Tolerance and Compromise in Social Networks.

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Abstract

In this paper, individuals are characterized by their *identity* – an ideal code of conduct – and by a level of tolerance for *behaviors* that differ from their own ideal. Individuals first choose their behavior, then form social networks.

This paper studies the possibility of compromise, i.e. individuals choosing a behavior different from their ideal point, in order to be accepted by others, to 'belong'. We first show that when tolerance levels are the same in society, compromise is impossible: individuals all choose their preferred behavior and form friendships only with others whose ideal point belong to their tolerance window. In contrast, we show that heterogeneity in tolerance allows for compromise in equilibrium. Moreover, if identity and tolerance are independently distributed, any equilibrium involves some compromise.

Key Words: Compromise, Social Capital, Tolerance, Homophily, Identity.

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1. Motivation

Bernard Crick defines tolerance "as the degree to which we accept things of which we disapprove" (Crick (1973), 63). It is the ability or willingness to tolerate something, in particular the existence of opinions or behavior that one does not necessarily agree with. The literature on homophily has long highlighted the role of preferences in explaining why people associate and bond more with others that are similar to them (see McPherson, Smith-Lovin, and Cook (2001) for a review). People prefer interacting with similar others.

However, there is an important distinction to be made between individuals caring about the true *identity* of their friends – their type – versus caring about the *conduct* of their friends – their behavior. In some settings, preferences depend others' types such as their religion, ethnicity, sexual orientation (see Currarini, Jackson, and Pin (2009)). But in other settings, individuals may not actually care about others' true identity but care about others' behavior: whether they act religious, whether they dress conservatively, whether they appear gay or even how 'white' they act (see Austen-Smith and Fryer (2005), Carvalho (2013), Berman (2000) and Lagunoff (2001)). This is this second type of preference that this paper is interested in studying.

In our model, individuals interact in social networks and their utility depends on their own conduct but also the conduct of others with whom they interact. Individual differ in their ideal behavior, their true *identity*, and potentially in their *tolerance* for conduct that differs from that ideal point. Prior to forming their social network, individuals choose their behavior and can therefore *compromise* in order to 'fit in'. For instance, a religious person might decide not to display any religious symbol to be accepted as a friend by a less religious person, or a non-religious person may sometime go to church to please a friend.

We show that there are strict limits to compromise and that these limits are strictly decreasing in the tolerance of the most intolerant members of the society. If all individuals have the same tolerance levels then no compromise is possible. They all choose their preferred code of conduct. Hence, individuals form links with each other if and only if the distance between their preferred conduct is less than the level of tolerance in the society. The intuition for this inability to compromise is simple. Since compromise is costly, individuals have the incentive to do the least as possible in order to be accepted. But this implies that their friendship is not very valuable to others who then have little incentive to compromise themselves.

Some heterogeneity is needed for compromise to happen. We show that introducing more intolerant individuals can allow compromise in equilibrium. What is more, this paper proves that, if compromise and tolerance are independently distributed, then there must be compromise in equilibrium. Relatively tolerant individuals compromise for relatively intolerant ones. Naturally the joint distribution of tolerance and Identity matters. If level of tolerance decrease towards the extreme, reciprocated compromise is not possible and behaviors tend to be polarized. In contrast, more tolerance at the extreme encourages a more connected society.

This work is clearly related to the general framework of Akerlof and Kranton (2000) since a person's identity matters for the gains and losses in utility from behaviors that conform or depart from her ideal, be it her own behavior or the behavior of others.[1] This paper studies how individual's identities and tolerance matter in their choice of social identities and social network. The inability to compromise to form the socially optimal networks identified in this paper also speak to the literature on diversity and social capital (see Putnam (2000), Dasgupta and Serageldin (1999) and Portes and Vickstrom (2011))

Finally, notice that this model can also be applied to other settings. For instance, political compromises and alliances between politicians can be important (Levy (2004)). Or it could be applied to the choice of technological standards and the formation of trade networks. Countries could be endowed with different initial technologies and the complementarities between their technologies could decrease in the distance between their standard while the cost of modifying a technology could be proportional to the extent to which it needs to be modified.

The next Section formalizes the model described above. Section 3 then introduces simple examples that provide the intuition for the main results. The latter are presented in Section 4. Section 5 discusses our results and Section 6 concludes.

[1]See also Cervellati, Esteban, and Kranich (2010).

2. Premises of the model

Individuals and Preferences:

Consider a population I consisting in a mass of size 1 of individuals i distributed over the interval $[0,1]$. Each individual has an ideal point $\iota_i \in [0,1]$, her *identity*. This identity represents the person's ideal code of conduct and is immovable. In contrast, individuals select a code of conduct, a *behavior*, a_i in the Euclidean space.

Individuals care about their own conduct and the behavior of the members of their social network and judge them in comparison with their ideal behavior. We assume that an individual's utility is strictly decreasing in the Euclidean distance from her ideal point. Individuals have potentially heterogenous preferences regarding these behaviors.

Consider an individual i with ideal point ι_i, behavior a_i and links with individuals in S whose profile of behavior is given by $\mathbf{a}_S$. Her utility is given by:

$$u_i(a_i, \mathbf{a}_S) = \int_{j \in S} v_i\left(d(\iota_i, a_j)\right) - g(d(\iota_i, a_j)), \tag{1}$$

where $d(\iota, a)$ is the Euclidian distance between ι and a; v_i is continuous, strictly decreasing with $0 < v_i(0) \leq F$ for some finite F; and g is continuous, strictly increasing and convex with $g(0) = 0$.

