Trade liberalization and inequality: a dynamic model with firm and worker heterogeneity*

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Abstract

A vast literature demonstrates that trade liberalization is associated with higher wage inequality. Nearly the entire literature considers comparative statics or steady states, which ignore dynamics and of necessity feature monotonic changes. I address these limitations by developing a micro-founded model that emphasizes the dynamics of reallocation between heterogeneous firms and workers in the presence of costly labor adjustments. Trade liberalization provides firms both new export markets and new sources of competition. Expanding high-paying firms increase wages to recruit better workers faster. Workers at firms threatened by competition accept wage cuts to delay their employers' exit and keep their job. This provides novel implications for both aggregate and within-firm inequality across a distribution of firm types. I show that key mechanisms of the model are consistent with a range of facts using matched firm-worker data from France. Results from the calibrated model suggest an overshooting of inequality on the path to a new steady state. This is consistent with evidence based on an event study of recent liberalization episodes. Inequality appears to peak about six years after liberalization, with one-fourth of the overshooting disappearing in the following ten years.

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1 Introduction

What are the dynamic effects of a trade liberalization reform on the wage distribution? The vast majority of studies have focused on comparative statics and comparative steady states and shown that trade liberalizations are associated with rises in inequality. Is the response of inequality over time monotonic? If it is, how fast does it unfold? If it is non-monotonic, what is the nature, magnitude and length of the transitory effect? By nature, comparative statics and comparative steady states approaches are not suited to address these questions directly. I study the dynamic effect of trade on inequality, developing and calibrating a micro-founded model with an explicit dynamic reallocation process.

To illustrate the dynamics of responses to liberalization episodes, I provide suggestive evidence from an event study. This exercise is motivated by country case studies showing rich dynamic responses[1]. In order to provide a more general picture, I follow the evolution of inequality in response to liberalization reforms in 37 countries. Consistent with previous work[2], Figure 1 shows that inequality rises after countries open to trade. The increase is gradual over the first six to seven years. From this point inequality stops increasing and the data even suggest that this increase gets partially undone. By contrast over the same period the ratio of trade to GDP rises steadily after the date of the liberalization. Of course, country panel studies of this type cannot identify the causal effects of trade. The heart of my analysis is, therefore, structural.

I develop a dynamic general equilibrium model with worker and firm heterogeneity that fits squarely with findings of the empirical literature that emphasizes the dominant role of intra occupation-sector inequality, and positive assortative matching between heterogeneous firms and workers[3]. I derive analytical predictions both about steady state comparisons and about the individual responses of firms and workers in the transition path equilibrium following a reduction in trade costs.

[1] Frias, Kaplan and Verhoogen (2009) for Mexico and Helpman, Itskhoki, Muendler and Redding (2012) for Brazil both document a hump-shaped response of inequality following trade-enhancing reforms.

[2] See Goldberg and Pavcnik (2007) for a survey of the effects of liberalization reforms on inequality in emerging countries.

[3] Card, Heining and Kline (2013) decompose changes in inequality in Germany over the period 1989-2009 and find that the bulk of the growth in inequality is explained by increasing dispersion in firm specific premia and rising firm-worker assortativeness. Additionally, Helpman, Itskhoki, Muendler and Redding (2012) find that two-third of the increase in wage inequality in Brazil between 1986 and 1995 is due to increasing wage dispersion within sector-occupation groups.

Then, I show that the model mechanisms are consistent with a wide range of salient facts, some of which are unexplained by current models. I use matched employer-employee data from France to calibrate the model and use the calibrated model to consider the general equilibrium transition path of the whole economy in response to a trade liberalization reform. The predictions of the model are qualitatively consistent with the findings of the event study: it predicts that inequality peaks after four years, then about one-fourth of the increase gets undone in the following ten years.

The model combines the screening of heterogeneous workers by heterogeneous firms with directed search and costly recruitment in a standard model of trade[4]. Following Kaas and Kircher (2015) I assume that hiring firms open job vacancies, announce the probability of getting the job and its expected wage payments. I enrich their mechanism by allowing for worker heterogeneity a la Helpman, Itskhoki and Redding (2010) in the sense that ex-ante identical workers draw a match specific productivity every time they are interviewed. Firms have incentives to screen interviewees and their incumbent workers alike because of complementarities between firm size and workers ability. In equilibrium job-seekers arbitrage between vacancy types that differ in job-matching rate, expected payments, and screening rates. Therefore high-wage jobs attract more workers and are harder to get as in Kaas and Kircher (2015) and firms that want to hire faster do so by offering higher wages. My model departs from theirs as worker heterogeneity requires intensive-screening firms to increase wage offers in order to compensate job-searchers for the higher probability of being screened out.

I show that under restrictive assumptions I can compare my dynamic model steady state predictions with the predictions of the canonical models of Melitz (2003) and Helpman et al. (2010). In a version of the model without screening, I show that the elasticity of consumption with respect to trade costs across steady states is related to assumptions about the distribution of firms and the elasticity of substitution across variety as in Melitz (2003). With screening, the model predicts that for a given cohort, more productive firms are larger, have higher levels of screening and therefore pay higher wages. I show that under restrictive assumptions the relationship between the steady state dispersion of firm average wages and the level of trade costs has the same inverted U-shape as in Helpman et al. (2012). Therefore the model extends steady state results of Melitz (2003) and Helpman et al. (2010) to a more general setting in which firms chose to grow progressively to their

[4] The model of screening builds on Helpman, Istkhoki and Redding (2010), the model of directed search builds on Kaas and Kircher (2015) and the standard model of trade builds on Melitz (2003).

optimal size.

Unlike previous work in the trade literature, the model features overlapping generations of firms that progressively adjust the size and average ability of their workforce. Firms do not leap to their optimal state; instead they grow progressively in order to save on vacancy posting costs that are assumed to be convex. By assumption firms start small with an unselected set of workers. In equilibrium, they grow production capacities by screening the least able of their workers and by replacing them with more and better workers. Firms decide to export only if they are productive enough and only when they are big enough to cover the fixed costs of export. Because decisions are based on current and future prospects, young small firms that will export in the future prepare to do so by hiring more and better workers.

A distinctive feature of the model is that wages depend on both firm productivity and firm growth rate: wages vary between firms, within firms and over time. In particular, since the distribution of firms and firm growth rates adjust along the transition path, the model allows for rich effects during transitions between steady states.

A once and for all reduction in trade costs has two effects. First it provides incentives for exporters to sell more and for the most productive domestic firms to start exporting. During the transition period, these expanding and already high-paying firms temporarily offer even higher wages to recruit better workers faster. Unlike previous models where the fate of workers is uniquely determined by their employer's fate, the model implies different outcomes in the transition path for different workers within the same firm. The least able workers at expanding firms is screened out while workers of the same ability keep their job at the less productive firms that do not raise their screening level.

Second, a reduction in trade costs also results in greater competition pressure. The model predicts that domestic firms grow more slowly, shrink or even exit depending on their productivity and stage of development. Domestic firms that still hire choose lower screening levels and lower wages. Furthermore the least productive firms may have committed to wage levels that they are not able to pay anymore now that increased competition reduced their revenues. Some firms may be able to negotiate wage cuts with their workers while others are bankrupt and exit.

I show that the novel predictions of the model match a large number of empirical findings on worker flows and wages. In particular, the model implies that separation rates increase with employment growth rates at the firm level because firms grow production capacity by increasing the number and average ability of workers by hiring and screening simultaneously. This is consistent with empirical work on hiring and separation rates in the US and in France; see Davis, Faberman and Haltiwanger (2013) and Duhautois and Petit (2015). Additionally, the model predicts that future exporters prepare their workforce by selecting better workers in advance. This echoes the empirical findings of Molina and Muendler (2013) about Brazilian firms that become exporters.

The model's novel implications about the wage dispersion within firms are also consistent with empirical findings. The model predicts that wage dispersion within exporting firms temporarily increases following a positive foreign demand shock because these firms hire workers at a premium to reach their new optimal state faster. This prediction is consistent with evidence in Frias, Kaplan and Verhoogen (2012) on Mexican exporters following the 1994 peso devaluation.

More generally the model predicts that similar workers in a given firm have different wages depending on what the firm growth rate was when they were hired. In steady states, the level of trade costs do not affect the growth rate profile of firms. Thus the model implies that the average within-firm wage dispersion is similar across steady states with different trade costs. This prediction is consistent with evidence in Helpman, Itskhoki, Muendler and Redding (2012) as they find little long run change in within-firm inequality before and after the trade reforms of the 80's and the 90's in Brazil.

To study the aggregate implications of trade on inequality and consumption along the transition path, I need to aggregate firm transitions. Aggregating across firms that differ in productivity and stages of development is complicated. Rather than imposing simplifying assumptions that would allow for analytical results, I calibrate the micro mechanisms of the model and obtain numerical results about aggregates.

I estimate and calibrate the firm-worker adjustment mechanisms with matched employer-employee data on the French manufacturing sector for 1995-2007. The elasticity of the wage of new hires with respect to firm employment growth is one of the central parameters of the model. While there are many theories that predict an upward sloping wage curve at the firm level, empirical evidence of the causal effect of firm growth are scarce (Manning 2011) because of the absence of good firm level instruments. I contribute to the labor literature by using the instrument developed in Hummels et al (2014) which is novel in this context. I restrict the analysis to exporting firms and use foreign demand shocks to instrument for firm growth. I find similar estimates as those in the previous work of Schmieder (2010) on German start-ups.

I compute the response of a closed economy that implements a once and for all reduction in trade costs. The main result is that overall inequality overshoots its long run increase. This pattern broadly matches the suggestive evidence of the event study of liberalizations. The model predicts that inequality peaks four years after the reform and that about one fourth gets undone in the following ten years. The evolution of the increase in inequality is mostly driven by the dispersion of firm average wages. Specifically it results from the temporary wage premium offered by high-paying expanding firms to speed growth and from the wage cuts at the least productive firms. Within firm inequality also contributes to a temporary increase in inequality as the wage dispersion increases at expanding firms but its contribution to total inequality is small.

The rest of the paper is organized as follows. I start by briefly outlining the relationship of this paper to the existing literature. Section 2 presents suggestive evidence about the evolution of inequality in countries that opened to trade. Section 3 lays out the mechanisms of the labor market, the trade-offs faced by workers and the optimal behavior of firms in steady states. Section 4 characterizes the steady state properties of the model and provides some comparisons with canonical models of the literature. Section 5 presents the equilibrium path following a once and for all reduction in trade costs. Section 6 describes estimations and calibrations of baseline parameters. It then discusses and reports the simulation results of the calibrated model's response to a counterfactual trade liberalization. Section 7 concludes.

Related literature

This paper contributes to a growing literature that focuses on the interplay between labor frictions, firm growth and trade. Fajgelbaum (2013) develops a steady state trade model with job-to-job mobility impeded by search frictions in the spirit of Burdett and Mortensen (1998) and shows that future exporters poach workers from currently exporting firms in order to prepare to export. A related result of my model is that future exporters prepare to export by screening more intensively before they actually serve the foreign market. In both models, exporters do so in order to smooth recruiting costs over time. Ritter (2012) and Felbermayr, Impullitti and Prat (2014) develop steady state frameworks that embed the directed search mechanism of Kaas and Kircher (2015) into trade models but find little impact of trade on wage inequality because they ignore the interaction between labor frictions and worker heterogeneity. Cosar, Guner and Tybout (2014) structurally estimate a rich steady state model with establishment-level data from Colombia to compare the relative importance of labor and trade reforms. They find that Colombia's opening to trade is the main driver of the observed increase in wage inequality and job turnover. While the common feature of these models is the existence of overlapping generations of firms, their analysis focus on steady states and ignore the dynamics that follow trade reforms.

Out-of-steady state models of trade have recently been developed along two dimensions. First, a branch of the literature examines the reallocation of workers with different levels of human capital along the transition path. Cosar (2010) and Dix-Carneiro (2010) develop models of sluggish accumulation of sector specific human capital and sector switching costs to study the distributional effects of trade and underscore the importance of sectoral mobility frictions. While sectoral differences can be important, previous empirical work has shown that most of the reallocation flows (Wacziarg and Wallack (2004)) and the increase in inequality occur within sectors (Helpman et al. (2012)). Emami and Namini-Lopez (2013) and Danziger (2014) develop models of skill acquisition examining the evolution of the skill premium in perfectly competitive labor markets. The dynamics are shaped by the time it takes for workers to acquire skills in response to a trade-induced shift in the relative demand for skills. They predict the overshooting of the skill premium. By contrast a defining feature of my framework is the focus on the evolution of within group inequality which is underscored as a key driver of inequality by the empirical literature[5].

Second, my model also belongs to the transitional dynamics literature that emphasizes the role of costly firm adjustments. The common feature of these models is the presence of frictions that prevent firms to instantaneously adjust to changes in the product market. A first generation of models starting with Ghironi and Melitz (2005) studies the implication of sunk costs and lagged entry on the evolution of the distribution of heterogeneous firms. In their framework as in mine, a reduction in trade costs generates consumption overshooting because there is a temporary reallocation of workers from the firm creation sector to the production sector[6]. Atkeson and Burstein (2010) develop a

[5] Card et al. (QJE 2013) for Germany: "Increasing workplace heterogeneity and rising assortativeness between high-wage workers and high-wage firms likewise explain over 60% of the growth in inequality across occupations and industries." Helpman et al. (2012) for Brazil: "The within sector-occupation component of wage inequality accounts for over two thirds of both the level and growth of wage inequality in Brazil between 1986 and 1995." Frias, Kaplan and Verhoogen (2009) also have similar findings for Mexico where most of wage changes result from changes in plant premia rather than changes in skill composition.

[6] Alessandria and Choi (2014), Alessandria Choi and Ruhl (2014) simulate rich models with productivity shocks, capital accumulation and sunk capital adjustment costs and they also find overshooting of consumption.

model of firm innovation showing how decisions on productivity improving investments interact with changes in trade costs. To some extent, my framework and theirs share the view that there are complementarities between export and investment decision if one is willing to interpret screening as a form of investment. Cacciatore (2014) introduces labor market frictions and unemployment in a two-country dynamic stochastic general equilibrium model to examine the synchronization of business cycles between partners with different labor market characteristics. This model has similar implications as mine about unemployment: both predict that trade liberalization generates a temporary increase in unemployment.

To this date, only one paper features changes in wage dispersion along the transition path in a model of overlapping generations of firms. Helpman and Itskhoki (2014) extend to a dynamic setting the trade model of Helpman and Itskhoki (2010) with search frictions and bargaining. In their model, changes in wage inequality are driven by wage cuts at the least productive of the declining firms and dynamics are driven by the exogenous quits and exits of these firms. A similar pattern can be found in my model. However the models differ along several dimensions. First the absence of worker heterogeneity in their model implies that there is no wage dispersion in steady states and that the fate of workers during transitions are tied to their employer's while my model has implications for between and within firm inequality in steady states and during transitions. Moreover in their framework, expanding firms leap to their long run optimal states and their wages are constant across firms and over time. Thus they rule out the wage dispersion and the wage changes at the top of the distribution that my model features.

Finally this paper relates to the literature on firm growth and human resource management. Two papers are particularly relevant. First Caliendo and Rossi-Hansberg (2006) and the subsequent empirical investigation of Caliendo, Monte and Rossi-Hansberg (2015) examine the internal organization of heterogeneous firms in hierarchical layers of occupations. The focus on occupations makes their work complementary to this paper. They show that firm expansions that do not lead to organizational changes result in wage raises for all workers as in my model, while the addition of a layer of management reduces the wages of incumbent workers. Second Haltiwanger, Lane and Spletzer (2007) use a matched employer-employee data from the US Census Bureau and investigate the choice of worker mix made by heterogeneous firms. They document a strong relationship between firm worker mix and firm productivity and find that initial deviations at young firms from this relationship tend to be corrected in the long run in a manner consistent with selection. The screening mechanism in my model provides a stylized version of this mechanism.

2 Patterns of inequality evolution following liberalizations

In this section, I present patterns of the dynamic response of inequality at the macro level by following the evolution of changes in inequality before and after liberalization reforms for a panel of 37 countries. Inequality gradually increases and seems to feature an overshooting response. I discuss the statistical significance of the latter. Before, I briefly outline the methodology which follows the work of Wacziarg and Welch (2008)[7] and the different dataset on which the methodology is implemented.

2.1 Country panel data and empirical approach

The analysis is based on country panel fixed effect regressions and focuses on within-country changes in Gini coefficients relative to country pre-reform trends. The event study analysis mainly draws from three data sources.

Firstly I use the liberalization dates of Wacziarg and Welch (2008). The authors carefully construct dates of trade liberalization for the 106 countries for which they have enough information. The determination of the liberalization dates relies on a comprehensive survey of country case studies. Specifically, the criteria used in chosing the dates are essentially based on major changes in annual tariffs, non-tariffs, and black market exchange rate premia[8]. In addition and whenever relevant, they used a variety of secondary sources to include the dates when state monopoly on major exports were abolished and when multi-party governance systems replaced the Communist Party's undivided rule. The varied nature of the reforms conducted at the liberalization dates may cast doubt on whether the event study really speaks about the effect opening up to trade. Panel A of figure 1 provides evidence that these reforms had a substantial effect on trade: country openness as measured by trade ratios to GDP starts trending up at the liberalization dates.

