t$; and all that matters of the level of $k_t$ is summarized in the price of investment $p_t$ as we will argue below. To this end, however, we shall need to express preferences as a function of variables observed at $t$. Since preferences are recursive, this amounts to express changes in welfare as a function of current consumption $c_t$ and net investment $x_t$.

In other words, we need a representation of preferences over current consumption and current investment, and this is what the Bellman equation gives us. The original problem is to maximize $U(_tC)$ subject to $(c_s, \dot{k}_s) \in \Gamma(k_s, \Theta_s)$ for all $s \geq t$, $k_t > 0$ given, where $\Theta_s$ is a vector of exogenous states that directly affect technology. The associated Bellman equation is

$$0 = \max_{(c,x)\in\Gamma(k_t,\Theta_t)} f(c, v(k_t, \Theta_t)) + v_1(k_t, \Theta_t)x + v_2(k_t, \Theta_t)\dot{\Theta}_t. \tag{2}$$

The intuition behind this equation becomes clear if one notes that along an optimal path $v(k_t, \Theta_t) = U(_tC)$ so $\mathrm{d}v(k_t, \Theta_t)/\mathrm{d}t = v_1(k_t, \Theta_t)\dot{k}_t + v_2(k_t, \Theta_t)\dot{\Theta}_t = -f(c_t, U(_tC)) =$

$-f\big(c_t, v(k_t, \Theta_t)\big)$. Note as well that, in a sense, with all past actions summarized in $k_t$, the objective function in (2) is giving us the preference relation over consumption and investment at instant $t$.[12]

## 2.2  Fisher-Shell true quantity index

In this section, we show that in the dynamic general equilibrium framework developed in the previous section, the Divisia index is a true quantity index. In regard of the Bellman equation (2), preferences of the representative consumer over consumption and investment at instant $t$ can be seen as represented by the function

$$w_t(c, x) \doteq f(c, v(k_t, \Theta_t)) + v_1(k_t, \Theta_t)x + v_2(k_t, \Theta_t)\dot{\Theta}_t.$$

To save notation, we write $w_t(c, x)$, but time enters this function only through the stock of capital $k_t$ and the exogenous states $\Theta_t$, both given at time $t$.

For a given state of the system, as represented by $k_t$ and $\Theta_t$, the function $w_t(c, x)$ can then be seen as a representation of individual preferences over consumption and invest-ment, the last summarizing postponed consumption. To the extent that the exogenous states and the stock of capital will change along an equilibrium path, these preferences are time-dependent. This is precisely the building block of the true quantity index in-troduced by Fisher and Shell (1971). Since welfare comparisons must be done within the same preference map, the Fisher-Shell true quantity index proposes to fix not only prices but also preferences. In particular, it compares income today with the hypotheti-cal level of income that would be necessary to attain the level of utility associated with tomorrow's income and prices with today's prices and today's preferences –as evaluated by $w_t(c, x)$. The remain of this section elaborates this idea.

In the following, we adopt the consumption good as numeraire. Of course, the choice of the numeraire is inconsequential. Let us define equilibrium nominal net income at time $t$, along an equilibrium path for $(c_t, x_t, p_t)$, as $m_t \doteq c_t + p_t x_t$. Under standard assumptions, optimal choices will lie in the boundary of $\Gamma(k_t, \Theta_t)$ so that there is a well-defined equilibrium price of net investment $p_t > 0$ expressed in units of consumption (see Figure 1). The constraint $(c, x) \in \Gamma(k_t, \Theta_t)$ can be replaced by the linear constraint

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[12]The planner solves a standard recursive program in which the state variable summarizes at each instant $t$ all past information that could be relevant for today's decisions. For a brief exposition of recursive techniques in continuous time see Obstfeld (1992).
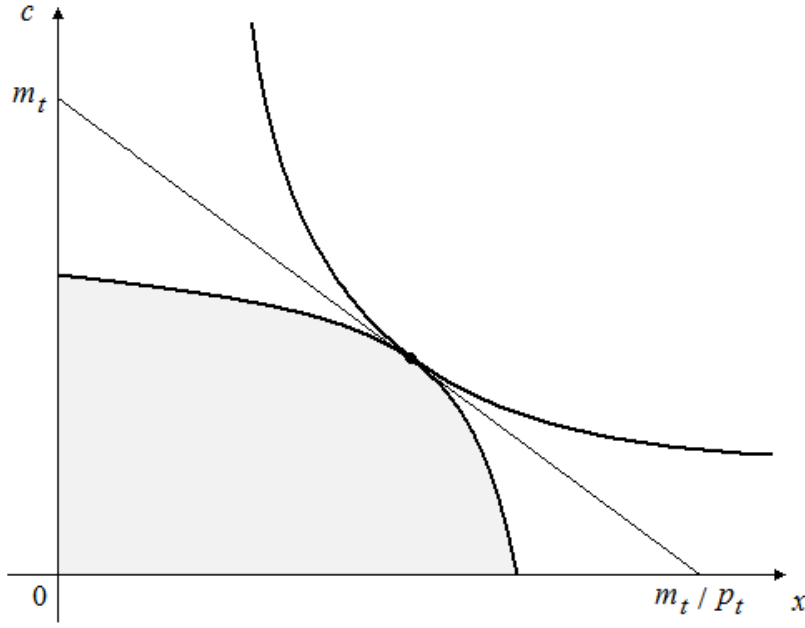
Figure 1: The production possibilities frontier and competitive prices

$c + p_t x \leq m_t$ in the problem of the Bellman equation (see Figure 1). Hence, the associated indirect utility function of the representative household problem is defined as

$$u_t(m_t, p_t) \doteq \max_{c + p_t x \leq m_t} w_t(c, x)$$

while the expenditure function is

$$e_t(u_t, p_t) \doteq \min_{w_t(c,x) \geq u_t} c + p_t x.$$

The fundamental idea behind *money metric utility* is to use the expenditure function to make welfare comparisons, by associating utility $u_t$ to observed income $m_t = c_t + p_t x_t$. Notice that at the equilibrium of our fictitious economy, the NSO observes $\{c_t, p_t, x_t\}$ at both the current period $t$ and the future period $t + h$, $h > 0$.[13] Since comparisons must be done within the same preference map, the Fisher-Shell true quantity index fixes both prices and preferences. In particular, it compares income today $m_t$ with the hypothetical level of income $\hat{m}_{t+h}$ that would be necessary to attain the level of utility $u_t(m_{t+h}, p_{t+h})$ associated with tomorrow's income and prices $m_{t+h}, p_{t+h}$ with today's prices $p_t$ and today's preferences as represented by functions $e_t(u, p)$ and $u_t(m, p)$. This artificial level