Individual i is said to *compromise* if her chosen behavior differs from her ideal point $d(\iota_i, a_i) > 0$.

Define individual i's *tolerance level* t_i as the largest tolerable distance t, that is

$$t_i = \{max \;\; t \in \mathbb{R}_+ | v_i(t) \geq 0\}.$$

As long as a person j's behavior is within a distance t_i of i's ideal point, $d(\iota_i, a_j) \leq t_i$, i would be happy to have j in her social group.

In this model, individuals are effectively characterized by two attributes: their *identity* ι_i or ideal code of conduct and their *tolerance level* t_i which represent the largest tolerable deviation from their ideal point. As we'll see, a person's identity and tolerance level are the relevant characteristics for our results.

We define as $\omega_i \equiv \{a \in \mathbb{R} | d(i, a) \leq t_i\}$ i's *tolerance window* and, if $a \in \omega_i$, we say that a belongs to i's tolerance window. Figure 1 illustrates these concepts.

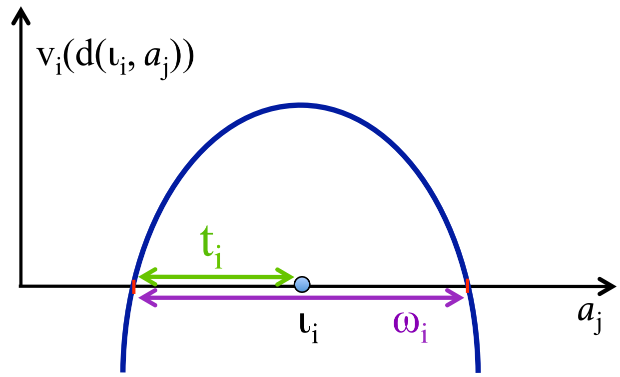

FIGURE 1. TOLERANCE

Timing:

This is a two-stage game. In a first stage, individuals choose their lifestyle (their behavior) and, in a second stage, they form their social network by choosing who to be friend with or which society to join. When people choose their lifestyles they have in mind this second-stage possibility and may want to compromise in order to be accepted, to 'belong'.

Network Formation:

There is no cost of link formation. As a result and since the benefits are additive, the network formation is trivial. Given a vector of behavior $\mathbf{a}$ in the population, an individual i is happy to form a link with an individual j if and only if $\mathbf{a}_j \in \omega_i$.

Following most of the network literature, we consider networks that are pairwise stable. Jackson and Wolinsky (1996) defined the following notion of *pairwise stability*: a network is pairwise stable if (i) no player would be better off if he or she severed one of his or her links, and (ii) no pair of players would both benefit (with at least one of the pair seeing a strict benefit) from adding a link that is not in the network.

Assume that if both players are indifferent, they would form a link. Then, for any given profile of action $\mathbf{a}$, there is a unique pairwise stable graph G so that i and j have a link $g_{ij} = 1$ if and only if $\mathbf{a}_j \in \omega_i$ and $\mathbf{a}_i \in \omega_j$.

3. Examples

This Section provides some intuition about the main results through simple examples with a discrete number of individuals. For these examples, we consider a discrete version of the model and assume linear payoffs

$$u_i(a_i, \mathbf{a}_{S(i)}) = \sum_{j \in S(i)} [F_i - b_i d(\iota_i, a_j)] - g d(\iota_i, a_i), \qquad g \geq 0; \tag{2}$$

where F_i represents the intrinsic value of a friendship for i while b_i captures her aversion for behavior that do not correspond to their ideal. In this case, i's tolerance level is given by

$$t_i = \frac{F_i}{b_i}.$$

Observe that this formulation captures well the fact that one's tolerance depends on both the benefit that she derives from a friendship and her dislike of differences. The more someone has to gain from social connections, the more she is willing to befriend individuals who differ from her ideal point.

3.1. No Compromise with Homogeneity.

Assume that $t_i = t$ for all i. When tolerance levels are symmetric, either both individuals's identities belong to the other's tolerance window $\iota_i \in \omega_j$ and $\iota_j \in \omega_i$ or both individuals's identities lie outside of the other's tolerance window$\iota_i \notin \omega_j$ and $\iota_j \notin \omega_i$.

In the first panel of Figure 2, where i and j are sufficiently tolerant or sufficiently similar that their ideal conduct already belong to the other's tolerance window. They therefore have no reason to compromise and can be friends in spite of their differences.

In contrast, the second panel of Figure 2 illustrates a situation where i and j do not belong to each other's window though their tolerance windows do overlap.

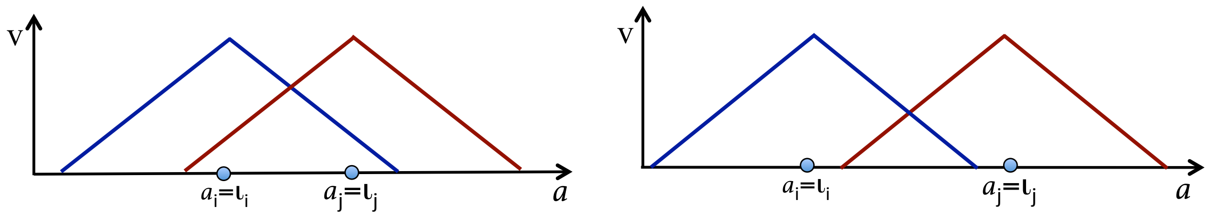

Figure 2. a. i and j are not friends; b. i and j are friends

The only way for them to become friend is for *both* to compromise. Since there is no incentive to unilaterally compromise, there is clearly an equilibrium without compromise. What this paper will show is that this equilibrium is unique. Since compromise is costly, individuals have the incentive to "minimally compromise": do the least as possible in order to be accepted. But this implies that their friendship is not valuable to others who then have little incentive to compromise themselves. Hence, two individuals cannot compromise for each other.