[7]They themselves provide an update of earlier work by Sachs and Warner (1995)

[8]The black market premium may seem unrelated to trade openness but the literature has actually made the case that it has effects equivalent to a formal trade restriction. The main argument is based on the observation that exporters often have to purchase foreign inputs using foreign currency obtained on the black market while they remit their foreign exchange receipts from exports to the government at the official exchange rate. As result, a black market premium acts as a trade restriction.

The second essential piece of data consists of country measures of inequality over time. The most comprehensive source of country panel data measures of inequality is the UNU-WIDER, "World Income Inequality Database (WIID3.0b), (2014)". This dataset is a compilation and harmonization of the Gini coefficients that were computed from detailed country specific micro-studies on inequality. Typically single micro-studies only span several years and I relied on overlapping series to construct long-time series of changes in inequality[9].

The last dataset used in the event study is the World Development Indicators provided by the World Bank. The WDI contains data on population, real and nominal GDP, inflation, import and export, investment and unemployment. I supplement these time series with the share of secondary and high education completed in the population aged 15 and over from the Barro-Lee Educational Attainment Data.

Once all sources are merged, I only keep countries for which the measure of inequality spans a period that includes the liberalization date. This leaves me with 37 countries spanning 6 regions (14 countries are in the Americas, 7 are in Asia, 6 are in Europe, 6 are Post-Soviet countries, two are in Oceania and two are in Africa) and with liberalization dates that range from 1964 to 2001 (four are in the 60's, two are in the 70's, four are in the 80's, 26 are in the 90's and one is in the 2000's)[10].

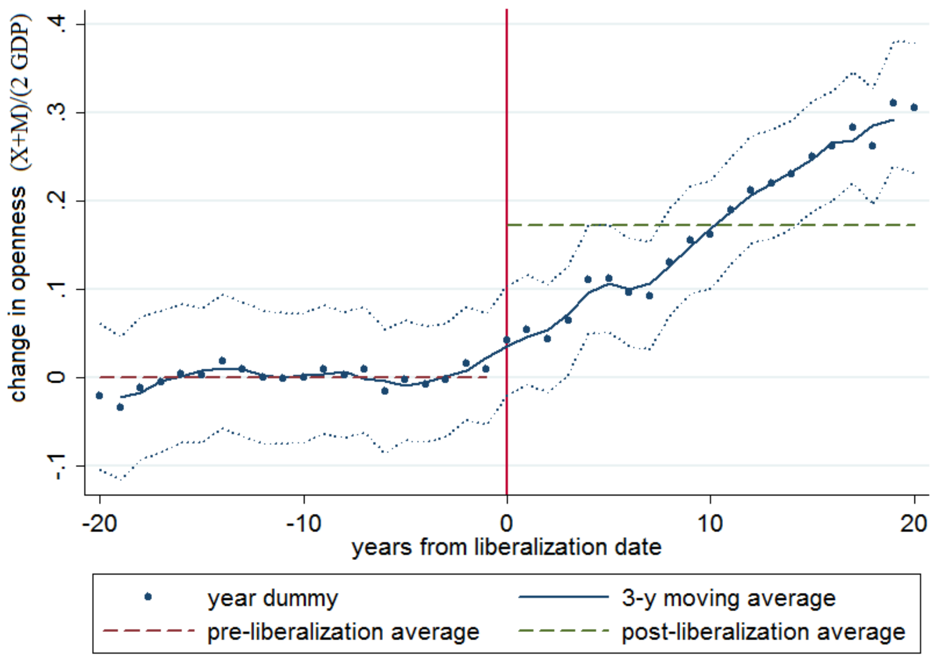
2.2 Event study results

Figure 1 displays the evolution before and after the liberalization dates of trade and inequality in the simplest way. Each variable is deviated from the country average over the pre-liberalization period. Openness is defined as the ratio of the sum of imports and exports over the GDP times two. In panel A, the evolution of openness features a kink at the date of liberalization and a steady upward trend thereafter. By contrast the evolution of inequality indexes looks non-monotonic. Inequality is on average higher after the liberalization but the evolution is hump shaped. Inequality reaches a peak 8 years after the liberalization and subsequently decreases.

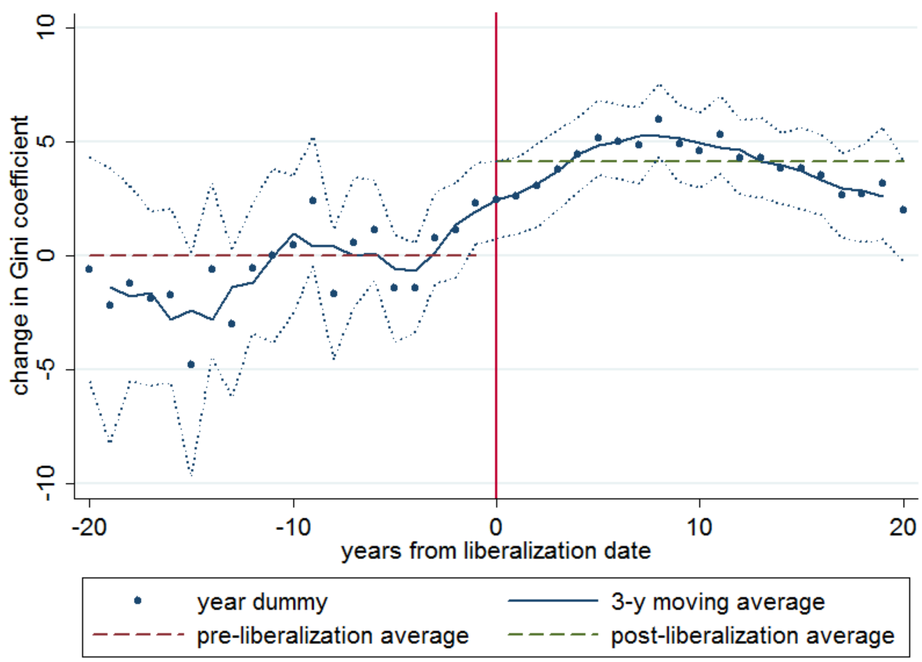
I formally estimate the height and timing of the peak and test for the non-monotonicity of

[9]Technical details about data construction can be found in the corresponding section of the appendix.

[10]More details can be found in the corresponding section of the appendix.



Panel A.



Panel B.

Figure 1: Average evolution of openness (the ratio of imports and exports over GDP) and Gini coefficients before and after liberalization dates. The dotted lines correspond to 95% confidence intervals of dummy variables. Sources: WIID, WDI.

inequality by fitting a flexible polynomial function to the data

$$gini_{c,t} = \alpha_c + \beta_c(t - t_{c,0}) + \gamma_0 \mathbb{I}_{\{t \geq t_{c,0}\}} + \sum_{n=1}^{4} \gamma_n (t - t_{c,0})^n \mathbb{I}_{\{t \geq t_{c,0}\}} + Z_{c,t} + \epsilon_{c,t} \tag{1}$$

where $t_{c,0}$ are country c specific liberalization dates, $\alpha_c + \gamma_c(t - t_{c,0})$ are country-specific trends, $\gamma_0 \mathbb{I}_{\{t \geq t_{c,0}\}}$ is a dummy that equals one in the post-liberalization period and the summation term corresponds to a polynomial of order 4 that is set to zero during the pre-liberalization period. Some additional country controls are included in $Z_{c,t}$. The dummy and the polynomial capture the response of inequality. If the response of inequality were immediate and permanent, one should expect the coefficients of the polynomial to be non-significantly different from zero while the liberalization dummy alone would capture the jump in inequality. This is a gross test of the performance of steady state models. The estimation results in table 4 shows that this is far from being the case, as figure 1.B already suggested.

Estimates of polynomial coefficients in table 4 are not easy to interpret directly but they have meaningful implications. Using simple computations, it is possible to derive that they all imply an inverted U-shape response of inequality in the 20 years following the reform. The bottom panel of table 4 provides the height of the peak and when it occurs. Results show that the peak is reached 6 or 7 years after the liberalization date. The first "F-test" line contains p-values of testing whether the height of the peak is equal to zero. The hypothesis is systematically rejected, implying that inequality does increase. However, the next line shows that 35% to 95% of the increase in inequality is estimated to be undone in the following 10 years depending on the specification. The bottom line is a one-sided test of whether the difference between inequality at the peak and 10 years after is equal to zero. I conclude from the p-values that the decrease is significantly different from zero. As shown by column (4.A-C), these results are robust to the inclusion of a large set of control variables and to the use of average population as weights.

The dynamic nature of the response of inequality could actually be explained by the steady state model of Helpman, Itskhoki and Redding (2010) if one were willing to interpret the data a sequence of steady states during which trade costs are gradually lowered. Unfortunately, I show that the explanation does not square with the data. The authors argue that trade induced changes in inequality stem from the existence of an exporter wage premium. For barely open countries, lowering barriers raises the small share of workers at premium-paying exporters and inequality rises. Conversely for very open countries with a large fraction of exporting firms, lowering barriers

VARIABLES	(4.A) Country trends		(4.B) Controls		(4.C) Pop. weights	
Polynomial terms:						
$(t - t_{c,0}).\mathbb{I}_{\{t \geq t_{c,0}\}}$	0.901***	(0.182)	0.870***	(0.199)	1.149***	(0.283)
$(t - t_{c,0})^2.\mathbb{I}_{\{t \geq t_{c,0}\}}.10^{-2}$	-10.908***	(2.080)	-9.433***	(2.336)	-13.716***	(3.165)
$(t - t_{c,0})^3.\mathbb{I}_{\{t \geq t_{c,0}\}}.10^{-4}$	37.228***	(9.221)	34.141***	(10.627)	56.397***	(13.893)
$(t - t_{c,0})^4.\mathbb{I}_{\{t \geq t_{c,0}\}}.10^{-6}$	-36.752***	(12.702)	-35.943**	(14.998)	-70.175***	(19.186)
Liberalization dummy:						
$\mathbb{I}_{\{t \geq t_{c,0}\}}$	1.189*	(0.625)	-0.296	(0.691)	0.457	(0.915)
Observations	753		609		440	
R-squared	0.950		0.962		0.964	
Country trends	YES		YES		YES	
Controls	NO		YES		YES	
Pop. weights	NO		NO		YES	
Peak increase	3.426	after 6 y.	2.260	after 7 y.	3.548	after 7 y.
F-test p-value	0.006		0.083		0.000	
Decrease	3.221	(year 6 to 17)	1.247	(year 7 to 17)	1.343	(year 7 to 17)
F-test p-value	0.000		0.083		0.162	

Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

Table 1: Within country evolution of Gini coefficients. Controls include the real GDP per capita and its square, the real GDP per capita growth rate, inflation, the openness index, the investment rate, the population growth and the share of secondary and high education completed in the population aged 15 and over. Sources: WDI, WIID, Barro and Lee (2010)

raises the large share of workers earning a premium and inequality decreases. Their model therefore predicts that the relationship between inequality and country openness is an inverted U-curve. Figure 2 shows that the relationship between changes in openness and changes in inequality is monotonically increasing in the data. Through the lens of the model, this means that countries are still on the left of the peak of the openness-inequality curve.

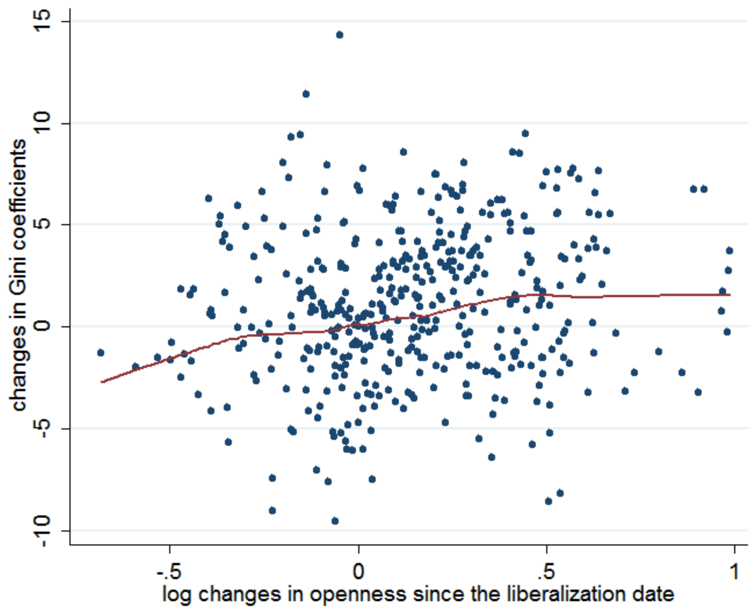


Figure 2: Average changes of Gini coefficients versus changes in openness following the liberalization dates. The red line corresponds to locally weighted regressions. Sources: WIID, WDI.

To conclude, the event study provides suggestive evidence that the evolution of inequality following liberalization dates is dynamic in nature and may feature an overshooting pattern. The present analysis should not be viewed as causal evidence of the effect of trade liberalization as it suffers from well-established limitations[11]. Therefore to pursue the examination of the dynamic response of inequality to trade reforms, I build a micro founded model of trade.

3 Recruitment and selection of workers by growing firms

The model is set in discrete time. I consider two symmetric countries, home (H) and foreign (F), in which there are two sectors. The first sector uses labor to produce and sell a homogeneous good to the other sector under perfect competition. Workers in this sector are hired in a perfectly competitive labor market and the competitive wage is the numéraire. The homogeneous good is non-tradable and can be thought of as services or capital. In the second sector, an infinite number of heterogeneous firms produce and sell varieties of the final good to consumers at home and possibly abroad. In what follows, I focus on the analysis the home differentiated good sector and its interaction with workers in a labor market characterized by search frictions. The foreign

[11]See the Rodríguez and Rodrik (2001) critic for instance.

differentiated sector is the same by symmetry.

3.1 Labor market interactions and timing

There is a mass of workers that is normalized to one. Workers have rational expectations, are assumed to be infinitely-lived and discount future income with factor $\beta \leq 1$. Workers can either be unemployed or employed in one of the two sectors. Only the currently unemployed can look for jobs. The unemployed can choose freely to look for a job in the service or the differentiated good sector. In the service sector workers get jobs without delay and earn a competitive wage.

The differentiated sector is characterized by frictions as job-searchers meet with firms according to a matching function that depends on the relative number of searchers to vacancies. Firms conduct interviews and screen workers whose ability is below a certain cutoff. Worker heterogeneity in ability follows the specification of Helpman, Itskhoki and Redding (2010) and is specific to firm-worker matches and independently distributed across workers and firms. Worker-firm specific ability is initially unobserved to all agents and revealed to firms during interviews.

The sequence of actions in every period is described in figure 3. It consists of three stages. First entrepreneurs may found start-ups by paying f_e units of the homogeneous good. A productivity x is drawn from a Pareto distribution with c.d.f $F(x)$ with shape parameter θ and lower bound x_{min}. x is revealed to entrepreneurs who then decide whether to carry on with their project. All firms (including start-ups) are hit by an exogenous exit shock with probability δ_0. Surviving firms may screen their workforce to get rid of some of their least able workers and may post detailed vacancy offers in order to grow or replace leaving workers. Vacancy posting is based on Kass and Kircher (2015) and their framework is extended to encompass the case of worker screening: job openings include the description of a contract specifying the probability of getting the job and the present value of expected wage payments. The different vacancy contracts are indexed by their type ω. Firms are assumed to comply with their posted contracts.

Simultaneously in the first stage, workers are hit by separation shocks with probability s_0. It is assumed that firms are large relative to workers in the sense that each firm employs a continuum of workers and that the law of large number applies for all probabilistic events. It implies that a fraction s_0 of employed workers exogenously quits their job every period at every firm. Unemployed and service workers decide whether to apply to a specific type of vacancy or get a job in the service sector.