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[13]Of course, the statistical office produces these measures after period $t + h$.
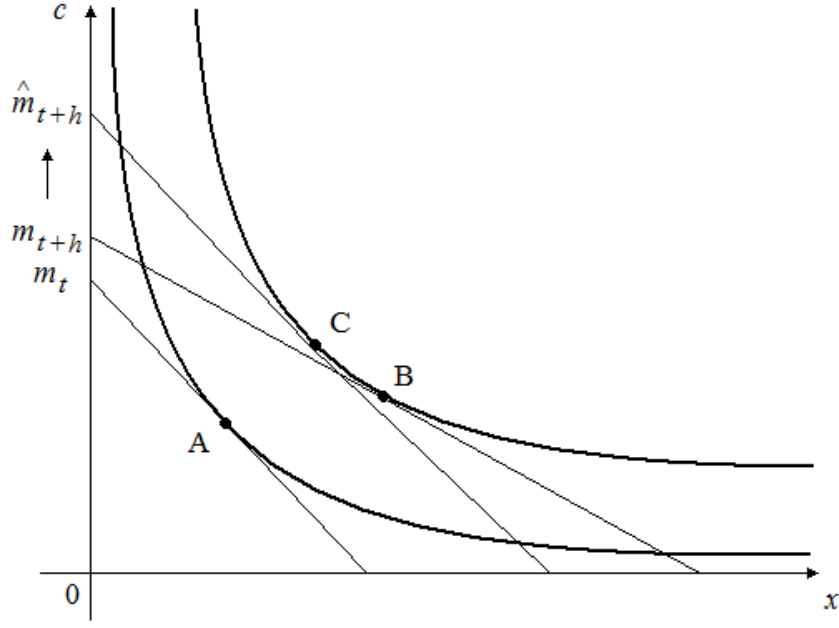
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Figure 2: The Fisher-Shell true quantity index

of income is given by

$$\hat{m}_{t+h} = e_t\big(u_t(m_{t+h}, p_{t+h}), p_t\big).$$

The idea is illustrated in Figure 2. The preference map corresponds to instant $t$ preferences as represented by $w_t$. Point A is the current situation at instant $t$. Point B is the hypothetical choice using instant $t$ preferences when facing prices $p_{t+h}$ and income $m_{t+h}$. Point C represents the choice that maintains such level of utility but with instant $t$ prices $p_t$. In the end, we compare two levels of income that correspond to the same price vector so it is clear that we are extracting price changes. In this particular case, the true quantity index is just reflecting the fact that the true output deflator is dropping with the price of investment, that is to say that income in real terms is growing more than nominal income $m_{t+h}/m_t$. The difference between $\hat{m}_{t+h}$ and $m_{t+h}$ is a compensating variation measure stating by how much income would have to increase to compensate for not having the price of investment dropping.[14]

In continuous time, the reasoning is the same and the time gap $h$ tends to zero. The

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[14]If alternatively, the investment good were the numeraire, nominal income will be growing faster than the hypothetical income $\hat{m}$ and consumption price changes should be subtracted from nominal income growth to get real income growth. Indeed, the real growth rate will remain unchanged, since it does not depend on the choice of the numeraire.

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instantaneous Fisher-Shell index is defined as

$$g_t^{\text{FS}} \doteq \frac{d}{dh} \left. \frac{\hat{m}_{t+h}}{m_t} \right|_{h=0} = \frac{1}{m_t} \left. \frac{d\hat{m}_{t+h}}{dh} \right|_{h=0},$$

that is, the instantaneous growth rate of the factor defined above as $h$ gets small.[15] To compute this index note that

$$\left. \frac{d\hat{m}_{t+h}}{dh} \right|_{h=0} = e_{1,t}\big(u_t(m_t, p_t), p_t\big)\Big(u_{1,t}(m_t, p_t)\dot{m}_t + u_{2,t}(m_t, p_t)\dot{p}_t\Big)$$

where subscripts denote the partial derivatives with respect to the corresponding arguments. To obtain an expression for all these derivatives let us go back to the dual and primal problems discussed above. Let $\mu$ be the Lagrange multiplier of the maximization problem in the definition of the indirect utility function, measuring the marginal contribution of income $m$ to welfare $w$. We have, from the the primal problem

$$u_{1,t}(m_t, p_t) = \mu$$
$$u_{2,t}(m_t, p_t) = -\mu x_t,$$

and, since the expenditure function is the inverse of the indirect utility function,

$$e_{1,t}(u_t, p_t) = \frac{1}{\mu}.$$

As expected, the marginal contribution of income to welfare, $\partial u / \partial m = \mu$, is equal to the inverse of the marginal contribution of utility $u$ to total expenditure, $\partial e / \partial u = 1/\mu$. Moreover, the negative marginal contribution of prices to welfare is $\partial u / \partial p = -x\mu$, since an increase in prices reduces income by $x$ units. These properties are critical for the result below and they are directly related to the *money metric utility* nature of the Fisher-Shell index, which defines the hypothetical income $\hat{m}$ using the expenditure function to valuate changes in utility after controlling for changes in prices.

Using the three conditions above in the definition of the Fisher-Shell index, we conclude that

$$g_t^{\text{FS}} = \frac{\dot{m}_t - x_t \dot{p}_t}{m_t} = \frac{\dot{m}_t}{m_t} - \frac{p_t x_t}{m_t} \frac{\dot{p}_t}{p_t}.$$

Notice that the marginal terms $e_1$, $u_1$ and $u_2$ in the definition of the Fisher-Shell index simplify as a direct consequence of the properties discussed in the paragraph above; all

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[15]Along an equilibrium path, in continuous time, it does not make a difference whether we define the true quantity index like we do or in terms of $m_t/\hat{m}_{t-h}$. See Appendix 5 for a rationale of this definition.

three are related to the marginal value of income $\mu$. It is in this sense that money metric utility operates in the Fisher-Shell index. Since gains in welfare are measured as a compensating variation by comparing the artificial level of income $\hat{m}_{t+h}$ with the nominal income $m_t$, and prices enter linearly in the budget constraint, gains in welfare are equal to the change in nominal income (arbitrarily measured here in units of the consumption good) minus the contribution of prices to it (which comes only from the change of investment prices, weighted by the equilibrium (net) investment share).

Finally, differentiate the definition of nominal income $m_t = c_t + p_t x_t$ with respect to time and define the equilibrium share of net investment to net income as $s_t \doteq p_t x_t / m_t$ to write

$$\frac{\dot{m}_t}{m_t} = (1 - s_t)\frac{\dot{c}_t}{c_t} + s_t\frac{\dot{x}_t}{x_t} + s_t\frac{\dot{p}_t}{p_t},$$

which implies that

$$g_t^{\text{FS}} = (1 - s_t)\frac{\dot{c}_t}{c_t} + s_t\frac{\dot{x}_t}{x_t} \doteq g_t^{\text{D}}$$

where $g_t^{\text{D}}$ denotes the Divisia index. We have then shown that, for all $t$, the Fisher-Shell index $g_t^{\text{FS}}$ is equal to the Divisia index $g_t^{\text{D}}$. In this framework, by definition, the Divisia index is the average of the growth rates of consumption and net investment, weighted by their corresponding equilibrium shares in total net income.

We have then shown that in this framework the Divisia index is a true quantity index, and as such it is a welfare measure. The interpretation is straightforward. It is clear that $g_t^{\text{FS}}$ is a measure of real growth since it is constructed as the growth rate of nominal income subtracting pure price changes, in this case the change of the relative price of investment $p_t$. The index only keeps changes in quantities. It is also clear that it is a true index because it is constructed from the representative household's preferences using standard theory.[16]

## 2.3  On household heterogeneity

The argument above was built under the assumption of a representative household. In this section, we show that the same reasoning applies to an economy where households

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[16]This equivalence would come as no surprise to index number theorists. The Fisher ideal index is known to approximate in general some sort of true quantity index because both are bounded from above and below by the Laspeyres and Paasche indexes respectively. In continuous time, these indexes tend to each other as the time interval $h$ tends to zero. Further, in general, the Divisia index coincides with the Fisher ideal index if the growth rates of consumption and investment are constant.

have both heterogeneous preferences and heterogeneous income. Critical in the result is the fact that the utility representation of preferences derived from the Bellman equation is quasilinear, belonging to the Gorman family.[17]

Let us assume that there is a continuum of heterogeneous households of unit mass with recursive preferences represented by the utility $U_i$ generated by the differential equation

$$\frac{1}{dt}U_i(_tC_i) = -f_i(c_{i,t}, U_i(_tC_i)),$$

where $_tC_i$ represents the consumption path of household $i$. Let function $f_i$ have the same properties as above. Let us also assume that, for this economy, an equilibrium exists and is unique. Notice that equilibrium will likely be different from the equilibrium with a representative household. In other words, the distribution of preferences and capital across individuals matters.