In both cases, all individuals choose their preferred actions and are friends only if they belong to each other's tolerance window.

3.2. **Heterogeneity Enables Compromise.**

To see how heterogeneity in tolerance levels enables compromise, take two individuals j and k who differ in tolerance levels. If j is more tolerant than k, a situation where $\iota_k \in \omega_j$ but $\iota_j \notin \omega_k$ is possible. This is illustrated in the left panel of Figure 3. Person j values a link to k even if k does not compromise. If such link is worth enough to j to compensate for her disutility from compromising and becoming acceptable to k, a link will be formed. If she compromised, j would clearly choose the smallest compromise needed to be friend with k: the action a_j in ω_k that is the closest possible to ι_j as shown in the left panel of Figure 3. Hence, j compromises and befriends k if

$$F_j - b_j|\iota_j - \iota_k| - g|\iota_j - a_j| \geq 0. \tag{3}$$

It follows naturally that the presence of a less tolerant individuals can allow more tolerant individuals to become friends. To show this, consider the example in Figure 2 where i and j have the same tolerance levels and add a more intolerant

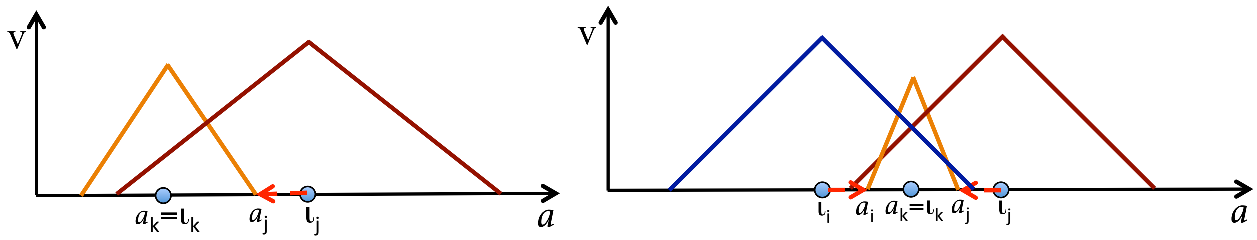

FIGURE 3. COMPROMISING FOR AN INTOLERANT PERSON

individual k right in the middle of them so that $\omega_k \subseteq \omega_i$, $w_k \subseteq \omega_j$, $\iota_i \notin \omega_j$ and $\iota_j \notin \omega_i$, as in Figure 3. If i compromises to be acceptable to k, she is attractive to j as well and vice versa. Let ℓ_k and r_k be, respectively, the left and the right extremities of k's tolerance window. There is an equilibrium where $a_i = \ell_k$, $a_k = \iota_k$ and $a_j = r_k$ and all three individuals are friends if

$$\begin{aligned}[F_i - b_i|\iota_k - \iota_i|] + [F_i - b_i|a_j - \iota_i|] &\geq g|\iota_i - \ell_k| \\ F_i - b_i|\iota_k - \iota_i| &\geq g|\ell_k - \ell_j|.\end{aligned}$$

The first equality requires the overall value of the compromise to be positive: the value of the friendships with j and k exceed the cost of compromise. In addition, the second inequality guarantees that i should prefer be at ℓ_k and be friend with both i and j to choosing the left extremity of j's tolerance window, ℓ_j, and be friend only with j. Both these constraint are satisfied for a sufficiently low cost of compromise g. This equilibrium is illustrated on the right panel of Figure 3.

EXAMPLE: assume that i and j have ideal positions $\iota_i = 0.2$ and $\iota_j = 0.8$, and are otherwise symmetric with $b_i = b_j = 1$ and $F_i = F_j = 0.5$, while k who has an ideal position just in between, $\iota_k = 0.5$, is less tolerant $b_k = 5$ and $F_k = 0.5$. With respect to their own action, they all have the same disutility from deviating from their ideal point $g = 1.1$. Interestingly, it can be checked that i would not compromise for k alone (condition 3 would not hold for i and k), but $a_i = \ell_k = 0.4$, $a_k = \iota_k$ and $a_j = r_k = 0.6$ is an equilibrium.

3.3. **Compromises builds on Compromise.**

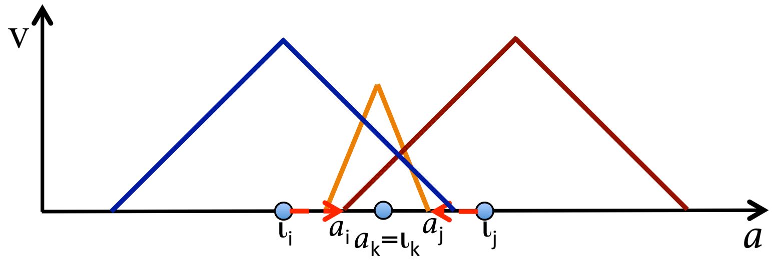

FIGURE 4. COMPROMISE BUILDS ON COMPROMISE

As we have just seen and will be shown more generally below, compromise originates with a individual who compromises to become friend with a less tolerant person. But it is worth noting that further compromise can be build from that initial effort. Indeed, other high tolerance individuals may compromise for the initial compromiser.

This is illustrated in Figure 4 where i and j are high tolerance, while k is a low tolerance person. In this example, k does not compromise, i compromises for k but j compromises for i who is now valuable to j. A complete network is achieved.