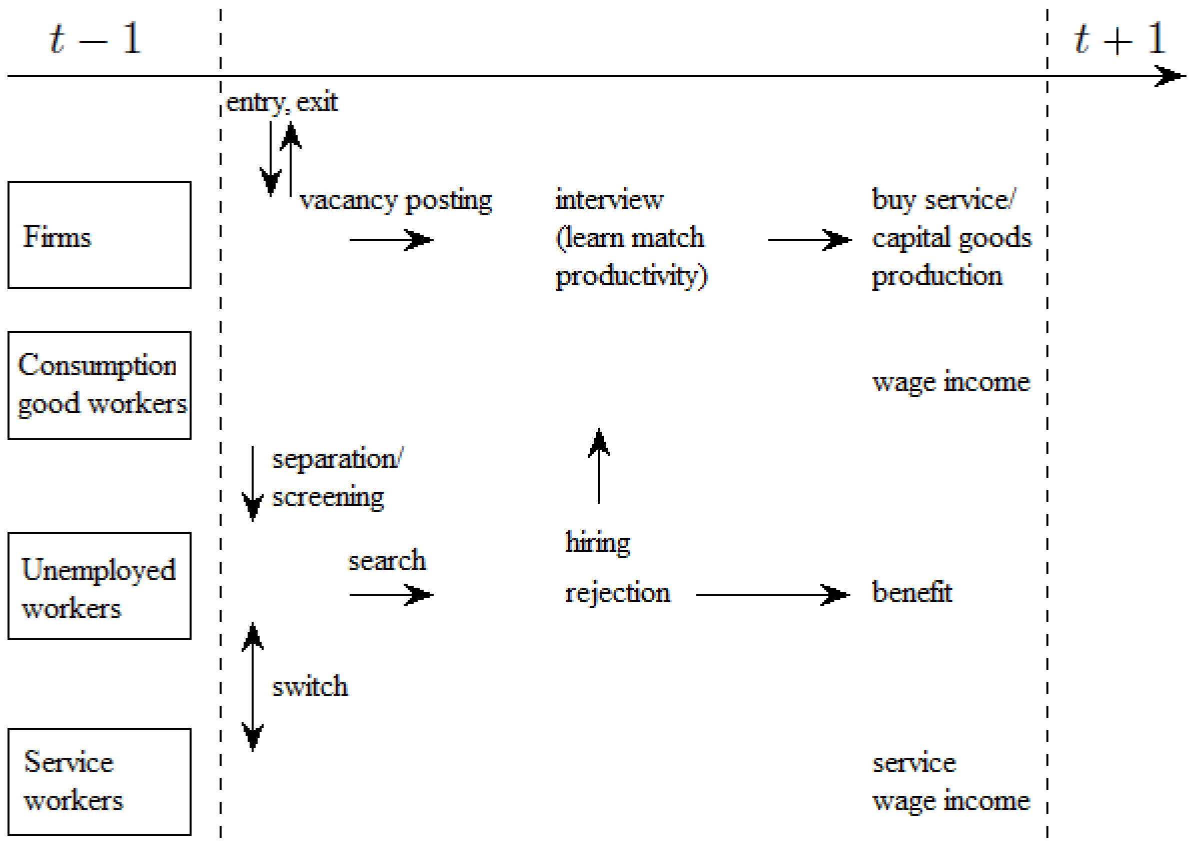


Figure 3: Within-period sequence of events and actions

In the second stage, job-seekers in the differentiated sector are interviewed by hiring firms with a probability $\mu(\omega)$ that is specific to each vacancy type. The worker-firm match specific productivity is revealed at interviews. Only a fraction $\chi(\omega)$ of interviewees are actually hired as firms only select the workers that have an ability level above the screening cutoff specified in the vacancy contracts.

In the third and final stage, unmatched job-seekers and rejected interviewees earn a benefit and wait for the next period. Inputs are purchased, production in the two sectors takes place and workers are paid according to contracts.

3.2 Labor supply

The key predictions of the model pertain to the evolution of the wage distribution and stems from the aggregation of individual wages. In this subsection, I derive the wage equation for new hires and the evolution of wages for previously employed workers. The main results are that wages are increasing with the firm level of selection and with the willingness of firms to fill-in vacant positions rapidly.

First I lay out the trade-offs faced by workers that must choose between employment options and then use arbitrage conditions to derive wage equations. Workers are assumed to have access to a complete insurance market. The value of employment options is specified in terms of the numéraire. The real value of employment options will therefore vary with the price index of aggregate consumption. Future monetary income flows are discounted using the equilibrium interest rate $R_{t,t'}$ which will be specified later.

Employment statuses are characterized by three value functions: the value of being employed in the service sector $V_{r,t}$ is indexed by r[12]; the value of being employed in the differentiated sector at a job of type ω is $V_{e,t(\omega)}$ and the value of being unemployed looking for a job ω is $V_{u,t}(\omega)$.
Service jobs – Workers incur a disutility c_r and earn a competitive wage w_r that is constant over time as it equals the constant normalized productivity of the sector.

$$V_{r,t} = w_r - c_r + \frac{1}{R_{t,t+1}} V_{r,t+1}$$

Unemployment – The labor market is segmented by vacancy type ω. Job-seekers can only search and apply to one type of vacancy at a constant disutility cost c_u. For each type, the probability

[12] I use "r" to characterize the "reservation option" rather than by s to avoid confusion with variables pertaining to screening

$\mu(\omega)$ for a job-seeker to get an interview is related to the ratio of searchers to vacancies $\Lambda(\omega)$ by a matching function $\mu(\omega) = m(\Lambda)/\Lambda$. The vacancy-filling rate is therefore equal to $m(\Lambda)$. The matching function is standard: I assume that the job-finding rate is convexly decreasing in the number of job-candidates per vacancy. It is important to note that the job-finding rate is then inversely related to the vacancy-filling rate. In other words, firms who decide to fill vacancies rapidly need many job-candidates and chose to open vacancies with low job-finding rates. To get a job with a promised option value $V_{e,t}(\omega)$, workers must also pass the interview screening process which they do with probability $\chi(\omega)$. Unsuccessful job searches leave workers with some unemployment insurance ui and the option value of searching next period. Hence the value of applying to jobs of type ω is

$$V_{u,t}(\omega) = -c_u + \chi(\omega)\mu(\omega)V_{e,t}(\omega) + (1 - \chi(\omega)\mu(\omega))\left(ui + \frac{1}{R_{t,t+1}}V_{u,t+1}(\omega)\right)$$

Because workers are free to direct their search to any type of jobs, including jobs in the service sector, the equilibrium value of searching is equalized across all vacancy types and equal to the value of working in the service sector $V_{u,t} = V_{r,t}$. The equalization mechanism is the following. If the value of searching for a job ω with a higher value $V_{e,t}$ or lower screening rates $(1 - \chi)$ entailed a higher value of applying, all job-seekers would direct their search towards it. As more and more workers apply to this job, the probability of getting an interview decreases up until the point when the value of searching is no higher than the value of working in services.

Differentiated sector jobs – At the beginning of every period, previously employed workers may lose their job either because of separations (with probability s) or because of their employer's exit (with probability δ). The probability $(1 - \eta)$ of keeping one's job is then $(1 - s)(1 - \delta)$ where s captures the probability of being separated from a firm because of screening, firing irrespective of ability, or exogenous quit shocks. The initial value of jobs in the differentiated sector is specified in vacancy contracts. Employment values result from the disutility of working c_e and the schedule of present and future wage and screening rates:

$$V_{e,t}(\omega) = w_t(\omega) - c_e + \frac{1}{R_{t,t+1}}\left[(1 - \eta_{t+1}V_{e,t+1}(\omega) + \eta_{t+1}V_{u,t+1}\right]$$

As in Kaas and Kircher (2015) there is an infinity of wage schedules that allow a firm to deliver a value $V_{e,t}(\omega)$ at the same costs because payments can be made sooner or later. Additional assumptions must be made to resolve this indeterminacy. I adapt the assumption[13] in Kaas and Kircher (2015) to the case of screening by imposing that (i) wages at stationary firms are constant over time and otherwise that (ii) expected wage changes are such that the expected value of keeping one's job is the same as in the stationary case:

$$(i) \quad V_{e,\text{sta.}}(w_t(\omega)) = V_{r,\text{sta.}} + \sum_{k\geq 0} \beta^k (1-\eta_0)^k (w_t(\omega) - c_e - (w_r - c_r))$$

$$(ii) \quad \frac{1}{R_{t,t+1}}(1-\eta_{t+1})\left[V_{e,\text{sta.}}(w_{t+1}) - V_{r,\text{sta.}}\right] = \beta(1-\eta_0)\left[V_{e,\text{sta.}}(w_t) - V_{r,\text{sta.}}\right]$$

In a stationary environment, the interest rate is $1/\beta$ and the probability of keeping one's job $(1-\eta_0) = (1-s_0)(1-\delta_0)$ comes from the combination of the exogenous exit and quit rates.

I show in the appendix that the wage of new hires is given by the following equation:

$$w(\omega) = \underbrace{\left(w_r - c_r + c_e - \frac{w_r - c_r - ui}{c_w}\right)}_{\equiv w_o} + \underbrace{\frac{1}{\chi(\omega)\mu(\omega)c_w}(w_r - c_r + c_u - ui)}_{\text{"the wage premium"}} \tag{2}$$

with $1/c_w = 1 - \beta(1-\eta_0)$. Equation (2) expresses that firms must at least pay workers the outside option w_o. In addition firms can only attract workers if they compensate them for search-related costs: the second term is a "wage premium" including $\tilde{c}_u \equiv w_r - c_r + c_u - ui$ the opportunity and net direct costs weighted by the average duration of search. Therefore the wage premium increases with the rejection rate and the average time to get an interview. It means that more productive workers that went through a tougher selection process are rewarded with higher wages. It also captures the fact that firms that want to fill their vacancies rapidly need to compensate workers for the length of their search with higher wages. From now on and without any loss of generality it will also be assumed that workers can also arbitrage between work in services and living on unemployment insurance while never searching. This simplifies exposition as it implies that $w_r - c_r - ui = 0$.

What happens to the wage of previously employed workers when firms raise the screening cutoff or when the interest rate varies? Firms are bound to deliver wage payments that are consistent with the commitments they made at the time of hiring. Assumption (ii) allows me to characterize the evolution of wages from one period to the next:

$$w_t - w_o = \frac{\beta R_{t-1,t}(1-\eta_0)}{1-\eta_t}(w_{t-1} - w_o) \tag{3}$$

[13] The authors note that their assumption corresponds to the limit case of risk-neutral firms and risk-averse workers as risk aversion vanishes.

First it means that the more productive workers being kept after more screening get a wage raise as w_t increases with the separation rate s through η_t. Second it means that the wage of workers grows with the size of the economy through changes in the interest rate[14].

To summarize, I obtained two results regarding wages. First the model predicts that wages are increasing in the vacancy-filling rate because workers need to be compensated for the corresponding lower job-finding rate. Second, wages increase in the screening rate because workers need to be compensated for the higher probability of becoming unemployed. The next step is to solve for labor demand and the behavior of firms given the labor supply equations (2) and (3).

3.3 Labor demand and the firm optimization problem

Firms make decisions about human resources and sales at every point in time in order to maximize the expected sum of discounted profits. Managing human resources means choosing which workers to separate from and deciding on the number and characteristics of job openings, including job wage offers. In each period, firm revenues are determined by the firm workforce capacity which stems from the number of workers and their average ability. Costs depend on current and past human resource management decisions.

I show in the appendix that the full dynamic problem of firms can be broken down in three steps and solved backward. In this subsection and for the sake of clarity, I use this result and present the three optimization problems and solutions sequentially starting with the last step. First, I consider the problem of choosing current vacancy and hiring wage policies to minimize recruiting costs taking as given a growth objective. Second, I consider production and sales decisions: firms choose their prices and whether to export taking as given their current workforce capacity and home and foreign product demand. Third, I consider the problem of choosing the growth profile of firm workforce capacity. The first two steps are static problems that can be studied the same way regardless of the nature of the general equilibrium (stationary or transitional)[15]. In the third step however, I assume in this section that the economy is in a steady state and I study the case of the transition path in

[14] Indeed, the interest rate reflects the relative desirability of present versus future consumption. Therefore in a growing economy, consumption-smoothing agents increase their demand for borrowing and drive the interest rate up. The above assumption then implies that wages increase.

[15] It may not be obvious that choosing wage offers in the recruiting cost minimization step can be done independently of other decisions. This property is one the results of the wage posting mechanism developed in Kaas and Kircher (2015) and its application to the present framework is derived in more details in the appendix.

section 5.

In what follows and whenever a distinction needs to be made, prime variables will denote end-of-period variables.

3.3.1 Sales and recruitment decisions

Recruiting costs minimization – Recruiting costs consist of vacancy management costs and wages.

Posting vacancies and actively searching for workers is a costly process that depends on the number of vacancies V and the number of workers available to help in the recruiting process $\check{l}$. For the sake of tractability, the cost function is assumed to take the following functional form:

$$C(V,\check{l}) = \frac{c_c}{1+\nu}\left(\frac{V}{\check{l}}\right)^{\nu} V, \qquad \text{with } \nu > 0 \tag{4}$$

This functional form has been used extensively in the theoretical labor work[16]. Vacancy costs are increasing in the number of vacancies as well as in the vacancy rate $V/\check{l}$. Costs are convex in the vacancy rate[17].

Consider the cost minimization problem[18] of a firm that wants to hire Δ workers with average productivity $\overline{\alpha}$. The distribution of worker-firm match productivity α is assumed to follow a Pareto distribution with shape parameter $\kappa > 1$ and lower bound α_{min}. The most efficient way to get workers of a certain average productivity is to minimize the rejection rate and select all workers that are above a cutoff α'_c. The Pareto distribution assumption implies that hiring workers of average ability $\overline{\alpha}$ requires selecting the fraction $\left(\frac{\alpha'_{min}}{\alpha'_c}\right)^{\kappa}$ of the best interviewees[19], with $\frac{\kappa}{\kappa-1}\alpha'_c = \overline{\alpha}$.

[16] Merz and Yashiv (2007), Kaas and Kircher (2015) are examples of papers using a similar function.

[17] These features are supported by many empirical studies. Manning (2006), Blatter, Muehlemann, and Schenker (2012) and Dube, Freeman and Reich (2010) all document increasing marginal recruitment costs. Blatter, Muehlemann, and Schenker (2012) and Barron, Berger, and Black, (1997) also find that larger firms have higher hiring rate costs. See Manning (2010) for a survey.

[18] While the problem is complicated by the addition of worker heterogeneity, the main steps for solving this problem follows Kaas and Kircher (2015).

[19] The fact that the fraction of selected interviewees only depends on the screening cutoff can be traced back to the following assumptions. First the infinitely small size of firms and the match specific nature of worker heterogeneity ensures that the productivity of workers in the unemployment pool follows the population distribution and is independent from the selection of employed workers at all other firms. Second, the fact that nobody knows worker productivity before interviews ensures that there is no self-selection.

Total recruiting costs consist of the vacancy posting costs $C(V,l)$ and the wage liability implied by the contract associated with the vacancy. The wage liability per new hire $c_w w_o + \frac{1}{\chi\mu}c_u$ is derived from equation (2) and is equal to the sum of the discounted wage payments[20]. Firms can trade-off between posting more vacancies and attracting more workers per vacancy with higher wages. Formally, the minimization problem is:

$$\min_{V,w} \quad C(V,\check{l}) + \Delta\left(c_w w_o + \frac{1}{\chi\mu}c_u\right)$$
$$\text{subject to} \quad \Delta = m(\Lambda)V\left(\frac{\alpha_{min}}{\alpha'_c}\right)^{\kappa} \quad \text{and with } \mu = \frac{m(\Lambda)}{\Lambda}, \quad \chi = \left(\frac{\alpha_{min}}{\alpha'_c}\right)^{\kappa}$$

where the condition of the program makes use of the law of large numbers to specify how many vacancies are to be posted in order to hire a number Δ of workers: V vacancies result in $m(\Lambda)V$ interviews and the selection of $m(\Lambda)V\left(\frac{\alpha_{min}}{\alpha'_c}\right)^{\kappa}$ new hires.

I assume that the matching function takes the standard form $m(\Lambda) = \frac{c_m}{1-\lambda}\Lambda^{1-\lambda}$ with $0 < \lambda < 1$. The first order condition of the cost minimization problem relates the number of vacancies and the wage offer to the desired number of hires and their productivity level:

$$\begin{aligned} V(\Delta,\alpha'_c,\check{l}) &= V_0\check{l}\left(\frac{\Delta}{\check{l}}\right)^{\frac{\gamma\xi}{\gamma-1+\xi}}\left(\frac{\alpha'_c}{\alpha_{min}}\right)^{\frac{\kappa\gamma\xi}{\gamma-1+\xi}} \\ w(\Delta,\alpha'_c,\check{l}) &= w_o + (1-\xi)(1-\beta(1-\eta_0))\frac{c_A}{\gamma}\left(\frac{\Delta}{\check{l}}\right)^{\gamma-1}\left(\frac{\alpha'_c}{\alpha_{min}}\right)^{\kappa\gamma} \\ &\text{with } \gamma = \frac{1+\nu}{1+\nu(1-\lambda)} > 1, \xi = \frac{\lambda}{1+\nu(1-\lambda)} \in (0,1) \end{aligned} \tag{5}$$

These equations make clear that the optimal behavior for a growing firm is to use the two recruiting channels by simultaneously posting more vacancies and raising wage offers[21]. These results also imply that choosing more productive workers leads to post more vacancies and to offer higher wages.

The presence of search frictions is the fundamental cause of wage dispersion and unemployment. At this point it is useful to define the "adjustment cost" function that represents total recruiting costs and includes vacancy costs together with contracted wage liabilities:

$$A(\Delta,\alpha'_c,\check{l}) = \frac{c_A}{\gamma}\Delta\left(\frac{\Delta}{\check{l}}\right)^{\gamma-1}\left(\frac{\alpha'_c}{\alpha_{min}}\right)^{\kappa\gamma} \tag{6}$$

[20]Assumption (ii) regarding the evolution of the wage of employed workers ensures that the sum of discounted wage payments is the same regardless of future firm decisions and the nature of the general equilibrium.

[21]New coefficients in these equations are functions of the model parameters: $V_0 = \left(\frac{\lambda c_u}{c_c}\right)^{\frac{1-\lambda}{1+\nu(1-\lambda)}}(1-\lambda)^{\frac{\lambda}{1+\nu(1-\lambda)}} c_m^{\frac{-1}{1+\nu(1-\lambda)}}$ and $c_A = \Gamma\frac{c_u^{1-\xi}c_c^{\xi}}{c_m^{\gamma}}$, with $\Gamma = \left(\frac{1-\xi}{\gamma}\right)^{\gamma-1}\left(\frac{1-\xi}{\gamma-1+\xi}\right)^{\xi}$

Adjustment costs are increasing in size Δ, in recruiting intensity $\frac{\Delta}{\check{l}}$ and in screening.