A distribution of capital maps any individual $i$ at any instant $t$ into a quantity of capital. We will denote by $\varphi_t$ such a distribution. In the recursive competitive equilibrium representation of this economy, with exogenous state $\Theta_t$ and a distribution of capital $\varphi_t$, the problem of a household $i$ with capital $k_{i,t}$ can be written as

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\begin{aligned}
0 = \quad &\max \quad f_i(c_i, v_i(k_{i,t}, \Theta_t, \varphi_t)) + v_{i,1}(k_{i,t}, \Theta_t, \varphi_t)x_i + \pi_{i,t} \\
&\text{s.t.} \quad c_i + p_t x_i = m_{i,t}
\end{aligned}
$$

where $p_t$ is the equilibrium price, common to all households, and $m_{i,t}$ is the equilibrium net income of individual $i$. $\pi_{i,t}$ represents the differential terms of $v_i(k_{i,t}, \Theta_t, \varphi_t)$ with respect to time that are exogenous to the problem of the consumer, i.e., those corresponding to $\Theta_t$ and $\varphi_t$.

As in section 2.2, the optimization problem of household $i$ is associated to the instantaneous utility function over consumption and net investment

$$w_{i,t}(c_i, x_i) \doteq f_{i,t}(c_i) + x_i,$$

where $f_{i,t}(c_i) \doteq f_i(c_i, v_i(k_{i,t}, \Theta_t, \varphi_t))/v_{i,1}(k_{i,t}, \Theta_t, \varphi_t)$. Notice that we have subtracted $\pi_{i,t}$ from the right hand side of the Bellman equation and then divided it by $v_{i,1}(k_{i,t}, \Theta_t, \varphi_t)$. Since non of these two terms depend on $c$ or $x$, such a transformation has no effect on the households program. Function $w_{i,t}(c_i, x_i)$ is maximized under the budget constraint $c_i +$

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[17]See Gorman (1953, 1961).

$p_t x_i = m_{i,t}$, where, as said above, $p_t$ is the equilibrium price and $m_{i,t}$, for all $i$, represents equilibrium household net income. Since this utility representation is quasilinear, it belongs to the Gorman family. It is easy to show that the indirect utility and expenditure functions become

$$u_{i,t}(m_{i,t}, p_t) = A_{i,t}(p_t) + m_{i,t}/p_t$$

$$e_{i,t}(u_{i,t}, p_t) = p_t\big(u_{i,t} - A_{i,t}(p_t)\big),$$

where $A_{i,t}(p_t)$ is defined below. In fact, from the household problem, optimal consumption $c_i$ solves

$$f'_{i,t}(c_i) = 1/p_t.$$

Let us denote the implicit solution for $c_i$ as $c_{i,t}(p_t)$. It is then easy to show that

$$A_{i,t}(p_t) = f_{i,t}\big(c_{i,t}(p_t)\big) - c_{i,t}(p_t)/p_t.$$

Let us define the artificial level of household $i$ income as in section 2.2, i.e.,

$$\hat{m}_{i,t+h} = e_{i,t}\big(u_{i,t}(m_{i,t+h}, p_{t+h}), p_t\big) = p_t\Big(A_{i,t}(p_{t+h}) - A_{i,t}(p_t)\Big) + p_t/p_{t+h}\, m_{i,t+h},$$

which is linear on income due to the fact that preferences are quasilinear. Consistently with National Accounts, let us define aggregate income as $m_t = \int_i m_{i,t}\mathrm{d}i$, which also measures per capita income since population has been normalized to unity. Let us now define aggregate hypothetical income consistently with the definition of per capita income as $\tilde{m}_t = \int_i \hat{m}_{i,t}\mathrm{d}i$. Using the results just above,

$$\tilde{m}_{t+h} = p_t\Big(\bar{A}_t(p_{t+h}) - \bar{A}_t(p_t)\Big) + p_t/p_{t+h}\, m_{t+h},$$

where

$$\bar{A}_t(p_t) = \int_i A_{i,t}(p_t)\mathrm{d}i.$$

Note that, in general, average hypothetical income $\tilde{m}_{t+h}$ at the equilibrium of the heterogeneous household economy, will be different from the hypothetical income $\hat{m}_{t+h}$ of the representative household economy at equilibrium, since these two economies will likely have a different equilibrium paths.

As in section 2.2, let us define the Fisher-Shell index for the economy with heterogeneous households as

$$\tilde{g}_t^{\mathrm{FS}} \doteq \frac{1}{m_t} \left.\frac{d\tilde{m}_{t+h}}{dh}\right|_{h=0}.$$

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Operating on the definition of $\tilde{m}_{it+h}$ above

$$\frac{d\tilde{m}_{t+h}}{dh}\bigg|_{h=0} = \dot{m}_t + \left(p_t \bar{A}'_t(p_t) - m_t/p_t\right)\dot{p}_t.$$

Notice that

$$\bar{A}'_t(p_t) = \int_i A_{i,t}(p_t)di = \int_i \big(\underbrace{f'_{i,t}c'_{i,t} - 1/p_t c'_{i,t}}_{=0, \text{ since } f'_{it}=1/p_t} + c_{i,t}/p_t^2\big)di = c_t/p_t^2,$$

where $c_t = \int_i c_{it}di$ is per capita consumption. Then

$$\tilde{g}_t^{\text{FS}} = \frac{\dot{m}_t}{m_t} - s_t\frac{\dot{p}_t}{p_t} = (1 - s_t)\frac{\dot{c}_t}{c_t} + s_t\frac{\dot{x}_t}{x_t},$$

where $s_t \doteq p_t x_t/m_t$ as before. The Fisher-Shell index is, indeed, equal to the Divisia index, meaning that the growth rate in NIPA is a welfare measure irrespective of households being either homogeneous or heterogeneous. Of course, at equilibrium, consumption and investment may be growing at different rates than in the representative household model, and the saving rate may also be different. Consequently, even when the growth rate, as measured by the Divisia index is a welfare measure in both economies, these two economies may be growing at different rates.

Two assumptions are critical for the main result in this section, i.e., that the Fisher-Shell index is a Divisia index under heterogeneous households. First, as in the case of homogeneous households, under money metric utility, nominal income is the metric used to measure households' utility, implying that gains in welfare are measured as gains in nominal income minus inflation; the main principle used by National Accounts. The second critical assumption is the use of the quasilinear representation of preferences that emerges from the Bellman equation representation of household preferences in the space of current consumption and current investment. This assumption is not critical at all in the case of a representative household; in facts, in Section 2.2, we show that the Fisher-Shell index is equal to the Divisia index for a general function $w(c, x)$. Indeed, it is critical in this section, since we profit from the quasi linearity representation of preferences to show that aggregate utility gains, as measured by the Fisher-Shell index, are equal to gains in nominal per capita income minus inflation.

This result comes at non surprise. By adopting aggregate nominal income as a norm for measuring aggregate output, the Fisher-Shell index implicitly assumes that the aggregate welfare function is utilitarian, giving to each household a weight proportional to its income. This clearly reflects in the definition of the artificial income measure $\tilde{m}_t$.

# 3 Embodied technical progress

As referred in the Introduction, following Gordon (1990)'s observation that quality adjusted equipment investment prices were permanently declining relative to the price of non-durable consumption goods and services, the Bureau of Economic Analysis (BEA) moved first to control for quality improvements in the measurement of investment prices, and second to a Fisher ideal chain index to measure output growth. The first change made investment to grow faster than non-durable consumption. As an undesirable consequence of trends in relative prices, the fixed-base quantity index used to measure GDP growth became obsolete fast enough to provided appropriate growth figures. In facts, in this case, fixed-base quantity indexes suffer from the well known substitution bias problem that tends to overestimate the weight of the fast growing items. The second change addresses this last problem by making the NIPA measure of output growth to be approximately equal to the Divisia index.