4. Main Results.

In this Section, we return to a continuum of agents with the general form preferences given in (1) and tolerance levels that can take values in $[\underline{t}, \overline{t}]$ with $\overline{t} \geq \underline{t} > 0$.

4.1. The Limits to Compromise.

We show that there are serious limit to compromise. Precisely, Proposition 1 shows that an individual i will never compromise by more than $t_i - \underline{t}$.

Before proceeding to these results, we introduce a piece of notation and three definitions that are useful for the proof and the rest of the paper. Denote as $I_i \equiv \{a \in \mathbb{R} | d(\iota_i, a) < t\}$ the interior of i's tolerance window.

DEFINITION **1.** j is *valuable* to i if $a_j \in I_i$.

DEFINITION **2.** i *compromises for* j if $d(\iota_i, \iota_j) > t_j \geq d(a_i, \iota_j)$.

Definition **3.** *i minimally compromises for j* if i compromises for j and $a_i = argmin_{a_i \in \omega_j} d(\iota_i, a_i)$.

That is, an individual i is said to compromise *for* another one j if her ideal point is outside of j's tolerance window while her chosen behavior is inside of j's window, and we say that she *minimally* compromises for j if she compromises for j while deviating the least as possible from her ideal behavior. We now turn to the proposition.

Proposition **1.** *An individual i with tolerance t_i will never compromise by more than $t_i - \underline{t}$ in equilibrium.*

By characterizing each individual's maximal compromise, this proposition naturally defines an overall limit to individual compromise: compromise can never exceed $T \equiv \bar{t} - \underline{t}$.

Corollary **1.** *Individual compromise cannot exceed $T = \bar{t} - \underline{t}$ in equilibrium.*

Another straightforward but very powerful Corollary of Proposition 1 is that there cannot be any compromise when all individuals have the same tolerance levels. Only individuals who already belong to each other's tolerance window can be friend in equilibrium. This result generalizes the example in Section 3.1.

Corollary **2.** *If all individuals have the same tolerance, $t_i = t$ for all i,*
[1] *compromise is not possible in equilibrium,*
[2] *i and j are friends if and only if $i \in w_j$ and $j \in \omega_i$.*

It also follows directly from Proposition 1 that the most intolerant individuals never compromise.

Corollary **3.** *The least tolerant type never compromises in equilibrium.*

Finally, bounds on compromise imply a maximal distance between any two linked individuals.

Corollary **4.** $=$ *In equilibrium, $|\iota_i - \iota_j| \leq t_i + t_j - \underline{t}$ for all pair $ij \in G$.*

Indeed, it follows from Proposition 1 that $d(\iota_i, a_i) \leq t_i - \underline{t}$ and $d(\iota_j, a_j) \leq t_j - \underline{t}$. If i and j are friends $d(\iota_j, a_i) \leq t_j$ and $d(\iota_i, a_j) \leq t_i$. Hence, $d(\iota_i, \iota_j) \leq t_i + t_j - \underline{t}$.

The proof of Proposition 1 is in Appendix but we outline the argument here.

The intuition for the proof is simple. Take homogenous individuals and assume that the claim is wrong and i compromises. The idea is that if someone compromises, there must be minimally compromising for a set of valuable individuals. Take one individual in that set. We show that he himself must be compromising and therefore must be minimally compromising for a different set of individuals who are valuable to him. Proceeding in this manner gives us sequences of compromising individuals. The proof shows that along these sequences, compromise must be ever expanding and cannot converge. This means that along the sequence compromise will at some point reach a level such that individuals would be better off compromising strictly less.

Similarly, with heterogenous individuals we show that if one individual compromised by more than $T = \bar{t} - \underline{t}$, it would imply the existence of sequences of individuals $m = 1, 2, ...$ who compromise and that $d(\iota_m, a_m) - t_m$ is ever increasing along these sequences. Again, there must be a point along any of these sequence where compromise becomes too costly and we reach a contradiction.

4.2. How Heterogeneity Helps Compromise.

The previous Section showed that compromise is bounded. At the extreme, we saw that no compromise is possible with homogenous tolerance levels. In contrast, the examples in Sections 3.2 and 3.2 demonstrated that heterogeneity in tolerance levels makes compromise possible. Assume heterogeneity in tolerance levels: $\bar{t} > \underline{t}$. Proposition 2 goes further than showing that heterogeneity is needed for compromise and proves that, if identities and tolerance levels are independently distributed and $g'(0) = 0$, then there is some compromise in *any* equilibrium.

PROPOSITION **2.** *Suppose that individuals' ideal points and tolerance levels* t_i *are independently distributed with non-degenerate interval support. If* $g'(0) = 0$ *then equilibrium involves compromise.*

The proof (in Appendix) builds on the intuition of example in Section 3.2. If there were no compromise in equilibrium, we could always find some relatively tolerant individuals at the extreme who would have incentive to unilaterally compromise. This is because while a little compromise is almost costless it allows a relatively tolerant individual at the extreme to become acceptable to a positive mass of more intolerant individuals that he values without loosing any friendship. Hence, there will be compromise in equilibrium. Now it is obvious that there can be many equilibria and that these equilibria may be difficult to characterize.

To prove that we have compromise in equilibrium when there is heterogeneity (Proposition 2), we assumed that an individual's level of tolerance and her ideal position are unrelated. This independence assumption in Proposition 2 could certainly be weakened but a mix of types at the extremes is crucial for the result.

This framework highlights the key role that relatively intolerant individuals play in compromise. This is also shown in Proposition 3 but first we introduce one more definition.