I show in the appendix that a fraction ξ of these costs goes to vacancy costs while the remaining part corresponds to the wage liability in excess of the outside option w_o, namely the "wage premia". The fraction of costs $(1-\xi)$ that goes to wage premia is decreasing in λ, the elasticity of the matching function to the vacancy rate, and increasing in ν, the degree of convexity of the vacancy costs. It is useful to re–write the wage equation (5) using the adjustment cost function:

$$w(\Delta, \alpha_c', \check{l}) = w_o + (1-\xi)(1-\beta(1-\eta_0))\frac{A(\Delta, \alpha_c', \check{l})}{\Delta} \tag{7}$$

Because of search frictions, the hiring behavior of firms also has implications for unemployment. Raising wage offers above market levels helps firms to grow faster but also generates longer unemployment queues. The number $u = \Lambda V$ of job-seekers looking for a given vacancy type can be related to the recruitment characteristics in a simple way thanks to the adjustment cost function:

$$u = \Lambda V = \frac{1-\xi}{c_u} A(\Delta, \alpha_c', \check{l}) \tag{8}$$

Consequently the number u' of unmatched workers associated to a vacancy type is given by $\Lambda V - \Delta$. Total end of period unemployment will be obtained by summing unmatched workers across all vacancy types. Additionally, these relations show that friction unemployment decreases with the efficiency and cost of searching (captured by c_u, c_m, and ξ) as workers favor faster matching rates. Conversely it increases with c_C the cost of posting vacancy as firms opt for longer queues and higher vacancy filling rates.

Revenue maximization across markets – Demand in the final good is defined over the continuum of horizontally differentiated varieties and takes the constant elasticity of substitution form with parameter σ. For exporters, foreign sales c^F require the production of τq^F units in order to cover for iceberg trade costs τ. On each market $M \in \{D, F\}$ (domestic or foreign), firms face the demand curve[22] $\frac{c^M}{C^M} = \left(\frac{p^M}{P^M}\right)^{-\sigma}$ given aggregate consumption C_M. Revenues on each market are:

$$r^D(q^D; \phi^D) = \left(q^D\right)^{\frac{\sigma-1}{\sigma}} \phi^D, \qquad \text{and} \qquad r^F(q^F; \phi^F, \tau) = \left(\frac{q^F}{\tau}\right)^{\frac{\sigma-1}{\sigma}} \phi^F \tag{9}$$

where $\phi^M = C^{M\frac{1}{\sigma}} P^M$ stands for market conditions on market $M \in \{D, F\}$. Exporters allocate their sales optimally. They equate the marginal revenues across markets $r_q^D = r_q^F$. This implies that the

[22] p^M is the price faced by consumers, meaning that it includes taxes and trade costs.

firm shares of production and revenues on the domestic market is $1/\Upsilon$ with $\Upsilon = 1 + \left(\frac{1}{\tau}\right)^{\sigma-1} \left(\frac{\phi^F}{\phi^D}\right)^{\sigma}$. Total revenues for exporters ($I_X = 1$) or domestic-only firms ($I_X = 0$) take the form:

$$r(q, I_X; \phi^D, \Upsilon) = q^{\frac{\sigma-1}{\sigma}} \phi^D \left(\Upsilon(I_X)\right)^{\frac{1}{\sigma}} \qquad \text{with } \Upsilon(I_X) = \Upsilon.I_X + (1 - I_X) \tag{10}$$

From now on the superscript ^{D} is dropped because countries are assumed to be symmetric.

Selling on the foreign market includes a fixed costs f_X in addition to the variable cost τ. This standard assumption in the trade literature implies that not all firms find it profitable to exports. Only the firms that sell enough to cover the fixed costs export. The formal condition is:

$$I_X = \begin{cases} 1 & \text{if } r(q, 1; \phi, \Upsilon) - f_X \geq r(q, 0; \phi, \Upsilon) \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

As in other dynamic models[23], only the firms that are productive and old enough to have accumulated enough production capacities export.

The production function of a firm of productivity x with l' workers of average ability $\overline{\alpha}$ is as in Helpman, Itskhoki and Redding (2010):

$$q = x\overline{\alpha}l'^{\rho} \tag{12}$$

This function features complementarities between worker ability in the sense that the productivity of a worker is increasing in the ability of coworkers. I assume that workers need some particular support technology to fully reach their potential productivity: firms need to pay a cost $\frac{c_S}{\psi}\overline{\alpha}^{\psi}$ every period to benefit from their workers' ability.[24]

Finally production also requires the payment of a fixed cost f_d. Per-period gross profits π are equal to the revenues net of the wage corresponding to the payments of workers' outside option, net of wage premia payments B', and net of the support technology and vacancy costs.

$$\pi = r(x\overline{\alpha}l'^{\rho}, I_X; \phi, \Upsilon) - w_o l' - B' - \frac{c_S}{\psi}\overline{\alpha}^{\psi} - C(V, \check{l}) \tag{13}$$

Net profits Π are obtained from subtracting the fixed costs from gross profit: $\Pi = \pi - f_d - I_X f_X$.

[23]Holzner and Larch (2011), Felbermayr, Impullitti and Prat (2015) and Fajgelbaum (2015) to name a few.

[24]This assumption departs from Helpman, Itskhoki and Redding (2015). Their model features a per-period cost that takes the same form which they call a "screening cost". They describe it as the cost of acquiring the technology to test and screen job-candidates. Despite the constant entry and exit of firms, their model has trivial dynamics. However if one is willing to take the dynamic nature of their model seriously, the "screening cost" interpretation is at odds with the fact that firms pay this cost every period, including when they are not hiring anybody. In the present explicitly dynamic framework, it is preferable to think of this cost as an expense that is related to the production process rather than the hiring and screening process.

3.3.2 Firm saddle path equilibrium in steady states

In this subsection I consider the case of the steady state and show how firms starting small with unselected workers build up production capacities by hiring and selecting the workers that are best suited for them.

I make two assumptions about screening. I first assume that firms set the same screening cutoff for new hires and incumbents alike, and second, that firms cannot decrease the screening cutoff. Another interpretation of the first assumption pertains to the definition of workforce productivity and could be micro-founded by an appropriate although more complex production function involving higher moments of the worker productivity distribution[25]. The first assumption simplifies the exposition of the firm problem greatly as a single variable, namely the screening cutoff, is then sufficient to characterize the distribution of worker productivity.

The second assumption is needed to formulate the firm problem. However it turns out to be not binding in steady state environments meaning that steady state results hold without the assumption. When considering the transition path triggered by a reduction in trade barriers, the assumption constrains firms to deviate from their optimal decision. In section 5, I then use an alternative approach[26].

Worker accumulation – The accumulation of workers depends on the number of separations and new hires. Separations happen for three reasons. First firms may decide on screening the least able of their workers by choosing an end-of-period screening cutoff α_c' greater than the beginning-of-period cutoff α_c: it implies the separation of a fraction $\left(\frac{\alpha_c}{\alpha_c'}\right)^\kappa$ of incumbent workers. Second, some workers exogenously quit with fixed probability s_0 irrespective of ability level. Finally, firms may also decide to scale down down their workforce without changing worker average ability and

[25]Indeed, the assumption imposes that firms improve productivity by getting rid of the least productive ones in addition to hiring better workers. Alternatively firms could improve productivity by only hiring workers and being even more selective in interviews. Hence the assumption could be justified by imposing that workforce productivity not only improves with average worker productivity but additionally decreases with the lowest worker productivity levels.

[26]These restrictions are not only important to derive results, they are also necessary to formulate the firm's problem in a parsimonious way. Keeping track of the distribution of worker ability within a firm can become very challenging in general as it may require knowledge about all the recruiting decisions of the firm since it was founded.

decide on a separation rate $s \geq s_0$. The law of motion of the number of workers is thus given by:

$$l' = \underbrace{(1-s)\left(\frac{\alpha_c}{\alpha_c'}\right)^{\kappa} l}_{\equiv \check{l}} + \Delta \tag{14}$$

The first term $\check{l}$ is the number of available workers after separations but before hiring that are involved in the recruiting process. At this point I can relate the continuation rate of jobs to the probability of being separated s and the probability of firm exit δ:

$$(1-\eta) = (1-\delta)(1-s)\left(\frac{\alpha_c}{\alpha_c'}\right)^{\kappa} \tag{15}$$

Why would firms ever fire workers irrespective of their productivity ($s > s_0$) rather than screen the least able of them, and why they would separate at all from some workers given the wage commitments they made? The answer lies in a careful analysis of costs. Firms commit to deliver a certain level of expected income to every cohort of workers. Commitments have two components: the wage premium and a "base wage" that corresponds to the workers' outside option w_o. Laid-off workers can always get the "base wage" from a job in services. This component of the commitment has not to be paid by firing firms anymore. This is the source of cost-reduction that firms may look for. Conversely firms are not able to save on the wage premium commitments: this part of the wage bill is reallocated to the remaining workers in accordance with equation (3) in order to comply with the overall expected wage commitment they made. Moreover while both firing and screening induce a reduction in the "base wage" bill, screening implies an increase in support technology costs. For this reason, declining firms may prefer firing over screening.

Wage premia liability accumulation – The dynamic nature of the model and the wage posting mechanism imply that wage payments depend on firms' past commitments. Let B be the beginning-of-period total amount of wage premium commitments accumulated by a firm. Every period, wage premium commitments vary in accordance with the wage equation (3), decrease with the number of separations and increase with new commitments. The end-of-period wage bill to be paid is:

$$B' = \underbrace{B\frac{(1-\eta_0)}{(1-\eta)}}_{\text{wage variation (3)}} \underbrace{(1-s)\left(\frac{\alpha_c}{\alpha_c'}\right)^{\kappa}}_{\text{separations}} + \underbrace{\Delta(w-w_o)}_{\text{new commitments}} \tag{16}$$

Equation (16) simplifies to $B' = B(1-s_0) + \Delta.(w-w_o)$ after substituting the continuation rate $(1-\eta)$ using (15). The wage liability carried over to the next period is $\beta R' B'$ in accordance with equation (3) but it simplifies to B' in the steady state.

I now express the lifetime problem of the firm in the steady state. The steady state environment implies that the interest rate is constant $R = 1/\beta$, and that the aggregate condition index ϕ is constant. The firm problem in recursive form is as follows:

$$
\begin{aligned}
G(\alpha_c, l, B, x; \phi, \Upsilon) \quad &= \max_{\delta, s, \Delta, \Lambda, w, \alpha'_c, I_X} (1-\delta) \left\{ r(\overline{\alpha}', l', x; \phi, \Upsilon) - C(V, \check{l}) - \frac{c_s}{\psi} \overline{\alpha}'^{\psi}_c - f_d ... \right. \\
&\qquad \left. ... - I_X f_X - B' - w_o l' + \frac{1}{R'} G\left(\alpha'_c, l', B', x; \phi, \Upsilon\right) \right\} \qquad (17) \\
\text{s.t.} \qquad & l' = (1-s)\left(\frac{\alpha_c}{\alpha'_c}\right)^{\kappa} l + \Delta, \qquad \Delta \geq 0, \;\; s \geq s_0, \;\; \delta \geq \delta_0, \;\; \alpha'_c \geq \alpha_c, \\
& \check{l} = (1-s)\left(\frac{\alpha_c}{\alpha'_c}\right)^{\kappa} l, \qquad B' = B(1-s_0) + \Delta.(w - w_o) \\
& w = w_o + \frac{\Lambda}{m(\Lambda)}\left(\frac{\alpha'_c}{\alpha_{min}}\right)^{\kappa} \frac{c_u}{c_w}, \qquad \text{and } \overline{\alpha}' = \frac{\kappa}{\kappa - 1}\alpha'_c
\end{aligned}
$$

and I show in the appendix that it simplifies to $G(\alpha_c, l, B, x; \phi, \Upsilon) = J(\alpha_c, l, x; \phi, \Upsilon) - c_B B$ with $c_B = \frac{1-\eta_0}{1-\beta(1-\eta_0)}$ and:

$$
\begin{aligned}
J(\alpha_c, l, x; \phi, \Upsilon) &= \max_{\delta, l', \alpha'_c, I_X} (1-\delta)\left\{ r - w_o l' - A - \frac{c_s}{\psi}\overline{\alpha}'^{\psi} - f_d - I_X f_X + \beta J\left(\alpha'_c, l', x; \phi, \Upsilon\right)\right\} \qquad (18) \\
\text{s.t.} \qquad & A = \frac{c_A}{\gamma}\Delta\left(\frac{\Delta}{\check{l}}\right)^{\gamma-1}\left(\frac{\alpha'_c}{\alpha_{min}}\right)^{\kappa\gamma}, \qquad l' = \check{l} + \Delta, \qquad \check{l} = (1-s)\left(\frac{\alpha_c}{\alpha'_c}\right)^{\kappa} l, \\
& w = w_o + (1-\xi)(1-\beta(1-\eta_0))\frac{A}{\Delta}, \\
& r = \left(x\overline{\alpha}' l'^{\rho}\right)^{\frac{\sigma-1}{\sigma}} \phi\left(\Upsilon(I_X)\right)^{\frac{1}{\sigma}}, \qquad \text{with } \Delta \geq 0, \;\; s \geq s_0, \;\; \delta \geq \delta_0, \;\; \alpha'_c \geq \alpha_c
\end{aligned}
$$

This statement makes use of results (6) and (10) obtained from the recruitment cost minimization and the optimal allocation of sales. The next steps are to characterize first the stationary equilibrium of firms and then the growth path that leads to it.

Proposition 1 (Firm stationary equilibrium)
In the steady state and if $\psi - \kappa\gamma \geq \frac{\sigma-1}{\sigma-\rho(\sigma-1)}$ and $\gamma\kappa\rho < 1$, the firm problem has a unique stationary equilibrium characterized by $\Delta_{SS}(x, \phi, \Upsilon), \overline{\alpha}_{SS}(x, \phi, \Upsilon), l_{SS}(x, \phi, \Upsilon)$. Moreover, the stationary employment l_{SS} and average worker productivity $\overline{\alpha}_{SS}$ are increasing in productivity x and the aggregate condition indexes ϕ and Υ. The hiring rate $\frac{\Delta_{SS}}{l_{SS}}$ and firing rate s_{SS} are equal to the exogenous quit rate s_0.

The formal proof is in the appendix and here I simply relate the above results to the assumptions of the model. More productive firms are larger because their cost advantage allows for a lower price that generates more sales. Firm employment increases with (ϕ, Υ) because these variables capture the size and the easiness of competition on each market. When a firm is at its stationary point, the hiring rate is just high enough to replace the workers that exogenously quit. Average worker ability increases with productivity and market size because of complementarities with firm size.

Corollary to proposition 1 (stationary wages)*: In the steady state, the wage offer* $w_{SS}(x, \phi, \Upsilon)$ *and vacancy rate* $\left(\frac{V}{l}\right)_{SS}(x, \phi, \Upsilon)$ *of stationary firms increase with* x *and macro-conditions* (ϕ, Υ)*.*
Proof. Apply equation (5).

In a stationary state, all firms only need to replace the fraction s_0 of exogenous quitters. Nevertheless, the number of interviews necessary to replace quitters vary with the screening cutoff. Larger firms screen more and therefore need to post relatively more vacancies. Higher screening levels must also be matched with higher wages in order to provide incentives for workers to apply. The empirical counterpart of this proposition would be the cross-section comparison of mature firms in a stable environment. The prediction is consistent with the empirical fact that larger firms pay higher wages.

In considering the equilibrium path of a firm to its stationary state, I assume that firms start small with an unselected set of workers. New firms are endowed with a number of workers $l_0 < l_{SS}$ with a distribution of ability that follows a Pareto distribution with shape parameter κ and lower bound $\alpha_{c,0} < \alpha_{SS}$.

Proposition 2 (Firm saddle path equilibrium)

*(A). [**The case of no-screening**] In a steady state economy, if* $\psi = \infty, \alpha \equiv \alpha_{min}$ *then the firm problem features a unique saddle path equilibrium. The hiring rate* $\frac{\Delta}{l}(l; x, \phi, \Upsilon)$ *is increasing with firm productivity* x *and macro-conditions* (ϕ, Υ)*, decreasing with employment* l *and independent of past wage commitments* B*. The firing policy* $s(l; x, \phi, \Upsilon)$ *is decreasing with* $(x, \phi\Upsilon)$ *and weakly increasing in employment* l*. The export decision* $I_X(l; x, \phi, \Upsilon)$ *is increasing in all variables. Successful entrants never choose to exit and* $\delta = \delta_0$*.*

*(B). [**The general case with screening**] In a steady state economy the firm problem features a local saddle path equilibrium if* ψ *is large enough. Along this path, the hiring, separation and exit rates have the same properties as in (A) above. The screening cutoff* $\alpha_c(l, x, \phi, \Upsilon)$ *is increasing in each of its variables and independent of past commitments* B*.*

The proofs are in the appendix. In the limit case where $\psi = \infty$, the support technology costs go to infinity unless firms do not screen. Hence firms don't screen and support costs are zero. This is the only qualitative difference from the general case of screening. The optimal path of a firm is to grow progressively to its stationary size. For given aggregate conditions (ϕ, Υ), firms are fully characterized by their employment and their productivity level. For a fixed (ϕ, Υ), firms' characteristics are equivalently fully determined by firm age and productivity[27]. With a slight abuse of notations, I will sometimes use age a and the quadruplet (a, x, ϕ, Υ) instead of (l, x, ϕ, Υ) as variables of the policy functions. Small young firms do not jump to their stationary employment level because of the convex vacancy costs: they save on these costs by smoothing recruitment over time. Firms progressively increase the screening cutoff along with employment for the same reason: young firms find it profitable to save on vacancy costs by screening job-candidates less intensively than in their stationary state.