Almost contemporaneously, a new literature developed in macroeconomics aimed to accommodate growth theory to this new evidence. Greenwood et al (1997), in their seminal paper, extend the Ramsey model to a two sector (consumption and investment) growth model with two sources of technical progress, consumption and investment specific technical change (disembodied and embodied in capital goods, respectively). This model is able to replicate the permanent decline in the relative price of equipment investment, as well as the fact that investment grows faster than consumption (implying that the investment to output ratio is permanently growing). In this context, it is particularly clear that the aggregation issue is far from trivial since consumption and investment grow at different rates.

In this section, we describe a simple version of the two-sector AK model proposed by Rebelo (1991) and apply to it the Fisher-Shell index proposed in Section 2.2 to show that the BEA had good fundamental reasons to move to a chained-type quantity index of output growth. As shown in Felbermayr and Licandro (2005), the two-sector AK model is the simplest endogenous growth model that replicates the observed permanent decline in the relative price of equipment and the permanent increase in the investment to output ratio.[18] We have preferred to use it instead of the original Greenwood et al (1997) model, since the AK model has the advantage of jumping to the balanced growth

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[18]See also Acemoglu (2009).

path from the initial time, which allows for an explicit solution of the value function. This is very useful to understand the role of money metric utility in the main statement of this paper that the growth rate in NIPA is a welfare measure.

## 3.1 The two-sector AK model

The model in this section is based on Rebelo (1991), follows Felbermayr and Licandro (2005) closely, and entails all the characteristics that are relevant to the present discussion in the simplest possible framework. The stock of machines at each instant $t$ is $k_t$, from which a quantity $h_t \leq k_t$ is devoted to the production of the consumption good. Consumption goods technology is

$$c_t = h_t^\alpha,$$

where $\alpha \in (0,1)$. The remaining stock $k_t - h_t \geq 0$ is employed in the production of new capital with a linear technology

$$\dot{k}_t = A(k_t - h_t)$$

where $A > 0$ is the marginal product of capital in the investment sector net of depreciation. There is a given initial stock of capital $k_0 > 0$. Again, we will write $x_t = \dot{k}_t$ for net investment.

The representative household has preferences over consumption paths represented by[19]

$$\int_0^\infty \frac{c_t^{1-\sigma}}{1-\sigma} \, e^{-\rho t} \, dt, \tag{3}$$

that is, the additive case mentioned above, where $\rho > 0$ is the subjective discount rate and $\sigma \geq 0$ the inverse of the intertemporal elasticity of substitution.

In the absence of market failures, equilibrium allocations are solutions to the problem of a planner aiming at maximizing household's utility subject to the technological constraints. The Bellman equation associated to the planner's problem is

$$\rho v(k_t) = \max_{x = A(k_t - c^{1/\alpha})} \frac{c^{1-\sigma}}{1-\sigma} + v'(k_t)x. \tag{4}$$

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[19]This is a particular case of the general preferences in Section 2.1. Here the correspondence $\Gamma$ is defined for every $k \geq 0$ as the set $\Gamma(k)$ of pairs $(c, \dot{k})$ such that there exists $h$ with $0 \leq h \leq k$, $c \leq h^\alpha$, and $\dot{k} \leq A(k - h)$.

As shown in Felbermayr and Licandro (2005), the equilibrium growth rate of capital is

$$\gamma = \frac{A - \rho}{1 - \alpha(1 - \sigma)}.$$

From the feasibility constraints, it is clear that the growth rate of investment is also $\gamma$, and that $\alpha\gamma$ is the growth rate of consumption. Competitive equilibrium allocations are balanced growth paths from $t \geq 0$.

Returns to scale differ between sectors. Since $\alpha < 1$, as the stock of capital grows the investment sector becomes more productive with respect to the consumption goods sector. This difference in productivity causes the decline in investment prices relative to consumption goods prices. This difference in returns to scale can be interpreted in terms of the investment sector being more capital intensive than the consumption sector or, as put forth by Boucekkine et al (2003), as a consequence of strong spillovers in the production of investment goods.[20] From the feasibility constraints, we can obtain the competitive equilibrium price of investment in terms of consumption units as the marginal rate of transformation:

$$p_t = -\frac{dc_t}{dx_t} = -\frac{dc_t}{dh_t}\frac{dh_t}{dx_t} = \frac{\alpha}{A}h_t^{\alpha-1}.$$

Since the stock of machines used in the consumption goods sector grows at the constant rate $\gamma$, the price of investment relative to consumption decreases at rate $(\alpha - 1)\gamma < 0$.

The competitive equilibrium allocation displays the regularities observed in actual data. Investment grows faster than consumption because $\gamma > \alpha\gamma$. The relative price of investment decreases at rate $(\alpha - 1)\gamma < 0$. Indeed, the nominal share of investment in income remains constant. To see this, let us take the consumption good as numeraire and define nominal income as in the general case as $m_t = c_t + p_t x_t$. From the equilibrium equations, one can show after some simple algebra that

$$s_t = \frac{p_t x_t}{m_t} = \frac{p_t x_t}{c_t + p_t x_t} = \frac{\alpha(A - \rho)}{\rho(1 - \alpha) + \alpha\sigma A} \doteq s$$

for all $t \geq 0$. To be precise, $s$ is the equilibrium share of investment in total income.

At this point it may be worth stressing that the choice of the consumption good as numeraire is inconsequential. The argument above follows equally if we choose to

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[20]Cummins and Violante (2002) observe that their measure of investment-specific technical change occurs first in information technology and then accelerates in other industries. They conclude that information technology is a "general purpose" technology, an interpretation that matches well with the spillovers' interpretation. See also Boucekkine et al (2005).

measure income in units of investment, $p_t^{-1} c_t + x_t$, or, for that matter, in any other arbitrary monetary unit provided that relative prices are respected –that is, that the price of investment relative to consumption is $p_t$. This is important because identifying real growth with growth of nominal income is as arbitrary as the choice of the numeraire in which nominal income is expressed.

## 3.2   Measuring output real growth

In this section, we apply the general theory proposed in Section 2 to the two-sector AK model. As in the general case, in regard of the Bellman equation (4), the function

$$w_t(c, x) = \frac{c^{1-\sigma}}{1 - \sigma} + v'(k_t)x$$

can be seen as representing preferences over contemporaneous consumption and investment. Again, the constraint in the Bellman equation (4) can be replaced by the budget constraint $c + p_t x \leq m_t$ because the budget line is tangent to the production possibilities frontier locally at the optimum. Notice that in this example the utility representation $w_t(c, x)$ changes over time only because the marginal value of capital does.

Let us define the indirect utility $u_t(m_t, p_t)$ and the expenditure function $e_t(u_t, p_t)$ as in Section 2. Recall that the Fisher-Shell true quantity index compares income today $m_t$ with the hypothetical level of income $\hat{m}_{t+h}$ that would be necessary to attain the level of utility associated with tomorrow's income and prices $m_{t+h}, p_{t+h}$ with today's prices $p_t$ and today's preferences as evaluated by $e_t, u_t$. Denote again this artificial level of income as

$$\hat{m}_{t+h} = e_t\big(u_t(m_{t+h}, p_{t+h}), p_t\big).$$

From the definition of $g_t^{\text{FS}}$ in Section 2, we conclude that, for all $t \geq 0$,

$$g_t^{\text{FS}} = (1 - s)\alpha\gamma + s\gamma = \frac{\alpha A(A - \rho)}{\rho(1 - \alpha) + \alpha\sigma A}$$

As already said, the Fisher-Shell quantity index is equal to the Divisia index. As in the general case, the interpretation is straightforward: $g^{\text{FS}}$ is a measure of real growth because it is constructed as the growth rate of nominal income substracting pure price changes, in this case the change of the relative price of investment $p_t$. The index only keeps changes in quantities. It is also clear that it is a true index because it is constructed from the representative household's preferences.