Definition **4.** We say that i and j *reciprocally compromise* if i compromises for j and j compromises for i.

Proposition **3.** *If i and j ($\iota_i < \iota_j$) reciprocally compromise, then there be some individual k in between ($\iota_i < \iota_k < \iota_j$) such that $\iota_k - t_k$ or $\iota_k + t_k$ in $\Omega_i \cap \Omega_j$.*

Proposition 3 tells us that in between any pair of individuals who reciprocally compromise, there must be *bridge* individuals who is strictly less tolerant than the most tolerant of the pair. It follows that for two agents of the same tolerance level to compromise for each other, we need a more intolerant person to serve as a *bridge* between them.

4.3. **Tolerance and Extremism.** In this Section, we investigate what happens when tolerance and ideal points are systematically related. First, we assume that there is more intolerance at the extremes. Formally, assume there there exist a mapping $M : [0, 1] \to \mathbb{R}_+$ that determines the tolerance of an individual based on

it's ideal behavior and that:
[T] tolerance levels increase then decrease over the interval $[0, 1]$.

PROPOSITION **4.** *Under assumption [T], reciprocal compromise is not possible in equilibrium.*

5. Discussion.

5.1. Welfare.

Unsurprisingly, the equilibrium compromise can be suboptimal.

Take the homogenous case. Corollary 4.1 tells us that no compromise is possible in equilibrium. However, it is easy to show that compromising could be optimal. A necessary condition for compromise to be optimal is that, for some behaviors, the gain in i's utility as j moves towards her must be higher than i's loss as i moves away from her ideal position, and the same for j. With linear payoffs as in (2), this is rather unlikely as it requires individuals to be more sensitive to the behavior of others than to one's own behavior as they move away from one's ideal point (b_i to be higher than g). Although some people are stricter with others than with themselves – finding unacceptable behaviors in others that they do themselves engage in[2] – it may not be the majority.

However, one expects the cost from deviating from one's ideal point to be convex. In which case, it is easy to construct examples where compromise would be optimal even if the deviations from one's ideal point by oneself are more or equally costly than others's deviations from that point.

EXAMPLE: Take the discrete case of 2 individuals $i = 0.1$ and $j = 0.9$. Assume their preferences to be

$$u_k(a_k, \mathbf{a}_S) = \sum_{l \in S} [F - bd(\iota_k, a_l)^2] - gd(\iota_k, a_k)^2, \qquad g \geq 0. \tag{4}$$

[2]Most people know of can imagine someone judging others for their lack of religiosity, their laxism with their children, their promiscuity while always having one good reason to oneself not attend service, give in to one's kids, enter a new relationship, etc.

where $F = 0.5$, $b = 1$ and $g = 1.1$. These preferences imply tolerance levels $t = 0.7$, so that i and j lie just outside of each other's tolerance window. In this example, any Pareto optimal solution involves compromise. For instance, the symmetric Pareto optimal point asks for each individuals to deviate from their optimal point by 0.38 and delivers a utility of 0.18 to each individuals (as opposed to 0 in the absence of compromise).

5.2. **Externalities.** To be sure, the assumption that individuals care only about the behavior of individuals with whom they are linked is strong. We are affected by others' behavior even if we do not interact frequently with them. However, the model could easily be modified by assuming that individuals care about every one's behavior but more strongly about people to whom they are linked. We believe that none of our results would be affected.

5.3. **Diversity in Social Network.** Notice that when compromise occurs, we can distinguish between the diversity in *behavior* as opposed to the the diversity in *identities* observed in social networks.

A measure of diversity in the network G based on the average distance in identities between the individuals who are friends:

$$\frac{1}{\sum_{ij \in G} 1} \sum_{ij \in G} |\iota_i - \iota_j|$$

will tend to depict a more diverse society than a measure of diversity based on individuals' actual behavior

$$\frac{1}{\sum_{ij \in G} 1} \sum_{ij \in G} |a_i - a_j|.$$

5.4. **Compromise and population density.** Note that increasing the density of population has non monotonic effect on compromise. To see this, consider the following example. Take two individuals located at 0 and 1 with the following preference:

$$u_i(a_i, \mathbf{a}_{S(i)}) = \sum_{j \in S(i)} [F - bd(\iota_i, a_j)] - gd(\iota_i, a_i) \; where \; \iota_0 = 0 \; \iota_j = 1,$$

where $g > 0$ but small. Their tolerance level is $\bar{t} = F/b < 1/2$. Now let's introduce on the interval relatively more intolerant individuals with a level of tolerance $\underline{t} < overlinet$. We assume that these intolerant individuals are equally spaced in terms of identities, that is we place them at location $\frac{1}{2^n}$, $\frac{2}{2^n}$,... $\frac{2^n-1}{2^n}$ and increase $n = 1, 2, ..$ to raise the density of population.

From Corollary, we know from 3 that the relatively intolerant will not compromise. Consider $i = 0$. Clearly, if she compromises at all, she must minimally compromise for a low tolerance person: $a_0 = 0$ or $a_0 = \frac{k}{2^n} - \underline{t}$ for some $k \in \{1, ...2^n - 1\}$. In addition, it is easy to see that if $a_0 \leq \frac{1}{2^n} + \underline{t}$. Otherwise, it means that by compromising minimally for $\frac{k}{2^n}$ 0 compromises so much that he misses on a friendship with $\frac{1}{2^n}$. By compromising instead minimally for $\frac{k-1}{2^n}$, he would have to compromise less, have as many friends and these would be more valuable to him. Hence, if 0 compromises, he will compromise minimally for $\frac{k}{2^n} \leq \frac{1}{2^n} + 2\underline{t}$.