A higher productivity level and better market conditions cause a firm to export at a younger age. In other words, exporters are the productive firms that are old enough to have grown to the point at which they can cover the fixed cost of exporting. There is not a single export cutoff as in Melitz (2003) but an export cutoff function of age $\underline{x}_{X,a}(\phi, \Upsilon)$ that can be obtained from equations (10)-(11) and proposition 2. Therefore exporters have the characteristics of old and productive firms. Consistently with empirical evidence, exporters are thus predicted to be larger and select better workers.

Furthermore, firms that are productive enough to export in their stationary state (when $\Upsilon > 1$) grow faster and choose higher screening levels than in autarky (when $\Upsilon = 1$). This is true even before these firms decide to export because they anticipate the fact that they will need to be bigger to serve the foreign market. This prediction echoes the findings of the literature on exporters showing that firms prepare export-market entry with organizational change (Helfat and Lieberman 2002), with productivity improvements and investments (López (2009)), and by poaching workers from current exporting firms (Molina and Muendler (2013)) if one assumes that these workers are likely to have a better match productivity.

The independence of policy variables from past commitments is a property inherited from the assumptions in Kaas and Kircher (2015): none of the firms' actions can affect the level of commitments

[27] Size is determined recursively by the history of growth rates, which depend on initial size and (x, ϕ, Υ), since age 0. Given (l_0, x, ϕ, Υ), it is equivalent to know the age or the size of a firm. This implies that a firm is small either because it is young or because it is not very productive.

and the only feedback of the latter is to impact the firm value.

The steady state assumption turns off a number of margins of adjustments. Firms would only fire if they happen to be significantly larger than their stationary employment. They would do so because of the diminishing returns in the revenue function and to save on the wage component that corresponds to the outside option. However this would never happen in a steady state environment where firms start small, grow to their optimal size and never exceed it. In the steady state, entrepreneurs may decide not to produce if they draw a bad productivity x but if they become active, they will never decide to exit and $\delta = \delta_0$. The absence of endogenous exit results from the absence of unexpected negative firm-level shocks (except for the exit shock).

Corollary to proposition 2 (wage offers and vacancy rates on the saddle path):
*(A). [**The case of no-screening**] On the saddle path, the firm wage offer and vacancy rate are decreasing with size and increasing with productivity x and macro-conditions (ϕ, Υ).*
Proof. Apply equation (5).

Proposition 2 states that firms have a higher hiring rate $\frac{\Delta}{l}$ when they are further away from their optimal stationary state. Firms achieve faster growth using two channels: they post more vacancies and attract more workers per vacancies with higher wages as shown by equation (5). In the case of no-screening or holding worker-productivity constant, wage offers are thus higher at younger firms. This prediction is supported by evidence from French data[28] and the findings of Schmieder (2013) from German data.

Without further assumption about parameter values, no results can be obtained for the evolution of wage offers at a given firm in the presence of screening because it results from two opposing effects. On the one hand young firms offer higher wages to attract more workers. On the other hand, they achieve faster growth with a lower rejection rate and having lower screening cutoffs than in the stationary state implies lower wage offers. Once I calibrate the model I find that the two effects are close to compensate each other and that the first effect slightly dominates.

4 The steady state equilibrium and the long run effects of trade

In this section, I define the steady state equilibrium and compare steady states with different levels of trade costs. I show that consumption is higher under trade than under autarky even though

[28] See section 6.

unemployment is higher. In addition, the distribution of wages in the differentiated sector is more dispersed. I show that larger wage inequality under trade results from a greater dispersion in firm average wages. In order to derive the results of this section analytically, I make some restrictive assumptions. Specifically, I consider two special cases of the model.

4.1 Steady state special cases and sectoral inequality

Special case #1 – I consider a steady state economy with no-screening ($\psi = \infty$), no impatience ($\beta = 1$) and I assume that exporters cannot choose *when* to export. Specifically I assume that a firm can only export from the beginning or not at all. I also assume that the number and characteristics of the workers of a new firm is proportionate to the firm's stationary workforce characteristics ($l_0(x; \phi, \Upsilon) = \iota.l_{SS}(x; \phi, \Upsilon)$ and $\alpha_{c,0}(x; \phi, \Upsilon) = \iota.\alpha_{SS}(x; \phi, \Upsilon)$).

Special case #2 – I consider a steady state economy where the outside option of workers is null[29] $w_o = 0$. This value can be obtained if the disutility of working in the service sector is high enough. I assume again that $\beta = 1$, that exporters cannot choose when to export and that new firms' workforce is as described in case #1.

These restrictive assumptions allow me to obtain a useful characterization of firms' saddle path.

Proposition 3 (Log-linearity of firm paths)
Define $X = x^{\frac{\sigma-1}{\sigma}} (\Upsilon(I_X))^{\frac{1}{\sigma}} \phi$ as "effective productivity" and $(l_{a,*}, \alpha_{c,a,*}, \Delta_{a,*}, w_{a,*}, r_{a,*})$ as the optimal choices of the firm with effective productivity $X = 1$ along its saddle path. In the two special cases, there exists a cutoff $\underline{x}_X^*$ such that all firms with $x \geq \underline{x}_X^*$ export ($I_X = 1$). Furthermore the optimal choices of any firm with effective productivity X along its saddle path are given by $(l_{a,*}.X^{\epsilon_l}, \alpha_{c,a,*}X^{\epsilon_\alpha}, \Delta_{a,*}X^{\epsilon_l}, w_{a,*}X^{\epsilon_w}, r_{a,*}.X^{\epsilon_r})$
*(A). In the **special case #1**, $\epsilon_\alpha = \epsilon_w = 0$, and $\epsilon_l = \epsilon_r = \left(1 - \rho\frac{\sigma-1}{\sigma}\right)^{-1}$*
*(B). In the **special case #2**, $\epsilon_\alpha = \left(\psi\left(1 - \rho\frac{\sigma-1}{\sigma}\right) - \frac{\sigma-1}{\sigma}(1 - \kappa\gamma\rho)\right)^{-1}$, $\epsilon_w = \kappa\gamma\epsilon_\alpha$, $\epsilon_l = (\psi - \kappa\gamma)\epsilon_\alpha$ and $\epsilon_r = \psi\epsilon_\alpha$*

The results in proposition 3 mean that firm variables follow the same sequence of growth rates at all firms irrespective of productivity. For a given cohort of firms however (i.e for a given age a),

[29] Helpman, Itskhoki and Redding (2010) rely on the same hypothesis throughout their model as it allows for cleaner analytical results.

the levels are scaled up or down by effective productivities. Two of the special case assumptions were particularly important in obtaining these results. First, It was necessary to restrict the timing of export because more productive firms choose to export sooner in the general case. Differences in the timing of exports would in turn generate differences in growth rates across firms of the same age: more productive firms would prepare earlier to export and would grow initially faster. Second, under the alternative assumption where all new firms have the same workforce, more productive firms start further away from their optimal stationary size. Therefore, the grow rate of new firms would be faster at more productive firms.

Proposition 3 provides a tractable characterization of the dispersion of variables across firms of the same cohort. In the special case #1, dispersion of revenues increases with diminishing returns (ρ) and the elasticity of substitution between variety (σ). In special case #2, the dispersion of wage offers increases with the dispersion of worker heterogeneity and decreases with the elasticity of the support technology cost ψ. The degree of convexity of adjustment costs also contributes to wage offer dispersion and acts as a multiplier of the effect of worker heterogeneity.

In order to characterize wage inequality among new and incumbent workers in the differentiated sector, I refer to the following three measures of inequality: total wage inequality S_T is defined as the normalized standard deviation of all wages [30], between-firm inequality S_B is defined as the employment-weighted average across cohorts of the normalized standard deviations of firm average wages, within-firm inequality S_W is defined as the employment-weighted average across firms of intra-firm normalized standard deviations of wages[31].

Firm average wage $\hat{w}_{a,X}$ and firm normalized standard deviation $nsd_{a,X}$ can be expressed in a simple way using the results of proposition 3. Specifically, I show in the appendix that the log-linearity property also applies to the size and wage of cohorts of workers: the size and wage of the cohort of workers hired at age c by a firm with productivity X and current age a can be expressed as $l'_{a,*,c}.X^{\epsilon_l}$ and $w_{a,*,c}.X^{\epsilon_w}$. Furthermore, I demonstrate:

$$\begin{aligned} \hat{w}_{a,X} &= \hat{w}_{a,*}.X^{\epsilon_w} \qquad \text{with } \hat{w}_{a,*} = \frac{\sum_{c=1}^{a} l'_{a,*,c} w_{a,*,c}}{l'_{a,*}} \qquad (19) \\ nsd_{a,X} &= \frac{1}{\hat{w}_{a,*}} \sqrt{\frac{\sum_{c=1}^{a} l'_{a,*,c} \left(w_{a,*,c} - \hat{w}_{a,*}\right)^2}{\sum_{c=1}^{a} l'_{a,*,c}}} = nsd_{a,*} \end{aligned}$$

[30] The normalized standard deviation is the ratio of the standard deviation over the average.

[31] These definitions are not a proper decomposition of inequality as $S_T \neq S_B + S_W$, but they allow for cleaner analytical results while remaining transparent. Formal definitions are in the corresponding section of the appendix.

Firm average wages depend on the firm screening level and the wage offer premia that are related to the history of firm growth rates. Because the growth rates are the same across firms with the same age, firm average wages only vary with firm screening levels through X^{ϵ_w} and with firm age as shown by equation (19).

Proposition 4 (Steady state comparison of between-firm wage inequality)
In the ***special case #2****, the dispersion of firm average wages in the differentiated sector is the same under free trade when all firms export and under autarky. Sectoral wage inequality between firm is strictly greater under trade when not all firms export than under autarky.*

In other words, proposition 4 means that the relationship between the dispersion of firm average wages S_B and trade costs has an inverted U shape. The proof of proposition 4 in the appendix borrows from Helpman, Itskhoki and Redding (2010) and the results are similar. For a cohort with a given age, the dispersion of firm average wage only depends on the dispersion of $X = x^{\frac{\sigma-1}{\sigma}} (\Upsilon(I_X))^{\frac{1}{\sigma}} \phi$ which is determined by the distribution of firm productivity and the export decision. The intuition behind proposition 4 is as follows. When a firm exports, it pays higher wages than what it would do under autarky because it screens workers more intensively. Thus, trade generates an exporter wage premium. When trade costs are high, only a small fraction of workers are employed at exporters and benefit from this wage premium. A decrease in trade costs then raises the share of workers at exporters and thereby increases wage dispersion. When trade costs are low, the majority of workers are employed at exporters and a reduction in trade costs implies that even more workers benefit from the wage premium: the wage dispersion decreases.

In the special cases, I show in the appendix that it is possible to obtain a parsimonious characterization of within-firm inequality:

$$S_W = \frac{\sum_a (1-\delta_0)^a l'_{a,*} nsd_{a,*}}{\sum_a (1-\delta_0)^a l'_{a,*}} \tag{20}$$

Proposition 5 (Steady state comparison of within-firm wage inequality)
In ***cases #1 and 2****, within-firm wage dispersion S_W is independent of the level of trade costs.*

The proof in the appendix is based on the log-linear property of firm saddle paths. Wages are pinned down by firm screening levels and the firm growth rate at the time of hiring. Within a firm, all cohorts have the same screening cutoff. Therefore the wage dispersion within any firm only results from the fact that different cohorts of workers were recruited at different times. Proposition 3 states that trade does not affect the sequence of growth rates of any firm in the steady state. Therefore trade does not affect within-firm inequality as measured by S_W. Proposition 5 is supported by evidence from Helpman, Itskhoki, Muendler and Redding (2012) which shows that there is little permanent change in the dispersion of wage within firms following the trade liberalization of the early 90's in Brazil.

4.2 The steady state general equilibrium

In this subsection I define and characterize the general equilibrium in the steady state. I start with the general case and then use the special cases to derive analytical results. In particular, I show that consumption, the unemployment rate and the share of labor in the service sector are higher under trade. I also provide comparisons with the benchmark trade model of Melitz (2003).

In order to close the model, the problem of consumers needs to be thoroughly defined. I assume that each worker has an equal participation in a national fund as in Chaney (2008). Hence profits are equally rebated among workers of the two symmetric economies. Because of the assumption of complete insurance for workers, the economy behaves as if there was a representative consumer. I assume that the representative consumer only consumes the differentiated good and that her utility function has a constant elasticity of intertemporal substitution (IES). Total utility derived from consumption given the streams of revenues and aggregate price indexes $(Y_{t'}, P_{t'})_{t'=t..\infty}$ is:

$$\mathcal{U}_t = \sum_{t'=t}^{\infty} \beta^t \frac{1 - C_t^{1-IES}}{IES - 1} \qquad \text{subject to } \sum_{t'=t}^{\infty} \frac{1}{R_{t,t'}} \left(P_t C_t - Y_t\right) = 0 \tag{21}$$

The equilibrium interest rate is given by a standard Euler equation $R_{t,t+1} = \frac{1}{\beta}\left(\frac{\phi_{t+1}}{\phi_t}\right)^{\sigma IES}\left(\frac{P_{t+1}}{P_t}\right)^{1-\sigma IES}$ where I use the definition of macro-condition $\phi_t = C_t^{\frac{1}{\sigma}} P_t$. In any steady state, $R = 1/\beta$.

When the productivity is revealed upon entry, the firms that are not productive enough to generate positive profits decide not to operate and exit. At the margin, the firm with productivity $\underline{x}$ has a value equal to 0. This is the analogue of the zero cutoff profit condition in Melitz (2003).

$$G(\alpha_{c,0}, l_0, 0, \underline{x}; \phi, \Upsilon) = 0 \tag{ZCP}$$

This condition implicitly defines a decreasing relationship between the entry cutoff and aggregate market conditions: easier conditions allow firms of lower productivity to successfully enter.

I assume free entry of new firms. It implies that the expected value of entering firms is at most equal to the sunk cost of entry f_E:

$$(1-\delta_0)\int_{\underline{x}} G(\alpha_{c,0}, l_0, 0, x; \phi, \Upsilon)dF(x) \leq f_E \tag{FE}$$

Equality is obtained when M_E the mass of firms that enters is positive. For a given level of trade costs indexed by Υ, the free entry condition implies a monotonic positive relationship between the entry cutoff $\underline{x}$ and the aggregate condition index ϕ. Thus, in the steady states with positive entry, the combination of the free entry and zero cutoff profit conditions pins down the values of $\underline{x}$ and ϕ.

Entry is related to aggregate conditions because of its impact on the labor market. More entrants implies that more workers are employed to produce the resources consumed during the firm creation process ($M_E f_E$). Firm creation is a source of labor demand that competes with existing firms for workers: a larger mass of entrants generates an increase in the average real wage, a reduction in the number of production workers and a reduction in production/consumption. From the point of view of firms in the differentiated sector, the overall effect is fully captured by a decrease in the market condition ($\phi \equiv C^{\frac{1}{\sigma}}P$).

Competition in the labor market is formally represented with the help of the market clearing condition. Total population POP must be equal to the sum of workers "attached" to every firm of every generation. Each generation of firms is characterized by its age a, a productivity cutoff $\underline{x}_a$ which is age invariant in the steady state and a mass M_a. In the steady state, the mass of a generation declines over time because of the exogenous exit shocks: $M_a = (1-\delta_0)^{a+1}M_E$. All workers are directly or indirectly "attached" to some firm of age a and productivity x, either because they work there, or because they are applying there, or because they are producing the services the firm uses. The labor market clearing condition is:

$$POP = M_E f_E + \sum_{a\geq 0} M_a \int_{\underline{x}} u_{a,x} - \Delta_{a,x} + l'_{a,x} + f_d + I_{X,a,x}f_X + \xi A_{a,x} + \frac{c_s}{\psi}\overline{\alpha}_{a,x}^{\psi} dF(x) \tag{LMC}$$

The terms under the integral represent the workers attached to every firm and consist of the unmatched job-candidates $u_{a,x} - \Delta_{a,x}$, the currently employed workers $l'_{a,x}$, and the workers employed in the production of the fixed costs $f_d + I_{X,a,x}f_X$, the support technology cost $\frac{c_s}{\psi}\overline{\alpha}_{a,x}^{\psi}$ and the vacancy cost $\xi A_{a,x}$. The dependence of these terms on (ϕ, Υ) is left implicit for clarity of exposition.

Nominal wages are pinned down by productivity in the service sector. Therefore adjustments in the real wages occur through adjustments of the aggregate price P. Given aggregate conditions

(ϕ, Υ), the aggregate price clears the differentiated good market:

$$\sum_{a\geq 0} M_a \int_{\underline{x}_a} r_{a,x} dF(x) = Y = P^{1-\sigma}\phi^{\sigma} \tag{GMC}$$

I now define the steady state equilibrium which is non trivial if there is a positive mass of entrants $M_E > 0$.