## 3.3 On money metric utility

What are the implications of money metric utility for the measurement of output growth? We argue in this section that in the two-sector AK model money metric utility, implicit in the Fisher-Shell index, selects a particular representation of preferences that makes welfare grow at the rate $g^{FS}$. The particular representation depends crucially on preferences and technology. Let us develop this argument.

Since the two-sector AK model jumps to the balanced growth path at the initial time, a constant fraction of total capital will be permanently allocated to the production of consumption goods and capital will be permanently growing at the endogenous rate $\gamma$. After substituting the optima consumption path in (3), the value function reads

$$v(k_t) = Bk_t^{\alpha(1-\sigma)}, \quad \text{with} \quad B = \frac{(A-\gamma)^{\alpha(1-\sigma)}}{(1-\sigma)(\rho - \alpha\gamma(1-\sigma))}. \tag{5}$$

Notice that the exponent of $k_t$ and $B$ depend on both preferences and technology.

When the Fisher-Shell index is applied to the equilibrium path of this economy, the growth rate of output is measure by $g^{FS}$, as shown above. The main argument of this paper is that the growth rate of output, when measured by the mean of the Fisher-Shell index, is a welfare measure. This section shows that in the case of the two-sector AK model, in facts, $g^{FS}$ measures the growth rate of welfare, in the sense that it is the growth rate of a particular representation of household preferences. In order words, money metric utility picks the particular representation of preferences that makes welfare, as measure by this particular representation, grow at the same rate as real output. The argument is the following. The utility function in the right hand side of (3) is one among many representations of the same preference order (constant intertemporal elasticity of substitution preferences). The Fisher-Shell index arbitrarily choses another, the one that at equilibrium grows at rate $g^{FS}$ and adopts nominal income at some base time as its benchmark. We build the argument in two steps.

First, let us denote by $\hat{v}_t$ the welfare at equilibrium of the representative agent at time $t$. Let us then make two assumptions concerning $\hat{v}_t$, consistently with the main implicit assumptions of the Fisher-Shell index. We assume first that at the initial time, $t = 0$, $\hat{v}_0 = (c_0 + p_0 x_0)/\rho$. This is the money metric utility assumption that the return to assets, the left hand side of the Bellman equation (4), is equal to nominal income at the base time (here, $t = 0$). The second assumption is that $\hat{v}_t$ grows at the rate $g^{FS}$,

meaning that $\dot{\hat{v}}_t/\hat{v}_t = g^{\text{FS}}$. Then, for all $t \geq 0$,

$$\hat{v}_t = \hat{v}_0 \, \text{e}^{gt},$$

where $g = g^{\text{FS}}$. Then, if a utility representation of the representative household preferences exists, such that, it is associated to an alternative representation of preferences in (3) consistent with the Fisher-Shell index, it has to be that at equilibrium welfare is a potential function of $k_t$ with exponent $g/\gamma$. In the following step we show that such a representation exists.

Second, we adopt the following alternative representation of the constant intertemporal elasticity of substitution preferences in (3)

$$\hat{v}(k_t) = \max C \left( \int_t^\infty \frac{c_s^{1-\sigma}}{1-\sigma} \, \text{e}^{-\rho(s-t)} \, \text{d}t \right)^{\frac{g}{\alpha\gamma(1-\sigma)}} , ,$$

which is maximized subject to the technological constraints in Section 3.1; with constant $C > 0$. Since this new utility function represents the same preferences as those of the original two-sector AK model, the equilibrium path is the same. Consequently, we can easily show that

$$\hat{v}(k_t) = C \, v(k_t)^{\frac{g}{\alpha\gamma(1-\sigma)}} = \hat{v}_0 \, \text{e}^{gt},$$

where the constant $C = \hat{v}_0 B^{-\frac{g}{\alpha\gamma(1-\sigma)}} k_0^{-\frac{g}{\gamma}}$ depends on the parameters of the model, and capital and nominal income both at the base time.[21] We have then proved that the growth rate as measured by the Fisher-Shell index is a welfare measure in the sense that it is equal to the growth rate of a particular representation of household preferences. The choice of this representation directly results from the key assumptions in money metric utility that welfare is measured in units of nominal income at some base time.

# 4 Discussion

In the framework of dynamic general equilibrium models, Section 2 shows that the Divisia index is, in fact, a true quantity index. This is of substantive interest since the Fisher ideal chain index used in actual National Accounts approximates well the Divisia index,

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[21]Notice that $\frac{g}{\alpha\gamma(1-\sigma)}$ may be positive or negative depending on $\sigma$ being smaller or larger than one, respectively. It is important to notice that $B$ contains $1-\sigma$, meaning that its sign is positive or negative, depending also on $\sigma$ being smaller or larger than one. This property of $B$ extends $C$.

implying that the growth rate of output in NIPA is welfare based. In this section, to make our main point clear, we refer to the two-sector AK model studied in Section 3 to explain what we mean by that, but most of the arguments directly apply to the general model in Section 2.

**More on money metric utility**. Notice that at equilibrium the welfare of the representative household, $v(k)$ in the Bellman equation (4), measures the value of capital. Then, $\rho\,v(k)$ is the return to capital. From (4), the return to capital is equal to the utility of current consumption plus the value of current investment, the latter being assessed at the marginal value of capital $v'(k)$. Of course, welfare as measured by $v(k)$ is defined in an arbitrary unit: monotonic transformations of $v(k)$ will change the level of utility leaving the preference map intact; consequently, the growth rate of different representations will not be necessarily the same. To overcome this problem, index number theory adopts current income a sensible norm to measure changes in welfare; i.e. money metric utility. In our context, it advocates for using observed income to measure the right hand side of the Bellman equation. Note that income as measured by National Accounts represents then the return to the stock of assets. Consequently, the Fisher-Shell quantity index and then the Divisia index are income compensating measures quantifying changes in the return to capital. Since the subjective discount rate in (4) is time independent, the Divisia index also measures changes in welfare. Indeed, in the more general framework of recursive preferences, the subjective rate of discount is not necessarily constant, implying that changes in real income may be also due to changes in the subjective rate of return. In Section 3.3, we formally analyze this issue for the two-sector AK model and show that the growth rate of output as measured by the Fisher-Shell index is equal to the growth rate of welfare, explicitly writing the particular representation of preferences that grows at this rate.

**Net National Product**. In connection with these considerations, the use of the Bellman equation makes it clear why production in National Accounts is measured as final demand. Since present and future consumption is all that matter for welfare, and net investment measures the value of the future consumption it will produce, a welfare measure of output growth has to weight the growth rate of both final demand components, consumption and net investment. This interpretation is consistent with Weitzman (1976)'s claim that "net national product is a proxy for the present discounted value of future

consumption."[22] In fact, his equation (10) is in spirit equivalent to the Bellman equations (2) and (4), which rationalize our choice of taking current net income as the proper norm in the Fisher-Shell true quantity index. It is important to point out that Weitzman (1976) is not about output growth and its relation to welfare gains in the growth process, but about the level of output and its relation to the level of welfare. In this sense, the non trivial question of the appropriate measurement of output growth has remained open until our days. The best result in this direction is in a subsequent paper by Asheim and Weitzman (2001). That paper builds a measure of the level of output and shows that output growth is a necessary and sufficient condition for welfare growth, but without providing any specific insight on how output growth should be measured. This papers gives a fundamental step ahead in this direction: by applying standard index number theory, we show that the precise way NIPA measures growth is welfare based.

At this point it may be worth clarifying that, as pointed out by Weitzman (1976), it is not GDP but NNP what matters for welfare. Depreciated capital is a lost resource that does not contribute to welfare. It is in this sense that some authors claim that NNP is relevant for welfare and GDP for productivity –see the discussion in Oulton (2004). If the depreciation rate is constant, however, net and gross investment grow at the same rate. Indeed, when investment growth faster than consumption, NNP grows slower than GDP since the share of net investment on net income is smaller than the corresponding share of gross investment.