While if this constraint does not bind, it must be that the additional benefit from compromising for k compared to $k - 1$ dominates the cost while it would not be the case at $k + 1$.

$$\begin{aligned} \frac{g}{2^n} &\leq F - b\frac{k}{2^n} \\ \frac{g}{2^n} &> F - b\frac{k-1}{2^n}. \end{aligned}$$

Clearly $\bar{t} - \frac{g}{b2^n}$ is increasing in n while the constraint $\frac{1}{2^n} + 2\underline{t}$ is decreasing in n. Hence, if g is not too large, increasing the density of the population n initially increases compromise then decreases it once the constraint binds. This initial increase then decrease in compromise is illustrated in Figure 5.

This finding is related to Rosenblat and Mobius (2004) who show that decreasing cost of communication allow heterogeneous agents to segregate along special interests rather than by geography.

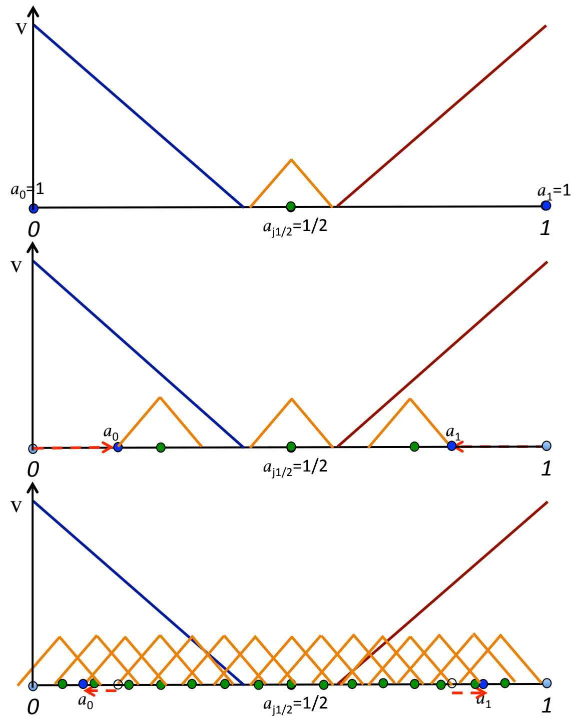

FIGURE 5. COMPROMISE AND DENSITY

6. CONCLUSION

In this paper, we study compromise in a model of social network formation. Individuals identities characterizes their preferred conduct for themselves and for others. People derive utility from links to others whose conduct is within their "tolerance window". They first choose a conduct then form their social network.

We show that compromise is strictly limited and that the bounds to compromise are decreasing in the tolerance level of the most intolerant. When all individuals have the same tolerance level, there cannot be any compromise in equilibrium. In contrast, with heterogeneity, any equilibrium has to exhibit some compromise if tolerance and identity are independently distributed.

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Appendix: Proofs

Lemma **1.** *If i compromises in equilibrium then the largest set of individuals who are valuable to i and for whom i minimally compromises X_i contains a positive mass of individuals.*

Proof. If i compromises, $d(\iota_i, a_i) > 0$ then there must be a positive mass of individuals who are valuable to i (in the sense of Definition 1) and for whom i is minimally compromising (in the sense of Definition 3). To prove this claim, assume that there was no such set. Since compromise is costly, there then would exist a small $\varepsilon > 0$ so that i can improve her utility by bringing her behavior a_i closer to her ideal point by ε while keeping all links any valuable individual j (for which $a_j \in I_i$) and deleting all others. ∎

Lemma **2.** *If i minimally compromises for j then $d(\iota_j, a_i) = t_j$ and $d(\iota_i, \iota_j) = d(\iota_i, a_i) + t_j$.*

Proof. The claim follows directly from $d(\iota_i, \iota_j) > t_j$ and $a_i = argmin_{a_i \in \omega_j} d(\iota_i, a_i)$. ∎

Lemma **3.** *If i minimally compromises for j and j is valuable to i, $d(\iota_j, a_j) - t_j > d(\iota_i, a_i) - t_i$.*

Proof. Indeed, if i minimally compromises for j, Lemma 2 tells us that

$$d(\iota_i, \iota_j) = d(\iota_i, a_i) + t_j$$

while $a_j \in I_i$ means that

$$d(\iota_i, \iota_j) < d(\iota_j, a_j) + t_i.$$

These two inequalities imply that

$$d(\iota_i, a_i) - t_i < d(\iota_j, a_j) - t_j. \tag{5}$$

∎

Proof of Proposition 1. Assume that the proposition does not hold, so that there is an equilibrium vector of behaviors $\mathbf{a}$ and an individual i whose behavior differs from her ideal by more than $t_i - \underline{t}$: $d(\iota_i, a_i) > t_i - \underline{t}$. Moreover, we can show that x_{m+1} compromises for x_m in the sense of **??**.

Step 1. We first prove that this implies that there are sequences of compromising individuals $\{x_m\}$ for $m \in \{1, 2, ...\}$ originating in i ($x_1 = i$) such that x_m minimally compromises for x_{m+1} and x_{m+1} is valuable to x_m. Along this sequence, let's denote x_m's ideal point ι_m, her tolerance level t_m and her choice of action a_m.

The first thing to notice is that $d(\iota_m, a_m) > t_m - \underline{t}$ (something that we assumed for $x_1 = i$) means that x_m compromises (since $t_m - \underline{t} \geq 0$). Lemma 1 tells us this implies the existence of a non-empty associated set X_m that is the largest set of individuals who are valuable to m and for whom m minimally compromises.