Definition of the steady state equilibrium –A steady state equilibrium is a list of policy functions $(\delta_{a,x}, \alpha'_{c,a,x}, l_{a,x}, \Delta_{a,x}, w_{a,x}, I_{X,a,x})_{\{a\geq 1, x\sim F()\}}$, an entry cutoff $\underline{x}$ and the constant macroeconomic variables (ϕ, Υ, P, M_E) such that:

1. unemployed workers make optimal decisions taking vacancy characteristics as given: (7) holds for all $(\alpha'_{c,a,x}, l_{a,x}, \Delta_{a,x}, w_{a,x})$) for all age a and $x \sim F()$ with $x \geq \underline{x}$
2. incumbent firms comply with wage commitments and maximize their value at all times taking aggregate conditions, labor supply (7), product demand and initial workforce characteristics $(\alpha_{c,0,x}, l_{0,x})$ as given: $(\delta_{a,x}, \alpha_{c,a,x}, \Delta_{a,x}, l_{a,x}, I_{X,a,x})$ solve (18) for all a and $x \sim F()$ with $x \geq \underline{x}$
3. there is positive free entry of new firms (FE) and the zero-cutoff condition is satisfied (ZCP)
4. the product and labor markets clear ((GMC) and (LMC))

Comparative steady state results – The general equilibrium conditions and the aggregation of variables across firms are greatly simplified under the assumptions of the special cases. In the special cases I can define appropriate averages of firm variables across ages in a simple way. For any productivity x, aggregate conditions ϕ, Υ and firm level variable z:

$$\overline{z}_x(\Upsilon, \phi) = \delta_0 \sum_{a\geq 0} (1-\delta_0)^a \, z_{a,x}(\Upsilon, \phi) = X^{\epsilon_z}.\overline{z}_* \qquad \text{with } \overline{z}_* \equiv \delta_0 \sum_{a\geq 0} (1-\delta_0)^a \, z_{a,*}, \quad \text{and } X = x^{\frac{\sigma-1}{\sigma}} \left(\Upsilon(I_X)\right)^{\frac{1}{\sigma}} \phi \tag{22}$$

Because I assume that the economy is in a steady state, firms across generations have the same growth profile. Therefore the above firm lifetime averages also correspond to the aggregation of firm variables across generations. Log-linearity allows me to aggregate firm variables separably across productivity levels or across generations.

Having restricted the timing of the export decision implies that the export cutoff function reduces to a single value: exporting firms are those with a productivity level x greater or equal than $\underline{x}_X$ irrespective of age. The equation that relates the fraction of exporting firms p_X to trade costs is the exact same as in Melitz (2003) up to the definition of elasticities (ϵ_π, σ). I show in the appendix that:

$$p_X = \min\left\{1, \left((\Upsilon^{\frac{\epsilon_\pi}{\sigma}} - 1)\frac{f_d}{f_X}\right)^{\frac{\theta}{\epsilon_\pi \frac{\sigma-1}{\sigma}}}\right\}$$

Thanks to log-linearity, the problem can be considered from the point of view of a representative firm of mass M_A, productivity $\tilde{x}_A(\epsilon_\pi)$ and whose variables are given by $\overline{z}_* \tilde{x}_A(\epsilon_z)^{\epsilon_z \frac{\sigma-1}{\sigma}} \phi^{\epsilon_z}$ for any $z \in \{\Delta, l', \alpha_c, r, \pi\}$, with:

$$\begin{aligned}
\tilde{x}_A(\epsilon_z) \quad &\equiv \left[\frac{1}{M_A}\sum_{a\geq 0} M_E(1-\delta_0)^{a+1}\left(\int_{\underline{x}}^{\underline{x}_X} x^{\epsilon_z \frac{\sigma-1}{\sigma}} dF(x) + \int_{\underline{x}_X}^{\infty} x^{\epsilon_z \frac{\sigma-1}{\sigma}} \Upsilon^{\frac{\epsilon_\pi}{\sigma}} dF(x)\right)\right]^{\frac{1}{\epsilon_z}\frac{\sigma}{\sigma-1}} \\
M_A \quad &\equiv \sum_{a\geq 0} M_E(1-\delta_0)^{a+1}(1-F(\underline{x})) + \sum_{a\geq 0} M_E(1-\delta_0)^{a+1}(1-F(\underline{x}_X))
\end{aligned}$$

This approach was initially pursued in Melitz (2003) and I solve for aggregates in the appendix by following similar steps.

I define welfare in the steady state Ω as consumption per capita and per period. I show in the appendix that in the special cases, the price index and welfare are related to the entry cutoff $\underline{x}$ and to trade costs in the following ways:

$$\begin{aligned}
\Omega \quad &= \quad \left(\frac{1}{POP}\right)^{\frac{1}{\sigma-1}} \left(\frac{\overline{r}_*}{b_*\left(1+b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{\overline{\pi}_*}{f_d}\right)^{\frac{1}{\epsilon_\pi}\frac{\sigma}{\sigma-1}} \underline{x} \qquad (23) \\
P \quad &= \quad \left(\frac{b_*\left(1+b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)}{\overline{r}_* POP}\right)^{\frac{1}{\sigma-1}} \frac{1}{\underline{x}} \left(\frac{f_d}{\overline{\pi}_*}\right)^{\frac{1}{\epsilon_\pi}\frac{\sigma}{\sigma-1}}
\end{aligned}$$

where b_* is a constant, $b(.)$ is null in the first special case and $b(.)$ is strictly lower under trade than under autarky in the second case.

Proposition 6 (Comparison of welfare and prices across steady states)
*In **case #1**, welfare is decreasing in the fixed and variable costs, the price index increases in both. In **case #2**, welfare is higher and the price index is lower under trade than under autarky.*

The proof of proposition relies on equation (23) and on changes in average firm productivity. As trade barriers fall, exporters make more profits from foreign sales. "Effective productivity" increases for exporters. Expected profits for entrants improve. In equilibrium, free entry implies that these expected gains must be offset by a lower probability of successful entry and consequently by a higher entry cutoff. Hence average firm productivity increases and so does welfare. In the special case #2, there is an additional effect. Trade fosters more screening as exporters increase in scale and get larger returns to screening. The associated productivity gains reflected by a lower $b()$ term are then another source of welfare increase.

The lower entry cutoff is mirrored by a tightening of market conditions as captured by a decrease in ϕ and a decrease in the price index. Therefore "effective productivity" falls for all domestic firms in contrast with exporters.

Labor allocation and unemployment – The sectoral unemployment rate at the end of each period U is the ratio of unmatched job-searchers to the sum of job-searchers and employed workers. I also define LF_p the labor share of production workers, including unemployed job-seekers. I show that:

$$\begin{aligned} U &= 1 - \frac{\overline{l'}_* \left(1 + b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)}{\overline{u}_* + (\overline{l'}_* - \overline{\Delta}_*)\left(1 + b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)} \\ LF_p &= 1 - \frac{\overline{\pi}_* + \xi\overline{A}_* + \frac{c_S}{\psi}\overline{\alpha}_*^{\psi}}{\overline{\pi}_* + \xi\overline{A}_* + \frac{c_S}{\psi}\overline{\alpha}_*^{\psi} + \overline{u}_* + (\overline{l'}_* - \overline{\Delta}_*)\left(1 + b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)} \end{aligned}$$

Proposition 7 (Steady state comparison of unemployment and labor allocation)
*In the **special case #1**, the sectoral unemployment rate U and the labor share of the differentiated sector LF_p are independent of the level of trade costs.*
*In the **special case #2**, the sectoral unemployment rate U and the employment share of the service sector $(1 - LF_p)$ are higher under trade than under autarky.*

Sectoral unemployment is related to the growth rates and the screening intensity of firms because faster growing and more screening intensive firms generate longer unemployment queues. Sectoral unemployment is constant in the first special case because the growth rates of firms in steady states are unchanged by trade. However in the special case #2, firms under trade screen more because they are on average bigger and this generates more unemployment[32]. Similarly the share of service sector workers is larger in the open economy because more screening requires spending more on the support technology and allows firms to produce more with a smaller share of production workers.

[32] Alternatively, one could have defined the unemployment rate as the ratio of end of period unemployed workers over total population. In this case, I show that the same result applies: unemployment is higher under trade than under autarky.

5 The equilibrium path following a reduction in trade costs

In this section I examine the effects of a once and for all reduction in trade costs. Specifically I define the equilibrium path along the transition from the steady state closed economy to the steady state economy with trade. The out-of-steady-state environment presents some challenges that I discuss. I then propose a new set of assumptions allowing for a full characterization of the individual responses along the transition path. The numerical results of the following section will confirm the generality of the results.

The firm problem in the transition path – The opening to trade has two effects. On the one hand it provides new market opportunity as reflected by the corresponding increase in Υ. On the other hand, it provides new sources of competition as reflected by the drop in ϕ in the new steady state.

Domestic firms that were in their stationary state at the time of the reduction in trade costs t_0 suddenly find themselves with a size and an average workforce productivity that are strictly above their new stationary levels ($l'_{t_0} > l'_\infty$ and $\alpha'_{c,t_0} > \alpha'_{c,\infty}$). Therefore the model implies that they should lower their screening level. Allowing these firms to gradually adjust their screening level would make the definition of the lifetime optimization problem extremely complicated: one would have to keep track of every cohort of new hires and their characteristics in order to compute average worker productivity.[33] To keep the problem tractable in terms of notations and to allow for the computation of numerical predictions with reasonable resources, I make the following assumption:

Assumption 1 – Declining firms are constrained to choose a new screening threshold equal to the new stationary screening level $\alpha'_{c,\infty}$ and keep it constant until they exit: $\alpha'_{c,\infty} \geq \alpha'_c \geq min(\alpha_c, \alpha'_{c,\infty})$

[33]When $\alpha'_{c,\text{new hires}} < \alpha_{c,incumbent}$ the new distribution of worker ability is no easy to characterize anymore as it is now the sum of Pareto distributions with different thresholds.

The new firm optimization problem is the generalization of equation (17) to a richer environment:

$$
\begin{aligned}
G(\alpha_c, l, B, x; \Phi) \quad = \quad & \max_{\delta, l', \alpha'_c, I_X} (1-\delta) \Big\{ r - w_o l' - B' - \xi A - \frac{c_s}{\psi} \overline{\alpha}'^{\psi} - f_d - I_X f_X ... \qquad (24) \\
& \qquad ... + \frac{1}{R'} G\left(\alpha'_c, l', \beta R' B', x; \Phi\right) \Big\} \\
\text{s.t.} \qquad & \Delta \geq 0, \;\; \delta \geq \delta_0, \;\; s \geq s_0, \quad \alpha'_{c,\infty} \geq \alpha'_c \geq min(\alpha_c, \alpha'_{c,\infty}) \\
& A = \frac{c_A}{\gamma} \Delta \left(\frac{\Delta}{\check{l}}\right)^{\gamma - 1} \left(\frac{\alpha'_c}{\alpha_{min}}\right)^{\kappa\gamma} \text{ with } \check{l} = (1-s) l \left(\frac{\alpha_c}{\alpha'_c}\right)^{\kappa} \\
& w = w_o + (1-\xi)(1-\beta(1-\eta_0)) \frac{A}{\Delta} \\
& r = \left(x \overline{\alpha} l'^{\rho}\right)^{\frac{\sigma-1}{\sigma}} \phi \left(\Upsilon(I_X)\right)^{\frac{1}{\sigma}} \qquad \text{with } \overline{\alpha} = \frac{\kappa}{\kappa - 1} \alpha'_c \\
& l' = (1-s) \left(\frac{\alpha_c}{\alpha'_c}\right)^{\kappa} l + \Delta, \qquad B' = B(1-s_0) + \Delta.(w - w_o) \\
& R' = \frac{1}{\beta} \left(\frac{\phi'}{\phi}\right)^{\sigma IES} \left(\frac{P'}{P}\right)^{1 - \sigma IES}
\end{aligned}
$$

The value function depends on the entire history and future path of aggregate conditions as represented by $\Phi = (\phi, R)_{t=-\infty..+\infty}$. Firms may decide to reduce the number of workers they employ through exogenous attrition ($\Delta < s_0 l$) or firing ($s > s_0$). At this point, I still assume that firms still pay their incumbent workers in accordance with initial commitments at all times.

However, the unexpected reduction in trade costs may result in wage commitments that some distressed firms cannot deliver anymore. There is a tension in the model between the commitment of firms to pay the promised wages and the unexpected negative impact of the reform. Specifically, if the trade-induced decrease in ϕ is large enough, the associated decline in the revenues of some domestic firms may be so large that they cannot cover their costs, including the wage payments. In other words the value of these firms can become negative $G_{t_0}(\alpha_c, l, B, x; \Phi) < 0$. These firms have no other choice than exit or renegotiate the wage of their incumbent workers and both options are a violation of initial commitments[34].

For some firms, there may be wage cuts that can both restore the firm value to non-negative levels while preserving a positive premium over the worker outside option. Whenever this is the case, renegotiation is a Pareto improvement over firm exit. If there is a renegotiation, the relative bargaining powers of the two sides, namely the firm and its workers, still needs to be determined.

[34]Voluntary firm exit is a violation of wage commitments as the workers would all lose the wage premium they earned when they go back to the unemployment pool.

Assumption 2 – All firms with values that are negative after having set wage premia commitments to zero exit: $\delta'_{a,x,t_0} = \mathbb{I}_{G_{t_0}(\alpha'_c,l',0,x;\Phi)<0}$. Otherwise firms that are such that $G_{t_0}(\alpha'_c, l', B', x; \Phi) < 0 \leq G_{t_0}(\alpha'_c, l', 0, x; \Phi)$ renegotiate wages with their incumbent workers. Specifically, wage cuts $cut'_{t_0,a,x}$ amount to the minimum decreases ensuring that the value of renegotiating firms is non-negative: $G_{t_0}(\alpha'_c, l', B' - cut'_{t_0,a,x}, x; \Phi) = 0$. The workers at a renegotiating firm (a, x) are all assumed to get the same percentage cut.

Assumption 2 means that the bargaining power of firm owners in the renegotiation is null. This is a conservative hypothesis in the sense that it minimizes wage cuts at already low-paying firms and therefore minimizes the change in inequality that results from renegotiation. In general, the importance of nominal wage cuts is a source of debate in the data. The wage cuts predicted by the model are in real terms: therefore they would not necessarily correspond to nominal decreases if other independent developments on the money market led to inflation. Furthermore there is some supportive evidence of nominal wage cuts under special circumstances in the literature. Akerloff, Dickens and Perry (1996) provide a survey of studies on wage rigidity. They document evidence of wage cuts in recessions or at firms which experience particularly harsh difficulties as is the case in the model.

I now define the equilibrium path resulting from a one-off reduction in trade costs at the end of t_0. I restrict my attention to the equilibrium in which there is a positive mass at all times during the transition. This ensures that the free entry condition is always binding.

Definition of the transition path equilibrium –An equilibrium path following a reduction of $(\tau_0, f_{X,0})$ to $(\tau_\infty, f_{X,\infty})$ at $t = t_0$ is a list of exit and renegotiation decisions at the time of the shock $(\delta'_{a,x,t_0}, cut'_{a,x,t_0})_{\{a>0,x\sim F()\}}$, policy functions $(\delta_{a,x,t}, \alpha'_{c,a,x,t}, l_{a,x,t}, \Delta_{a,x,t}, w_{a,x,t}, I_{X,a,x,t})_{\{a\geq 1,x\sim F(),t>t_0\}}$, a sequence of entry cutoffs $\underline{x}_{t>t_0}$ and a sequence of variables $(\phi_t, P_t, M_{E,t})_{\{t\geq t_0\}}$ such that:

1. $(\alpha'_{c,a,x,t_0}, l_{a,x,t_0}, \Delta_{a,x,t_0}, w_{a,x,t_0}, I_{X,a,x,t_0})_{\{a\geq 1,x\sim F()\}}$ and $(\phi_{t_0}, P_{t_0}, M_{E,t_0})$ is a steady state equilibrium with $(\tau_0, f_{X,0})$.
2. At $t = t_0$, firms with negative value negotiate wage cuts cut'_{a,x,t_0} or exit δ'_{a,x,t_0} in accordance with assumption 2
3. unemployed workers make optimal decisions taking vacancy characteristics as given: (7) holds for all $(\Delta_{a,x,t}, l_{a,x,t}, w_{a,x,t})$ for all a, all $x \sim F()$ and all $t > t_0$ such that $\delta_{a,x,t} = 0$
4. incumbent firms comply with wage commitments and maximize their value at all $t > t_0$ taking aggregate conditions, labor supply (7), product demand and initial workforce characteristics

$(\alpha'_{c,a,x,t_0}, l_{a,x,t_0}, \Delta_{a,x,t_0}, w_{a,x,t_0}, I_{X,a,x,t_0})$ and $(\alpha_{c,0,x}, l_{0,x})$ as given: $(\delta_{a,x,t}, \alpha_{c,a,x}, \Delta_{a,x}, l_{a,x}, I_{X,a,x})$ solve (33) for all a, all $x \sim F()$ and all $t > t_0$ such that $\delta_{a,x,t} = 0$

5. there is positive free entry of new firms (FE) at the zero-cutoff condition (ZCP) is satisfied for all $t > t_0$
6. the product and labor markets clear for all $t > t_0$ ((GMC) and (LMC))

In order to obtain analytical predictions for the transition path, I consider the special case in which the inverse elasticity of intertemporal substitution takes a particular value:

Special case #3 – I assume $IES\sigma = 1$ and that the reduction in trade costs is "not too big".