**Investment matters**. The following example makes it more clear why investment matters in the definition of output growth. Consider a world with embodied technical progress –as the one in Greenwood and Yorukoglu (1997), for example. Let the consumption path in this economy be depicted as in Figure 3. In period $T$ there is an unexpected permanent technological shock to the investment sector: embodied technical progress accelerates. New machines, if produced and added to the capital stock, can make the productivity in the consumption goods sector grow faster indefinitely. In our example, hence, after observing the unexpected acceleration of investment specific

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[22]Weitzman's argument is developed in a simple optimal growth model with linear utility and the proof is based on the assumption that current income remains constant over time. In its own words, he gets "the right answer, although for the wrong reason." To be precise, using the main argument of the paragraph above, Weitzman's claim should be restated as "net national product is a proxy for the return to capital, which value is equal to the present discounted value of future consumption."
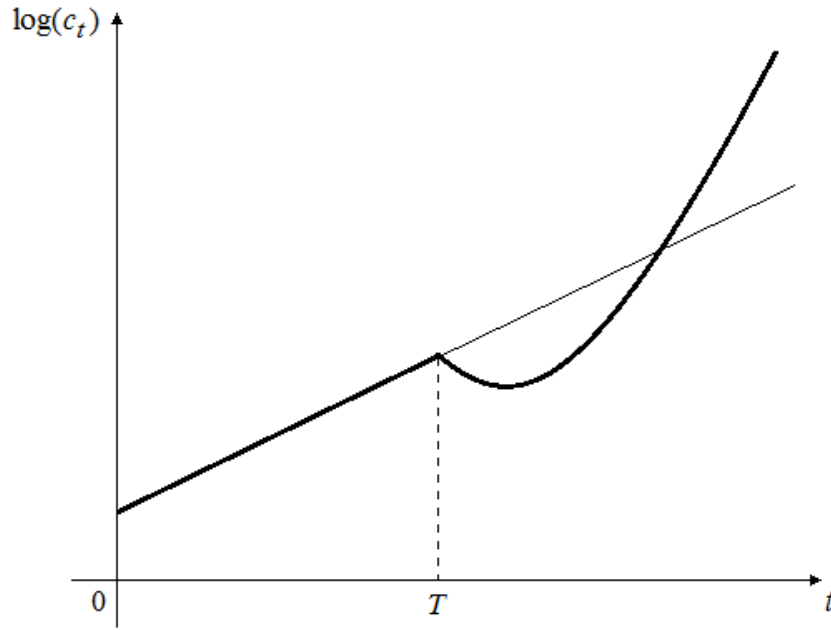
Figure 3: The manna economy versus embodied technical change

technical change in $T$, the consumer finds optimal to initially reduce consumption in order to increase investment and, then, profit from technical progress. In this world, at time $T$ households welfare increases: the drop in consumption reflects the interest of the consumer in benefiting from faster growth thereon; if this move would have not increased her welfare, she would have chosen not to increase investment and remain in the original path with lower growth. Then, the consumption growth rate at time $T$ does not measure welfare correctly. In fact, it has the opposite sign! However, the growth rate of output as measured by the Divisia index does, since it captures well the gains in welfare coming from the acceleration of technical progress and the associated optimal increase in investment. Remind that technical progress is assumed to be investment specific. Then, gains in productivity require new investments. The discussion above helps to illustrate why the growth rate of investment matters for output growth measurement. Faster growing investment today represents our best proxy for the preference for faster consumption growth tomorrow.

**Paradox of endowment vs production economies**. Moreover, it is very important to understand that a true quantity index of output growth is a welfare measure conditional on both preferences and technology, simultaneously. In other words, it does not reflect changes in welfare independently of the possibilities allowed by technology. We

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present below an example that shows the interplay between technology and preferences in the definition of output growth emerging from index number theory applied to this family of problems.

Consider the following example that clarifies further the meaning of measuring welfare changes. For the two-sector AK model in Section 3, take any configuration of parameters such that, for example, the growth rate of investment at equilibrium is 6% and the investment share is 20%. Let $\alpha$ be equal to 1/3. The Divisia index tells us that this economy will be growing at 2.8%, since consumption represents 80% of output and will grow at 2%. Alternatively, consider an *endowment* economy with exactly the same preferences and the same equilibrium consumption flow. In this economy, consumption is mana from haven. Indeed, a household would be indifferent between living in the AK or in the endowment economy, since she will get the same consumption path, that she will evaluate using the same preference map. In the endowment economy, indeed, index number theory will associate income to current consumption; the Divisia index will then measure output grow as consumption growth; 2% in our example. Why is it the case that two economies where people have identical preferences and face exactly the same consumption path do not grow at the same rate? The reason is that a true quantity index takes current income as a norm and current income is defined differently; at any time, both economies share the same consumption utility, but investment goods are produced only in the production economy. These seemingly paradoxical example illustrate well the intimate relation between preferences (what we want to do) and technology (what we can do) when measuring output growth. Indeed, in this particular example, both measures of output growth are welfare based and consistent with NIPA methodology. The example makes also clear the implications of measuring production as final demand: since there is no investment in the endowment economy, output growth becomes identical to consumption growth.

**Growth accounting**. To end this discussion, let us review the implications for growth accounting. In terms of model representations of actual economies, the introduction of more than one sector with different growth rates raises the practical and conceptual issue of how output growth has to be measured. The choice of the appropriate output growth rate affects every quantitative exercise based on the measurement of growth. This is the case in the literature on growth accounting under embodied technical change, the so-called Solow-Jorgenson controversy. To measure the contribution of investment specific

technical change to growth, Hulten (1992) measures growth (his equation (7)) following Jorgenson (1966). He suggests a raw addition of consumption and investment units, calling the outcome quality-adjusted output. Using our notation, this strategy amounts to $c_t + x_t$. Greenwood et al (1997) note that, in their setting, adding consumption and effective investment turns the economy into a standard Solow (1960) growth model with no embodied technical change.[23] Greenwood et al (1997) correctly state that any aggregation requires the different quantities to be expressed in a common unit and they adopt the consumption good as their standard. For this purpose, investment has to be multiplied by its relative price, in our notation their choice of output level would be $y_t = c_t + p_t x_t$.[24] Oulton (2004) generalizes the argument and suggests that output components have to be deflated by the consumption price index in order to measure growth. But this is indeed what Greenwood et al (1997) suggest when they identify non-durable production with real output and the real growth rate with the growth rate of consumption. What the present paper clarifies is that the issue is not the units used to measure real output *levels* but the choice of the right index of real output *growth*. In this sense, we follow Licandro et al (2002) and conclude that the "true" contribution of ETC to output growth, reflecting welfare changes, has to be measured using NIPA methodology as in Cummins and Violante (2002).

**A word of caution**. We have to be careful in the way we interpret the output growth rate. Since raising the growth performance of an economy is costly, it is well-known in endogenous growth theory that there exists an optimal growth rate. In the case of the two-sector AK model above, the optimal growth rate of capital is $\gamma$. Let us then assume, for example, that the two-sector AK model is at equilibrium growing at its optimal growth rate but an uninformed government decides to introduce some incentives to promote growth, for example by subsidizing capital production and then distorting the private return to capital. The economy will be then growing faster at the cost of a welfare reduction at the initial time. The growth rate of output in the distorted economy, like in Section 3.3, will measure welfare gains, which will be larger than in the efficient economy. However, the initial welfare losses will not be captured by National Accounts,

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[23]See Hercowitz (1998) for a review of the Solow-Jorgenson controversy.