It follows from Lemma 3 that $d(\iota_m, a_m) > t_m - \underline{t}$ implies $d(\iota_{m+1}, a_{m+1}) > t_{m+1} - \underline{t}$ for $x_{m+1} \in X_m$. Since $t_{m+1} - \underline{t} \geq 0$, x_{m+1} compromises.

Finally, we show that x_{m+1} compromises for x_m in the sense of **??**. We already know that x_{m+1} is valuable to x_m so all that remains to show is that $d(\iota_m, \iota_{m+1}) - t_{m+1} > 0$. This holds since $d(\iota_m, a_m) - t_m + \underline{t} > 0$ means that $d(\iota_m, \iota_{m+1}) - t_{m+1} - t_m + \underline{t} > 0$ and therefore that

$$d(\iota_m, \iota_{m+1}) - t_{m+1} > t_m - \underline{t} \geq 0.$$

Step 2. Denote as $\mathcal{S}_i$ the set of all the sequences identified in Step 1 that originates in i. Lemma 3 tells us that $d(\iota_m, a_m) - t_m$ strictly increases along any sequence in $\mathcal{S}_i$.

Now assume that, along one of the sequences in S_i, $\{x_m\}_{m=1,2...}$, the distance $d(\iota_m, a_m) - t_m$ did not converge. Then there would exist n so that $g(d(\iota_n, a_n)) > v_n(0)$. Even if compromising to a_n allowed x_n to become friend with everyone and if everyone was choosing her favorite behavior, it would not be worth it.

This means that $d(\iota_m, a_m) - t_m$ needs to converge along every sequence in $\mathcal{S}_i$. Pick one of these sequence originating in i: $\{x_1, x_2, ...\} \in \mathcal{S}_i$. For each individual x_m, let X_m denote the largest associated set of individuals who are valuable to x_m and for whom x_m minimally compromises and let μ_m denote the density of this associated set. For any $\varepsilon > 0$ there exists n so that $(d(\iota_y, a_y) - t_y) - (d(x_n, a_n) - t_n) < \varepsilon$ or

$$t_n + d(\iota_y, a_y) - t_y - d(\iota_n, a_n) < \varepsilon \tag{6}$$

for all $y \in X_n$. Using the facts that $d(\iota_y, a_y) \geq d(\iota_n, \iota_y) - d(\iota_n, a_y)$ (since y compromises for x_n) and that $d(\iota_n, \iota_y) = d(\iota_n, a_n) + t_y$ (since x_n minimally compromises for y) in (6), we get that, for any $y \in X_n$,

$$t_n - d(\iota_n, a_y) < \varepsilon.$$

It follows that

$$v_n(d(\iota_n, a_y)) < v_n(t_n - \varepsilon) \forall y \in X_n, \tag{7}$$

where v_n represent the utility Let $\eta > 0$ be the smallest compromise along the sequence $\{x_m\}_{m=1,2...}$. We can pick ε to be such that $v_n(t_n - \varepsilon) < g'(\eta)$. In which case

$$g'(d(\iota_n, a_n)) > \mu_n v_n(t_n - \varepsilon),$$

and x_n would strictly increase her utility by choosing a behavior slightly closer to her ideal point. ∎

Proof of Proposition 2. Assume not. This implies that $a_i = \iota_i$ for all i. However pick an individual i located at one extreme $\iota_i = 0$ with tolerance $t_0 > \underline{t}$. Let F denote the distribution of ideal position and G the distribution of tolerance levels. If i does not compromise, he will be friend with all individuals j in his tolerance window $[0, t_0]$ who tolerate him $t_j \geq \iota_j$: a proportion $F(t_0)[1 - G(\iota_j)]$. If he chooses a code of conduct $a_i = \varepsilon > 0$ instead, the individuals j in his tolerance window $[0, t_0]$ who tolerate him are now such that $t_j \geq \iota_j - \varepsilon$: a proportion $F(t_0)[1 - G(\iota_j - \varepsilon)]$. Hence, the gain from compromising is given by

$$\int_0^{t_0} v_i(\iota_j)[G(\iota_j) - G(\iota_j - \varepsilon)]F(j)$$

while the cost is $g(\varepsilon) - g(0) = g(\varepsilon)$. Since g is continuous and $g'(0) = 0$, it must be that for a sufficiently small $\varepsilon > 0$

$$\int_0^{t_0} v_i(\iota_j)[G(\iota_j) - G(\iota_j - \varepsilon)]F(j) > g(\varepsilon).$$

Hence, compromise must arise in equilibrium. ∎

Lemma 4. *If $ij \in G$ then a_i (and a_j)$\in \Omega_i \cap \Omega_j$.*

Proof For j to accept i's friendship it must be that $a_i \in \Omega_j$. $a_i \in \Omega_i$ follows directly from Proposition 1. ■

Proof of Proposition 3

Assume that the proposition is not true. Then there must exist individuals i and j (with $\iota_i < \iota_j$) who reciprocally compromise with no intermediary individual k with $\iota_i < \iota_k < \iota_j$ and an extremity, either $\iota_k - t_k$ or $\iota_k + t_k$, in $\Omega_i \cap \Omega_j$.