In the special case #3, I show in the appendix that there is a solution such that the aggregate market condition drops instantaneously to the future steady state value: $\phi_t = \phi_\infty$ for all $t > t_0$. This result is a consequence of the fact that the firm problem (33) only depends on the index ϕ in that case. Hence the entry cutoff also jumps instantaneously to its long run level because this condition is purely forward looking. All other variables are solved forward given the allocation of resources at time t_0. In particular, the mass of entrants evolves according to the labor market clearing condition (LMC). I need to assume that the reduction is "not too big" in order to ensure that there is always a positive of entrants. The price index (in terms of the numéraire) adjusts to clear the product market (GMC).

New entrants behave exactly as they would in the future steady state. They start small with low screening levels and progressively expand according to the policy functions of proposition 2.

The individual transitions of incumbent firms are fully determined by productivity and the age at the time of the shock. Patterns can be studied with the help of cutoff curves $x(a)$. Figure 4 graphically illustrates the discussion below and proofs are provided in the appendix.

The firms that exit after the announcement of the reforms are the firms that have the lowest values even after a hypothetical wage renegotiation. These firms are the youngest and the least productive of firms. Specifically, there exists an exit cutoff function $x_{exi}(a)$ decreasing in age a: all firms with productivity and age such that $x < x_{exi}(a)$ exit. Unproductive firms experience declines in revenues that do not allow them to cover the fixed cost of operation anymore. This is more true for young unproductive firms that still have small production capacity and would have to pay prohibitive recruitment costs were they to expand in size and revenues.

Among the firms that do not exit, there are two cutoff functions $x_f(a)$ and $x_{gth}(a)$ increasing in a that determine firms evolution. Domestic firms only suffer from the reform as they face an increase

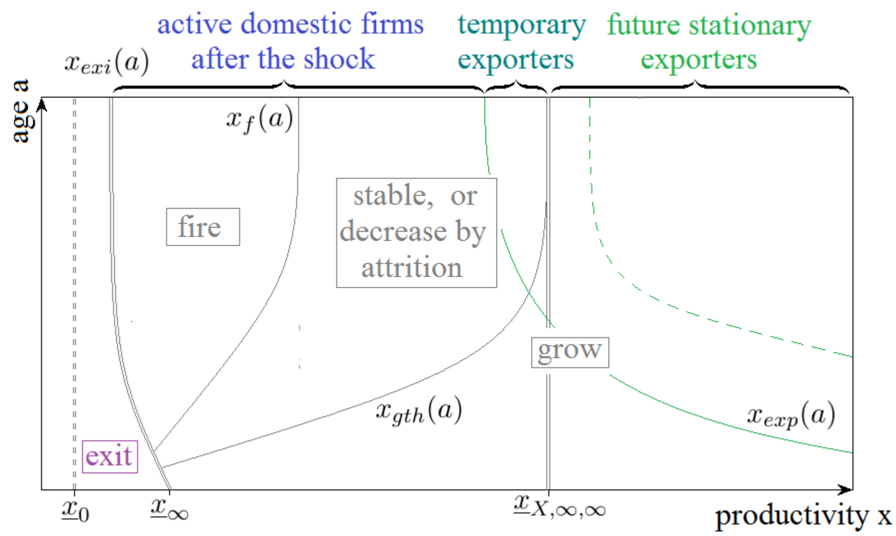


Figure 4: Illustrative diagram of cutoff functions in the period after the reduction in trade costs. The plain gray line delimit the areas (from left to right) where firms exit, fire, stay stable or decline by attrition, grow. The firms above the green line export. The dashed lines are respectively the exit and the export cutoffs before the reform.

in competition from expanding exporters and foreign firms and as they do not benefit from lower trade costs. In steady state models of trade, the discussion only focuses on a single productivity cutoff value. In the current dynamic setting, age also matters. Some young domestic firms are small enough that reaching their new optimal stationary state still requires them to grow. Hence, the older and less productive firms decline when x and a are such that $x < x_{gth}(a)$. If these firms still hire some workers to mitigate the effect of quits, they offer very low wage since they do not need to attract many, and since they may have reduced their screening level. It may be the case that the oldest and least productive of firms ($x < x_f(a)$) are so much above their new optimal stationary size that they chose to fire some workers irrespective of their productivity levels in order to save on the "base wage" component of labor costs ($w_o l'$). The young and more productive firms with x and a such that $x \geq x_{gth}(a)$ grow.

The reduction in trade costs has both short and long term effects on the export decision of firms. The firms that export in the period $t_0 + 1$ that follows the reform are firms with x and a such that $x \geq x_{exp}(a)$ where the cutoff function is decreasing in age a. Indeed, equation (11) implies that it is the old and productive firms that export.

Among the exporting firms at $t_0 + 1$, there can be firms that will decide not to export in their new stationary state. These firms grew large before the reform when the environment was not as competitive. They are now large enough to cover the fixed export cost. If they chose to decline, these firms will stop to export as they reach their new stationary size. The newborn firms with the same productivity do not chose to grow as large and do not export in the more competitive environment.

The dependence of firm variables by age and productivity is obviously different from what is the case in the future new steady state. This is what drives the temporary effects of the reform. Let $\underline{x}_{X,a=\infty,t=\infty}$ be the export cutoff of stationary firms in the steady state with lower trade costs. For all $t > t_0$, domestic firms (x, a) with productivity x lower than $\underline{x}_{X,a=\infty,t=\infty}$ have lower growth rates and, if still hiring, they offer lower wage premia than firms (x, a) in the new steady state. Before the reform, these firms were on a saddle path equilibria featuring larger sizes at all ages. They now grow more slowly or decline in order to reach their lower new stationary state.

By contrast, firms with $x \geq \underline{x}_{X,a=\infty,t=\infty}$ that were in their stationary state before the reform now decide to grow by hiring better workers faster in order to reach their new optimal state. They screen the least able of their workers, while other workers get wage raises as they raise their screening cutoff. These firms' growth rates are higher and their screening levels are lower than at their counterparts of similar productivity and age in the new steady state.

The model implies that workers with the same ability can experience different outcomes. Workers of any ability that are employed at firing or exiting firms lose their job while similar workers at growing firms keep theirs. Furthermore, worker outcomes also result from the *interaction* between firm and worker productivity. The workers employed at an exporter that are close to the screening cutoff at the time of the reform have a higher probability of becoming unemployed than workers of similar ability at a non-firing domestic firm because the domestic firm does not raise its screening level in contrast with the exporter.

The model has novel predictions for the evolution of the wage dispersion within firms. The model predicts that old exporters experience an increase in the dispersion of wages among their workers. Firms that were close or at their stationary state just before the reform have not experienced big adjustments for some time. Their incumbent workers have similar wages as they were all hired under the same small growth rate. Upon the shock, exporters suddenly need to speed growth with high wage offers and the premia they offer generates some internal wage dispersion. This prediction is consistent with the findings of Frias, Kaplan and Verhoogen (2012) about the Mexican exporting firms that experienced the positive shock of the 1994 peso devaluation. Furthermore, they also find that the dispersion increase is temporary. This is also consistent with the model which predicts that wage premia offer is only needed during the transition. The within-exporter wage dispersion increase is predicted to vanish as workers hired at a premium quit and are replaced by new hires with lower premia.

The type of firms that renegotiate wages is ambiguous because both age and productivity have two opposite effects. Older and more productive firms have accumulated more and better workers and therefore, they are more likely to maintain profitable operations. However, older and more productive firms also have accumulated larger wage liabilities and this makes the necessity of wage negotiation more likely. Thus the type of firms that renegotiate depends on the relative importance of these two effects. The calibrated version of the model of section 6 predicts that the former effect dominates: young and less productive firms are then predicted to be more likely to renegotiate.

Depending on parameter values, some of the cutoff curves may connect at different points and some of the firm behaviors presented may not be observed[35]. The variety of firm experiences render the economy-wide analysis intractable analytically and I turn to numerical solutions to obtain more results.

6 Counterfactual exercise

In this section I consider the counterfactual experiment of having a country opening to trade. I obtain numerical results from the calibrated model for aggregates in the general case. First I briefly present the data that I use to recover some specific parameters. Next I outline the sources from which I draw the standard parameter values and then focus on the estimation and calibration of the non-standard parameters of the labor market. Lastly I obtain and discuss the results of the calibrated model on the transition path following a liberalization reform.

[35] While it is possible to prove that $x_X(a) \geq x_{exi}(a)$, and that $\underline{x}_{X,\infty,\infty} \geq x_{gth}(a) \geq x_f(a) \geq x_{exi}(a)$ for all a, it is not possible to locate the cutoff curve of temporary exporters $x_X(a)$ with respect to $x_{gth}(a)$ and $x_f(a)$ and when these inequality are strict.

6.1 Estimation and Calibration

Data – For some parts of the calibration, I will use a combination of datasets from France covering the period 1995-2007. First the matched employer-employee DADS[36] is based on mandatory employer reports of the earnings of each employee. While it covers the full universe of the private sector I only use information on the individuals that worked once for a manufacturing firm. The unit of observation is the match between a firm and an employee over two years. Workers cannot be followed for more than two consecutive years. However, a firm identifier allows me to follow firms over the entire period. The DADS provides information on workers' gender, age, and location as well as information on jobs' occupation type, earnings and full or part time status. I match the DADS with the EAE[37] census of manufacturing firms of 20 employees and over. This allows me to complement information on the workforce with firm characteristics including value added, profits, capital, and investment. Lastly, firm level quantities and values of imports and exports by partner country and product category is matched from customs data[38].

Calibration overview – In the counterfactual experiment, I consider a country that is assumed to be initially under autarky and examine the response of a trade reform. I assume that the trade reform is a once and for all reduction in trade costs such that 10% of firms export and they export 30% of their sales in the future steady state[39].

The model presents a number of standard features that are characterized by parameters that have been extensively studied in the literature. Therefore and whenever readily available I use standard values and calibration methods. Table 2 summarizes the parameter values and the corresponding source or the corresponding moment of the data used in the calibration. I chose periods to correspond to years. The discount rate $\beta = 0.95$ and the intertemporal elasticity of substitution $IES = 2$ are standard values. I chose the elasticity of substitution between varieties to be $\sigma = 4$[40]. The cofficient of diminishing returns to labor $\rho = 0.7$ is set to obtain a wage to valued added ratio of 50%.

[36]The Déclarations Annuelles de Données Sociales (DADS) dataset is put together by the French National Institute of Statistics and Economics Studies (INSEE)

[37]Enquête Annuelle d'Entreprise also collected by the INSEE

[38]The Données import/export du commerce extérieur are collected by the Direction Générale des douanes et des droits indirects.

[39]These numbers correspond roughly to the characteristics of trade in France.

[40]These values are similar to the ones chosen by Ghironi and Melitz (2005)

I follow the methodology of Head, Mayer and (2014) for the calibration of the firm productivity distribution and the levels of trade costs in the terminal steady state. Specifically I recover the shape parameter from the right tail of the distribution of value added of French manufacturing firms. Under the assumption of Pareto, my model shares with Melitz (2003) the property that the entry cost f_E has no other impact than determining the fraction of successful entrants. The operation costs f_d and the coefficient of the technology costs c_S can be arbitrarily chosen by choosing units of the match and firm productivities[41]. I also normalize population to one and use the outside option $w_o = 1$ as the numéraire. The exogenous exit rate of firms $\delta_0 = 2.5\%$ is based on the observed exit rate in the EAE data[42].

The remaining parameter values pertain to characteristics of the labor market. The triplet (κ, ρ, ψ) is over-identified as the distribution of match-specific productivity cannot be directly observed from the data[43]. Therefore I chose to normalize $\psi = 7$ and this value is such that none of the constraints outlined in the theory section are binding. I directly obtain the worker-firm separation rate $s_0 = 0.19$ from the data by assuming it corresponds to the fraction of separations when there is no firm growth. I chose the search cost coefficient $c_u = 4$ in order to obtain a sectoral unemployment rate of 9%. I chose the share of vacancy costs to wage offers $\xi = 10\%$ in order to match the magnitude of recruitment costs surveyed in Manning (2011).

As a result there are three remaining parameters to calibrate: γ the elasticity of wage offers with respect to firm growth, c_A the coefficient of the adjustment cost function, and κ the shape parameter of the Pareto distribution of match productivity. The next subsection is dedicated to show how I directly estimate γ from the data as it is an important in shaping the dynamics. Lastly I calibrate c_A and κ in order to match two relevant empirical moments for the model. The first moment is the elasticity of firm average wage with respect to firm value added. In the EAE data, firm average wage increases by 10% when firm size is doubled. Second I use the elasticity of the separation rate with respect to firm growth. After controlling for firm characteristics as well as firm and time effects,

[41] I chose $f_d = 0.1$, and $c_S = 0.05$ for convenience of exposition

[42] In the EAE data, 7% of firm identification numbers disappear every year but that rate includes administrative restructuring and relabeling of firms. I then correct this rate according to the findings of Hethey-Maier and Schmieder (2013) showing that only about one third of identification number terminations corresponds to real exits.

[43] Intuitively, the elasticity of the cost function ψ, the strength of complementarities and the dispersion of abilities have offsetting effects on the net returns to screening: greater net returns are obtained from raising the cutoff if the cost-elasticity is lower, if ability dispersion is greater or if complementarities are stronger

Parameters			Source or data target	Target
Labor market				
γ	1.1	Elasticity of adjustment costs	Estimation	
κ	1.05	Shape of match productivity distribution	Separation-growth rate elasticity	0.13
ξ	0.1	Vacancy costs to wage premia ratio	Manning (2011)	
c_A	5	Adjustment cost function coefficient	Wage-size elasticity	0.1
c_u	4	Worker search cost	Unemployment rate	0.09
s_0	0.19	Worker exogenous separation rate	Worker separation rate	
Melitz-type firm/trade				
θ	2.5	Shape of firm productivity distribution	Head et al. (2014) QQ method	
τ	1.3	Iceberg trade cost	Share of exporters' foreign sales	0.3
f_X/f_d	1.4	Export fixed cost	Share of exporters	0.1
Standard macroeconomics				
β	0.95	Discount rate	Average real interest rate	
δ_0	0.025	Firm exogenous exit rate	Firm exit rate	
σ	4	Elasticity of substitution between variety	Ghironi and Melitz (2005)	
ρ	0.7	Decreasing returns to labor	Wage share in firm value added	0.5
IES	2	Elasticity of intertemporal substitution	Havranek et al. (2015)	

Table 2: Parameter values and calibration strategy.

I find an elasticity of 13%. I am able to match these two moments as the parameters κ and c_A directly govern the returns to screening and how it translates into wage premia. Precisely, a lower κ means a larger dispersion in worker ability and greater screening returns: raising the cutoff results in a lower fraction of screened out workers. Ultimately a larger κ also results in more screening at larger firms and this is reflected by higher wages. Larger adjustment costs steming from a higher c_A translate into larger wage premia but also discourage screening.

Wage elasticity to firm growth – With a simple extension, the model can encompass worker observable characteristics. I assume that individuals are endowed with characteristics that result in heterogeneity in efficiency units of labor e_i that are orthogonal to ability. These efficiency units can be thought of as differences in the supply of hours. I additionally assume that the vacancy costs are proportionate to the efficiency units of workers. In that case the analysis of the theory section carries through and the wage equation can be extended to:

$$w_{i,t} - w_{o,t} = e_{i,t} \frac{(1-\xi)}{\gamma} \frac{c_A}{c_w \alpha_{min}^{\kappa\gamma}} \left(\frac{\Delta}{\breve{l}}\right)_{t,j}^{\gamma-1} (\alpha_{j,t})^{\kappa\gamma}$$

In log, this equation can be taken to the data in a few steps. First I partial out the effect of observable worker characteristics and obtain the residual wage of new hires $\ln \overline{w}^{hires}_{\text{res.},j,t}$ by computing firm averages. The wage equation at the firm j level becomes:

$$\ln \overline{w}^{hires}_{\text{res.},j,t} = \ln \left(\frac{(1-\xi)}{\gamma} \frac{c_A}{c_w \alpha_{min}^{\kappa\gamma}} \right) + (\gamma - 1) \ln \left(\Delta/\breve{l}\right)_{j,t} + \kappa\gamma \ln \left(\alpha_{j,t}\right)$$

There are two challenges in estimating this equation. First the screening level $\alpha_{j,t}$ is not observed. For this reason I include in the regression firm fixed effects as well as firm time-varying characteristics as proxies. In particular I use the wage of incumbents since the model predicts that it should be directly related to the current screening level. Second there might be firm level shocks to the recruiting technology through c_A or to the labor supply that generate an omitted variable bias. Specifically one should expect a more efficient recruiting technology or a local shock to the labor supply to simultaneously raise firm growth and lower the wage of new hires, implying a downward bias of the coefficient estimate.