[24]In their setting, this choice looks somewhat natural because the investment sector uses as input the consumption good. In their notation $y_t = c_t + p_t x_t$ is total output in the non-durable sector, even if only $c_t$ is consumed and the remaining production $p_t x_t$ is allocated to the investment sector.

since changes in the value of assets are in general not registered.

Let us formalize the previous statement by following the same steps as in Section 3.3. The value function of the distorted economy reads

$$v_d(k_t) = B_d \; k_{d,t}^{\alpha(1-\sigma)}, \quad \text{with} \quad B_d = \frac{(A - \gamma_d)^{\alpha(1-\sigma)}}{(1-\sigma)\big(\rho - \alpha\gamma_d(1-\sigma)\big)}.$$

where

$$\gamma_d = \frac{\tau A - \rho}{1 - \alpha(1-\sigma)} \quad \text{and} \quad k_{d,t} = k_0 e^{\gamma_d}.$$

The distortion introduced by the subsidy is represented by the wedge $\tau > 1$. It is easy to see that $B_d < B$ and decreasing with $\tau > 1$, meaning that at the initial time the policy generates welfare losses, which are larger the larger is the distortion. Paradoxically, since capital is growing faster than in the optimal economy, there exists a finite time $t_d > 0$ from which $v_d(k_{d,t})$ becomes larger than $v(k_t)$, which is the reason why welfare in the distorted economy is growing faster.

# 5   Conclusions and extensions

This paper shows that a Fisher-Shell true quantity index is equal to the Divisia index when applied to a two-sector dynamic general equilibrium economy with general recursive preferences and general technology transforming production factors (capital and labor) into consumption and investment. Indeed, it turns out that the chained-type index used by National Accounts to compute real output growth is well approximated by the Divisia index. Consequently, real output growth in NIPA is a welfare measure. This result is illustrated in the framework of the two-sector AK model. This model replicates the well-know stylized facts that investment grows faster than consumption and that the relative price of investment permanently declines. Hence, it is the appropriate context to evaluate the shift to chain indexes by National Account. More important, changes in the growth rate of investment induced by changes in embodied technical progress turn out to be a relevant part of welfare increases along an equilibrium path. Investment then matters in the measurement of output growth. In general, this paper can be seen as a recall that index number theory has an important role to play clarifying the criteria with which we construct our indexes. In particular, this approach may be of great relevance for the recent debate on the use on index number theory to rationalize the use of the Penn World Tables (see Neary (2004) and van Veelen and van der Weide (2008)).

Let us finally comment on those dimensions in which this approach could be extended and those in which it will be hard to do. Broaden it to many durable and non-durable goods seems straightforward. The approach could also be applied to many forms of non-optimal equilibria. Notice that, in this case, the production possibility frontier will not be tangent to an indifference curve at equilibrium, and hence the generalization will not be straightforward. However, if the representative household is price taker in all markets, irrespective of the fact that prices are distorted, at equilibrium the budget constraint will be tangent to an indifference curve. Under theses circumstances, index number theory could be applied to compare different points in the equilibrium path in a similar way we did in Section 2. In particular, for a stationary economy moving from a distorted to a non distorted equilibrium, the Divisia index could be measuring the welfare gains period by period.

Note that this paper understands welfare changes as income compensating variations of a representative household. Yet, one could interpret the Divisia index to be measuring welfare changes of the average household in an economy with many different consumers, but neglecting any consideration regarding unequal effects.[25] Actually, in the Bellman equation representation (4) utility is quasilinear on investment. Quasilinear preferences belong to the more general family of Gorman preferences, which can be aggregated and represented by those of a representative household.[26] In this sense, the growth rate in NIPA may be understood as a welfare based measurement even in worlds with heterogenous households. Indeed, things will be more complicated in overlapping generations economies.

# Appendix: Quantity indexes in continuous time

## A1. Quantity indexes in continuous time

In continuous time, let us define a growth factor $\Gamma_{t+h}^{t}$, interpreted as the gross rate of growth of an arbitrary variable between a base time $t$ and a current time $t + h$. In the

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[25]The interpretation of welfare in this paper is not related to the notion of social welfare. With heterogeneous households some authors accept interpersonal comparisons of utility when interpreting the economy as an artifact for a normative discussion. See, for example, Dasgupta (2011) or the attempts to go beyond GDP surveyed in Fleurbaey (2009).

[26]See Gorman (1953, 1961).

jargon of National Accounts, $\Gamma_{t+h}^t$ is referred as a volume index. Let us then define the instantaneous growth rate of the underline variable at time $t + h$ when the base time is $t$ as

$$g_{t+h}^t = \frac{\mathrm{d}\Gamma_{t+h}^t}{\mathrm{d}h}. \tag{6}$$

Notice that in continuous time the derivate of a growth factor at current time $t$ is equal to the growth rate of the variable itself. Let $z_t$ be a continuous-time variable and fix some base time $t$. The growth factor in this case is $\Gamma_{t+h}^t = z_{z+h}/z_t$. Let take the first derivate of it

$$\frac{\mathrm{d}\Gamma_{t+h}^t}{\mathrm{d}h} = \frac{\dot{z}_{t+h}}{z_t}.$$

When evaluated at $h = 0$

$$g_t^t = \left.\frac{\mathrm{d}\Gamma_{t+h}^t}{\mathrm{d}h}\right|_{h=0} = \left.\frac{\dot{z}_{t+h}}{z_t}\right|_{h=0} = \frac{\dot{z}_t}{z_t}.$$

This way of defining the instantaneous growth rate may look odd but it may be useful in those cases in which we have an index like $\Gamma_{t+h}^t$ but no explicit variable giving rise to this index like $z_t$ in this example. The Fisher ideal chain index is one of these cases.

Using the notation introduced in Section 2, the starting point is some nominal aggregate $c_t + p_t x_t$. Remind that consumption is the numeraire so that its price is normalized to one while the price of investment in consumption units is $p_t$. Laspeyres quantity indexes use time $t$ (the base time) prices as weights based on the following growth factor

$$\mathcal{L}_{t+h}^t = \frac{c_{t+h} + p_t x_{t+h}}{c_t + p_t x_t}.$$

It does allow to compute the growth rate of output by putting all nominal values at base time prices. Paasche indexes take current prices as weights by defining the growth factor as

$$\mathcal{P}_{t+h}^t = \frac{c_{t+h} + p_{t+h} x_{t+h}}{c_t + p_{t+h} x_t}.$$

Real output growth is measured at current $t + h$ prices.

Let us now use (6) to define the Laspeyres and Paasche indexes for the corresponding definitions of the growth factors. It is easy to see that in continuous time both Laspeyres and Paasche quantity indexes are equal to the Divisia index when evaluated at $t$:

$$\left.\frac{\mathrm{d}\mathcal{L}_{t+h}^t}{\mathrm{d}h}\right|_{h=0} = \left.\frac{\mathrm{d}\mathcal{P}_{t+h}^t}{\mathrm{d}h}\right|_{h=0} = (1 - s_t)g_{ct} + s_t g_{xt},$$

where $s_t = \frac{p_t x_t}{c_t + p_t x_t}$ is the investment share, $g_{ct} = \frac{\dot{c}_t}{c_t}$ the growth rate of consumption and $g_{xt} = \frac{\dot{x}_t}{x_t}$ the growth rate of investment.[27]

The Fisher ideal growth factor between $t$ and $t+h$ is defined as

$$\mathcal{F}_{t+h}^t = \left(\mathcal{L}_{t+h}^t \mathcal{P}_{t+h}^t\right)^{\frac{1}{2}}. \tag{7}$$

Given that in continuous time, both Laspeyres and Paasche chain quantity indexes are equal to the Divisia index at $t$, it is easy to show that the Fisher ideal chain index is equal too.

The definition in equation (6) is also useful applied to the Fisher-Shell quantity index since we have a well-defined index $\hat{m}_{t+h}/m_t$.