Since i compromises, there must be a non-empty set X_i of valuable individuals for whom i minimally compromises. Any $k \in X_i$ has an extremity $(\iota_k - t_k) \in \Omega_i \cap \Omega_j$. Hence, by assumption it must be that $\iota_k \notin (\iota_i, \iota_j)$. And since Lemma 4 tells us $a_k \in \Omega_i$, k compromises for i. $\Omega_i \cap \Omega_k \subset \Omega_i \cap \Omega_j$. Using the same logic, we can show that there are sequences of compromising individuals $\{x_m\}$ for $m \in \{1, 2, ...\}$ originating in i, $x_1 = i$ (and we can do the same for j), such that x_m minimally compromises for x_{m+1} and x_{m+1} is valuable to x_m. Along this sequence, we denote x_m's ideal point ι_m, her tolerance level t_m and her choice of action a_m. If x_m for $m \geq 2$ compromises, then X_m is non empty (Lemma 1). Since $a_m \in \Omega_i \cap \Omega_j$, any $\ell \in X_m$ has an extremity in $\Omega_i \cap \Omega_j$ and therefore $\iota_\ell \notin (\iota_i, \iota_j)$. From Lemma 4 $a_\ell \in \Omega_m \cap \Omega_{m+1} \subset \Omega_i \cap \Omega_j$. Hence, $x_{m+1} \in X_m$ compromises for m.

We can then apply the second Step of Proposition 1 to reach a contradiction. ■

Proof of Proposition 4. Assume that the claim does not hold so that there is a non empty set P of pairs of individuals (x, y) who engage in reciprocal compromise. Next, select a pair (i, j) in P according to the following criteria
(a) one of the two individuals, has the lowest level of tolerance among all members of P;
(b) if multiple pairs satisfy (a), select among these pairs one where the most intolerant individual is closest to the extreme in the following sense: (i, j) minimizes $\delta(x, y)$ defined as follows

$$\begin{aligned}\delta(x,y) &= d(\iota_y, 1)\ if\ either\ t_y < t_x\ or\ t_y = t_x\ \&\ t_z \leq t_y \forall z\ such\ that\ \iota_z \geq \iota_y \\ &= d(\iota_x, 0)\ otherwise;\end{aligned}$$

(c) if multiple pairs satisfy (a) and (b), choose the pair, or one of the pairs, with the largest distance between the two individuals.

In what follows we assume that j is the most intolerant of the two: $t_i \geq t_j$ and if $t_i = t_j$ $t_k \leq t_j$ for all k with $\iota_k \geq \iota_y$. The argument can easily be adjusted to the case where i is the most intolerant.

Step 1. Since i compromises, there must be a non-empty set X_i of valuable individuals for whom i compromises. Take any $k \in X_i$. If $t_k > t_j$, then

$$d(\iota_i, \iota_k) = t_k + d(\iota_i, a_i) > t_j + d(\iota_i, a_i) \geq d(\iota_i, \iota_j)$$

where the first inequality follows from Definition 2 and the last inequality follows from the fact that i and j become friends and therefore a_i must be in j's tolerance window. But this means that $\iota_k > \iota_j$ while $t_k > t_j$: a contradiction to [T]. Now, if $t_k < t_j$ then [T] implies that $\iota_k > \iota_j$ and $d(\iota_i, \iota_k) > d(\iota_i, \iota_j)$. Hence, to be valuable to i k must be compromising. But then $t_k < t_j$ contradicts our selection criterium (a). Hence, $\iota_k = \iota_j$ and $t_k = t_j$ for any $k \in X_i$ and any $k \in X_i$ compromises. Either $j \in X_i$ or j minimally compromises for i.

Step 2. Take any $k \in X_i \cup j$. Since k compromises, there is a non-empty set X_k of individuals for whom k compromises. Take any $l \in X_k$. We show that $\iota_l \in [\iota_i, \iota_k]$. Since j compromises towards i, $\iota_l < \iota_k$, and if $\iota_l < \iota_i$ then l and k would be engaged in reciprocal compromise with $d(\iota_l, \iota_k) > d(\iota_i, \iota_k)$, in contradiction with part (c) of our selection. Following the same logic as before, it must also be the case that $t_l \leq t_i$, as otherwise it would also imply that l and k would be engaged in reciprocal compromise while $d(\iota_l, \iota_k) > d(\iota_i, \iota_k)$, in contradiction with part (c) of our selection. Hence, $\iota_l \in [\iota_i, \iota_k]$ and $t_i \geq t_l \geq t_k$ for all $l \in X_k$ (where the last inequality follows from [T]).

Step 3. Assume that $t_l < t_i$ for some $l \in X_k$. To be valuable to k, l must compromise (as $t_k \leq t_l$). Let X_l be the set of individuals for whom l compromise and $m \in X_l$. If $t_m < t_j$ then [T] implies that $d(\iota_l, \iota_m) > d(\iota_l, \iota_j)$ so that l and m would be engaged in reciprocal compromise while $t_m < t_j$: a contradiction of selection criterion (a). Hence, $t_m \geq t_j$. Since l is valuable to k, it implies that

$$d(\iota_l, \iota_m) = d(\iota_l, a_m) + t_m > d(\iota_l, \iota_j)$$

which implies that $\iota_m > \iota_j$. From [T] this means that $t_m = t_j$. But then, that there is a pair of individuals (l, m) with the lowest tolerance individual m located more at the extreme than the pair (i, j). This directly contradict criteria (b) of our selection. It follows that $t_l = t_i$ for all $l \in X_k$.

Step 4. Since $t_l = t_i$ for all $l \in X_k$, $\iota_k - \iota_i$ and $d(\iota_i, a_k) = t_i$ and this for all $k \in X_i \cup j$. None of the individuals for whom i is compromising have an action in the interior of i's tolerance window: a contradiction. ∎