## A2. Fixed-base quantity indexes in continuous time

Traditional measures of real growth stem from fixed-base quantity indexes. The most common among them are the Laspeyres and Paasche indexes referred in Appendix A1. From Appendix A1, the Laspeyres factor of change between $t$ and $t+h$ is

$$\mathcal{L}_{t+h}^t = \frac{c_{t+h} + p_t x_{t+h}}{c_t + p_t x_t},$$

for all $h \geq 0$, where the superindex $t$ in $\mathcal{L}$ designates the base time $t$ and the subindex the current time $t+h$. In continuous time, the Laspeyres index $g_{t+h}^{\mathcal{L}\,t}$ is the instantaneous growth rate of factor $\mathcal{L}_{t+h}^t$ as a function of $h$ (see Appendix A3). That is,

$$g_{t+h}^{\mathcal{L}\,t} = \frac{d\mathcal{L}_{t+h}^t}{dh} \frac{1}{\mathcal{L}_{t+h}^t} = \frac{\dot{c}_{t+h} + p_t \dot{x}_{t+h}}{c_{t+h} + p_t x_{t+h}},$$

which measures the real growth rate at $t+h$ for the given base time $t$. The Laspeyres index is popular because it is conceptually simple.

However, if the relative price of investment permanently declines and substitution makes real investment permanently grows faster than real consumption, as observed in the data, the Laspeyres index tends to give too much weight to investment as we depart from the base time $t$. In order to illustrate it, let us assume the economy is at a balanced growth path with constant investment and consumption shares, $s$ and $1-s$ respectively, $s \in (0,1)$, the relative price of investment goods $p_t$ declining at a constant rate $\gamma > 0$

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[27]In discrete time, the weights of consumption and investment growth rates in the Laspeyres and Paasche indexes are different from current income shares.

and investment and consumption growing at the constant rates $g_x$ and $g_c$, respectively, $g_x > g_c > 0$. The growth rate of output, at any time $t + h$, $h > 0$, as measured by the Divisia index is

$$g = (1 - s)g_c + sg_x.$$

The main result of this paper is that a welfare based measure of output growth is equal to the Divisia index.

Note, indeed, that the Laspeyres fixed-base index reads

$$g_{t+h}^{\mathcal{L}\,t} = \frac{c_{t+h}}{c_{t+h} + p_t x_{t+h}} g_c + \frac{p_t x_{t+h}}{c_{t+h} + p_t x_{t+h}} g_x. \tag{8}$$

Since $p_{t+h}$ declines with $h$ at the rate $\gamma$, it is easy to see that along a balanced growth path, the weight of consumption in the Laspeyres fixed-base index decreases and the weight of investment increases with $h$. This effect is known in the index numbers literature as the *substitution bias*. Fast growing items when weighted using past (relatively high) prices are overweighted, overstating the real growth rate of output. The effect is larger the farther we are from the base time, converging to the growth rate of investment as $h$ goes to infinity.

The Paasche index uses current prices as a base, and hence tends to understate real growth as we go back in time. The Passche factor is

$$\mathcal{P}_{t-h}^t = \frac{c_t + p_t x_t}{c_{t-h} + p_t x_{t-h}}$$

for all $h \geq 0$ and the growth rate

$$g_{t-h}^{\mathcal{P}\,t} = \frac{d\mathcal{P}_{t-h}^t}{dh} \frac{1}{\mathcal{P}_{t-h}^t} = \frac{c_{t-h}}{c_{t-h} + p_t x_{t-h}} \frac{\dot{c}_{t-h}}{c_{t-h}} + \frac{p_t x_{t-h}}{c_{t-h} + p_t x_{t-h}} \frac{\dot{x}_{t-h}}{x_{t-h}}. \tag{9}$$

As $h$ grows, so $t - h$ decreases, the weight of consumption increases because $x_{t-h}/c_{t-h}$ decreases, converging to the growth rate of consumption as $h$ goes to infinity.

For the arguments developed above, both Laspeyres and Paasche fixed-base indexes yield poor measures of real growth when output components grow at different rates because of changing relative prices.[28]

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[28]Updating regularly the base is not a solution because it would imply a permanent revision of past growth performance. It posses the additional problem of multiple real growth measures for each period, each of them affected differently for the substitution bias depending on the associated base period.

## A3. Chained-type quantity indexes in continuous time

In this appendix, we use our simple framework to review the BEA methodology.[29] The introduction by the BEA of quality corrections in equipment prices in the mid-eighties revealed a persistent declining pattern in the price of equipment relative to the price of non-durable consumption goods. Since then, real investment appears to be growing much faster than real non-durable consumption. In this new scenario, fixed-base quantity indexes face the severe substitution bias problem explained in Appendix A2 above. For this reason, the BEA moved to a chained-type index based on a Fisher ideal index computed for contiguous periods.[30] Let us first define a Fisher ideal index to them define a Fisher ideal chained index both in continuous time.

A Fisher ideal growth factor $\mathcal{F}_{t+h}^t$ in the interval $(t, t+h)$ is the geometric mean of a Laspeyres growth factor and a Paasche growth factor both defined in the same interval, that is

$$\mathcal{F}_{t+h}^t = \left( \mathcal{L}_{t+h}^t \mathcal{P}_{t+h}^t \right)^{\frac{1}{2}}.$$

The Fisher ideal index is the growth rate of the factor $\mathcal{F}_{t+h}^t$ as a function of $h$. Computing the average compensates the overstatement of the Laspeyres index with the understatement of the Paasche index, thus reducing the impact of the selection bias.

Let us now define a Fisher ideal chained (factor) index for the time interval $(0, T)$, where $t = 0$ represents now the *reference time* (in contraposition to the *base time*). The key assumption of chained indexes is that the base time moves with $t$, by taking $t$ as the base time when computing the growth rate at time $t$. From Appendix A1, for any time $t \in (0, T)$, the instantaneous growth rate of the Fisher ideal index is

$$g_t^{\mathcal{F}} = \left. \frac{\mathrm{d}\mathcal{F}_{t+h}^t}{\mathrm{d}h} \right|_{h=0} = (1 - s_t) g_{ct} + s_t g_{xt}.$$

Even if there is a trend in relative prices, inducing the substitution of one good for another, the chained-type index allows weights to change continuously to avoid the emergence of any substitution bias.

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[29]Young (1992) is a non-technical presentation of the methodological changes introduced in NIPA. Whelan (2002, 2003) provides a more detailed guide into the new methods in use at BEA to measure real growth. For economic index number theory see Diewert (1993), Triplett (1992), Fisher and Shell (1998) and IMF (2004, chapter 17).

[30]Diewert (1993) provides a clear explanation of the index suggested by Fisher (1922).

Let us assume that $s_t$, $g_{ct}$ and $g_{xt}$ are continuous function of $t$, then the Fisher ideal index $g_t^{\mathcal{F}}$ is continuous too. A Fisher ideal chained (factor) index $\mathcal{C}_t^{\mathcal{F}}$ is defined by the differential equation

$$\dot{\mathcal{C}}_t^{\mathcal{F}} = g_t^{\mathcal{F}} \mathcal{C}_t^{\mathcal{F}},$$

$\mathcal{C}_0^{\mathcal{F}} = 1$, which solution is

$$\mathcal{C}_t^{\mathcal{F}} = e^{\int_0^t g_s^{\mathcal{F}} \, ds}.$$

A chained factor index for a time interval $t \in (0, T)$ is build in two stages. First, at any time $t \in (0, T)$ a growth rate is computed using $t$ as the base time. Second, the time $t$ growth rates computed at the first stage are chained in order to build growth factors in an interval of time $t \in (0, T)$. Notice that fixed-base factor indexes are equal to one at the baes time. In the case of chained indexes base times are changing. The time at which the factor index is set equal to one is now called the reference time.

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