To address endogeneity concerns, I instrument firm growth with the firm demand shock developed in Hummels et al. (2014). The instrument consists of changes in firm foreign partners' demand. In practice I use the weighted changes in the imports of foreign countries from the rest

Dependent variable: new hires avg. residual wage

	3.A	3.B	3.C	3.D
	OLS-FE	IV-FE	additional controls	Single estb.
log hiring rate $\ln\left(\Delta/\bar{l}\right)_{j,t}$	0.019***	0.083*	0.099*	0.09
	(0.002)	(0.043)	(0.057)	(0.089)
incumbents' avg. wage	0.540***	0.511***	0.527***	0.498***
	(0.017)	(0.028)	(0.028)	(0.042)
Observations	96588	96588	89376	70122
Validity tests of the first stage:				
– F-test of excluded IV		0.0012	0.0092	0.17
– Weak identification		10% max IV size	10% max IV size	20% max IV size

Firm-level clustered standard error in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 3: . Weighted firm fixed effect estimation of the elasticity of wage offers with respect to firm growth. In a preliminary step, individual wages are partial-ed out from worker observable characteristics. In all specification I include year and firm fixed effects, firm-level controls and I weight firm-year observations by the number of hires. Controls include the shares of women employed, part-time jobs, and occupation types, as well as the average log hourly wage, log revenues, log capital/employment ratio, log inputs, the investment rate, export and imports dummies and log import and export values. Sources: DADS-Postes and EAE. Manufacturing sector, 1995-2007.

of the world but France. The weights are at the product-destination level and are computed as the export share in the first observed year of export.

Results are presented in table 3. The first stage is valid as shown by the test results and instrumenting correct for the expected downward bias. The corresponding estimated value of γ is about 1.08 or 1.10 and correspond to modestly convex adjustment costs. Although not very precise, the estimates are consistent across the different specifications considered. These values are somewhat larger but close to the 5% estimates in Schmieder (2013) and the difference in results could come from the fact that he focuses on small new firms rather than on exporters which are more relevant for my analysis.

6.2 Counterfactual liberalization

Now equipped with parameter values calibrated on micro data, I turn to the study of the numerical responses predicted by the model.

First I briefly outline the numerical implementation and the algorithm used to solve for the equilibrium path. I simulate an economy populated with firms whose productivity levels can take 100 different values with probabilities that are given by a Pareto distribution. I first solve for the general equilibrium in the initial and terminal steady state. Then I guess a transition period ($T = 140$) and use an iterative fixed point algorithm to solve for the transition path equilibrium. Starting from a guess about the sequence of market conditions $\phi_{t=1..T}$ and price $P_{t=1..T}$, I update the price indexes P using the free entry condition by backward iteration. Along the way I also derive the policy functions of firms given present and future guesses of ϕ and P. Then I update the market conditions ϕ forward by applying the policy functions from the initial allocation of labor across firms. The algorithm converges when the distance between the guesses and the updates fall below an arbitrary threshold.

Consumption, price and trade – The predictions of the model with respect to these central variables are consistent with previous results of the literature. Specifically I find that production overshoots its long run value as in Ghironi and Melitz (2005) and Alessandria and Choi (2014). This might come at a surprise given the presence of adjustment frictions in all these models. However the presence of sunk costs causes inertia and allows unproductive firms that would not successfully enter in the new steady state to stay active. These unproductive firms entered before the reform when the lower level of competition that prevailed allowed them to do so. The production of these firms more than compensates the sluggish adjustments of expanding firms and production overshoots. Because of the absence of a storage technology and the assumption of balanced trade, consumption and production are equal at all times.

Consumption overshooting and the resilience of unproductive firms worsen competition as is reflected by a drop in the price index which falls below its long run value. This results in higher real wages and deter entrepreneurs to create as many new firms as would be the case in the future steady state. Therefore there is a reallocation of labor away from firm creation activities to production and a temporary drop in the mass of entrants.

The adjustment of trade as a share of GDP is gradual but the vast majority of the adjustment happens instantaneously. Two effects drive this pattern. First while recruitment frictions prevent the expansion of exporters, the allocation of sales across markets is costless. Therefore firms that are already big and productive enough at the time of the reform to cover the fixed costs of exports immediately export. The subsequent gradual increase in trade comes from the extensive margin.

Some firms need a few periods to expand and be able to cover the fixed costs. Moreover the share of firms that exports also increase because of the gradual replacement of unproductive firms by more productive ones that are more likely to be or become exporters.

Evolution of inequality – The evolution of inequality is the distinctive new feature of the model. Figure 5 shows that it overshoots its long run value. The drivers are manifold. First there is a big temporary increase in within-firm inequality. However the contribution of within inequality to total inequality is small. The main driver of inequality is the dispersion in firm average wage. At the top of the wage distribution, the largest and more productive firms raise their screening levels and offer temporary wage premia. At the bottom of the distribution, workers at unproductive firms accept wage cuts to delay the exit of their employer and preserve their jobs. These workers will eventually separate from these firms and they will be replaced by workers that get jobs at better firms with better wage on average.

The calibrated model is successful in predicting the overall increase in inequality and some of the overshooting feature. It highlights the role of wages in shaping worker flows during the reallocation process. However labor frictions barely explains the sluggish increase in trade and in inequality. This limitation could come from the fact that liberalization reforms may have been progressive unlike what I assumed. Alternatively this could come from the absence of other adjustment frictions. For example destination specific frictions would slow the expansion of exporters on the foreign market. These observations highlights a path for future research.

7 Conclusion

The objective of this paper was to examine the evolution of inequality and its drivers when a country opens to trade. To this end, I first document the response of inequality following recent liberalization reforms. In this study, I note that the increase in inequality is gradual for about 7 years and that some of the increase may have unraveled thereafter. Because of the limited availability of controls or instruments, it is not possible to rigorously identify the causal effect of liberalization.

Therefore I develop a dynamic general equilibrium model of trade to examine the evolution of inequality in response to a reduction in trade costs. A distinctive feature of the model is the incorporation in a dynamic framework of elements that the literature underscored as key determinants of the effect of trade on inequality. Specifically the model features a positive assortative matching

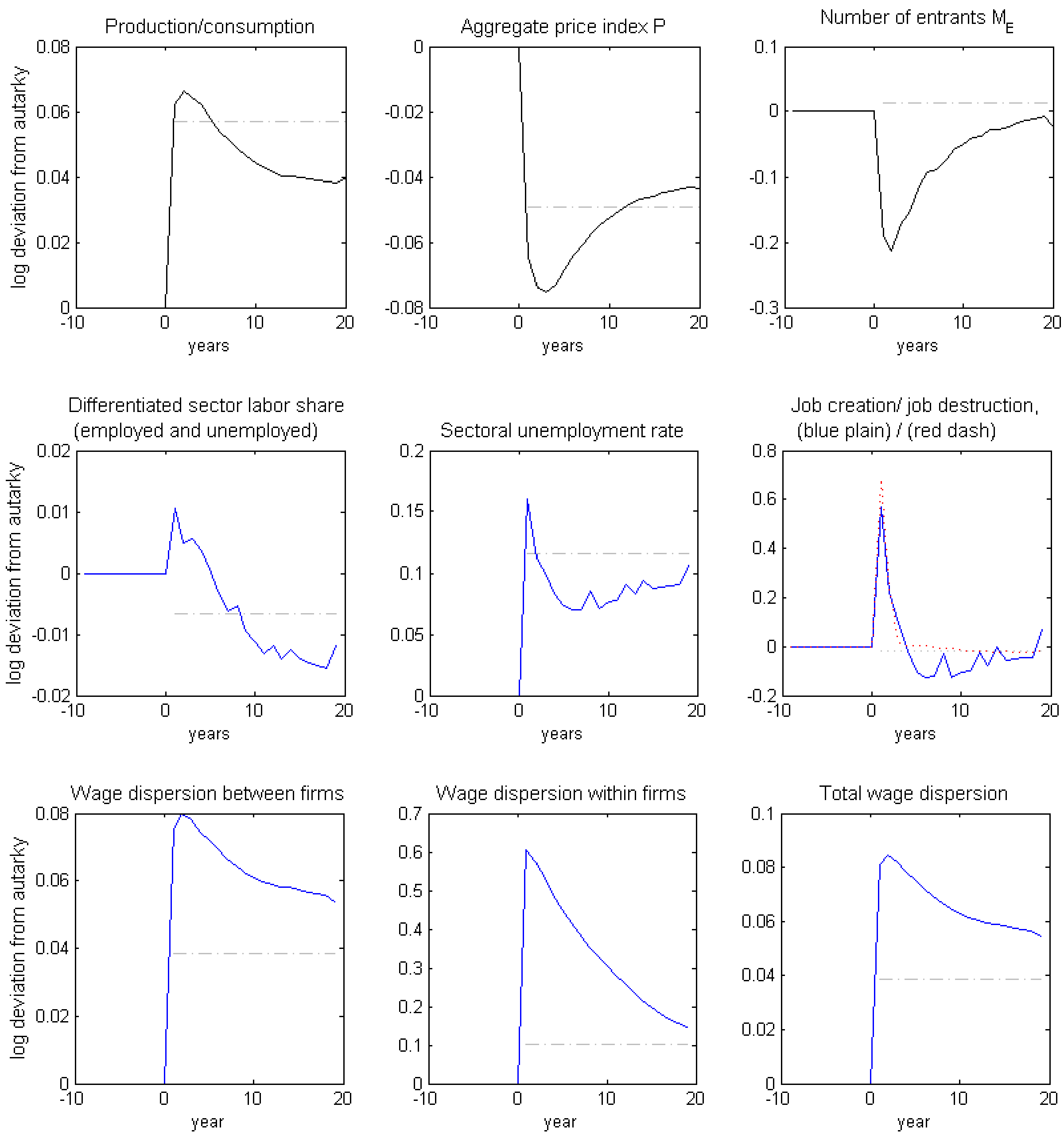


Figure 5: Evolution of selected aggregates in log-deviation from the autarky level. Inequality is measured by normalized standard deviations. The dashed line are the future steady state values towards which the economy converges.

between heterogeneous firms and workers: more productive firms employ more productive workers and they pay higher wages to compensate workers for the higher probability of being screened out. Furthermore, firms adjust the size and average productivity of their workforce progressively because of the convex costs of recruiting. The model successfully rationalizes features of the data. Specifically it explains why bigger firms pay higher wages, and why faster growing firms have higher separation rates and offer higher wages.

The framework is tractable as I demonstrate by deriving results about welfare, inequality and unemployment. First I show that welfare, inequality and unemployment are higher in the steady state under trade than under autarky because of selection effects. Tougher competition resulting from trade leads to the exit of the least productive firms: average productivity improves and so does welfare. Additionally firms screen more intensively because of complementarity between screening and productivity. This translates in more unemployment as workers get separated more often from firms. Inequality is larger under trade because the fraction of firms that accesses the foreign market raises their wage and consists of the firms that already pay higher wages in autarky. Exporters raise their wage because of a composition effect: the expansion that is necessary to serve foreigners is achieved with more able and better paid workers.

Second I show that a reduction in trade costs generates rich dynamic effects about firm and worker outcomes. Expanding firms do not leap to their new stationary level but they speed growth by offering wage premia and attracting better workers faster. As these firms were already paying higher wages before the reform, this effect contributes to an increase in inequality that is temporary because it only operates during the reallocation process. A novel prediction of the model is that the fate of workers depends on the interaction between their ability and the productivity level of their employer. While some workers at better firms get wage raises along with the expansion of their firm, some of them get screened out depending on their ability level. By contrast, workers with the same ability at less productive firms that survives keep their job. A fraction of unproductive firms may struggle to operate profitably in the more competitive open environment. Workers at these firms may have to accept wage cuts to delay the exit of their employer and preserve their jobs.

I use a calibrated version of the model to conduct a counterfactual experiment to compute the dynamic effects of a liberalization reform on aggregates. The model is micro-founded and that allows me to calibrate the novel mechanisms to micro data. The model predicts the overshooting of inequality in response to a reduction in trade costs. The temporary increase in inequality comes from greater dispersion between firm average wages.: high-paying firms offer wage premia to speed the reallocation process while unproductive firms cut wages to temporary survive. I find that the prediction about the overshooting response of inequality qualitatively coincides with the patterns documented in the country panel study.

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Appendix

A Country panel study of inequality

A.1 Country panel data and empirical approach

The analysis is based on country panel fixed effect regressions and focuses on within-country changes in Gini coefficients relative to country pre-reform trends. The analysis mainly draws from three data sources.

I first use the liberalisation dates of Wacziarg and Welch (2008). The authors carefully construct dates of trade liberalization for the 106 countries for wich they have enough information. The determination of the liberalization dates relies on a comprehensive survey of country case studies. The dates are essentially based on the major changes in annual tariffs, in nontariffs, and in the black market exchenge rate premium[44][45]. In addition and whenever relevant, they used a variety of secondary sources to include the dates when state monopoly on major exports were abolished and when multiparty governance systems replaced the Communist Party's undivided rule.

The second essential piece of data consists of country measures of inequality over time. The most comprehensive source of country panel data measures of inequality is the UNU-WIDER, "World Income Inequality Database (WIID3.0b), (2014)". This dataset is a compilation and harmonization of the Gini coefficients that were computed from detailed country specific micro-studies on inequality. However, the data available is not readily useable for the purpose of the event study: several choices and adjustments must be made. Very few countries feature an inequality index that is directly comparable over time for entire periods of interest. Therefore it was sometimes needed to extrapolate and/or retropolate inequality time series. Indeed several measures of inequality might be available for the same country and the overlaps allowed me to extend time series. However

[44] Although seemingly undirectly related to trade openness, the literature has made the case that a black market premium on the exchange rate has effects equivalent to a formal trade restriction. The main argument is based on the observation that exporters have to purchase foreign inputs using foreign currency obtained on the black market, but remit their foreign exchange receipts from exports to the government at the official exchange rate. As result, the black market premium acts as a trade restriction.

[45] The quantitative criteria they used were as follows. At the date of liberalization, countries must have i) an average tarff rates of less than 40%, ii) nontariff barriers covering less than 40% of trade, and iii) a black market exchange rate that is depreciated by no more than 20% relative to the official rate

the coexistence of different indexes also required some choices. The ideal data for the project would have been a measure of wage dispersion. Thus I chose the closest possible measure whenever there is the possibility to do so. To summarize, I use the following algorithm for every country separately. I start with the time series measure of inequality whose coverage spans before and after the liberalization date. I then proceed to extend the series backward and forward using inequality growth rates provided by other time series. Whenever there are more than one available time series, the priority was given to consistent measures, longer time series, and measures that were likely to better capture dispersion in wage inequality.

The last dataset used in the event study is the World Development Indicators provided by the World Bank. The WDI contains data on population, real and nominal GDP, inflation, import and export, investment and unemployment. I supplemented these time series with the share of secondary and high eduction completed in the population aged 15 and over from the Barro-Lee Educational Attainment Data. Eventually the dataset only includes the countries for which a Gini time series that included the liberalization date could be constructed[46]. This leaves 37 countries spanning 6 regions (14 in the America, 7 in Asia, 6 in Europe, 6 Post-Soviet, 2 in Oceania and 2 in Africa) whose liberalization dates range from 1964 to 2001 (4 in the 60's, 2 in the 70's, 4 in the 80's, 26 in the 90's and 1 in the 2000's).

TABLE of country characteristics

I formally compute the heigth and timing of the peak and test for the non-monotonicity of inequality. I first run the following specification

$$gini_{c,t} = \alpha_c + \beta_c(t - t_{c,0}) + \gamma_0 \mathbb{I}_{\{t \geq t_{c,0}\}} + \sum_{n=1}^{4} \gamma_n (t - t_{c,0})^n \mathbb{I}_{\{t \geq t_{c,0}\}} + Z_{c,t} + \epsilon_{c,t} \tag{25}$$

where $t_{c,0}$ are the country c specific liberalization dates, $\alpha_c + \gamma_c(t - t_{c,0})$ are country-specific trends, $\gamma_0 \mathbb{I}_{\{t \geq t_{c,0}\}}$ is a dummy that equals one in the post-liberalization period and the summation term corresponds to a polynomial of order 4 that is set to zero during the pre-liberalization period. Some additional country controls are included in $Z_{c,t}$.

[46]Table A.1 in appendix A.?? summarizes the main characteristics of the 37 countries over the period 1960-2010.

VARIABLES	(4.A) Country trends		(4.B) Controls		(4.C) Pop. weights	
Polynomial terms:						
$(t-t_{c,0}).\mathbb{I}_{\{t\geq t_{c,0}\}}$	0.901***	(0.182)	0.870***	(0.199)	1.149***	(0.283)
$(t-t_{c,0})^2.\mathbb{I}_{\{t\geq t_{c,0}\}}.10^{-2}$	-10.908***	(2.080)	-9.433***	(2.336)	-13.716***	(3.165)
$(t-t_{c,0})^3.\mathbb{I}_{\{t\geq t_{c,0}\}}.10^{-4}$	37.228***	(9.221)	34.141***	(10.627)	56.397***	(13.893)
$(t-t_{c,0})^4.\mathbb{I}_{\{t\geq t_{c,0}\}}.10^{-6}$	-36.752***	(12.702)	-35.943**	(14.998)	-70.175***	(19.186)
Liberalization dummy:						
$\mathbb{I}_{\{t\geq t_{c,0}\}}$	1.189*	(0.625)	-0.296	(0.691)	0.457	(0.915)
Observations	753		609		440	
R-squared	0.950		0.962		0.964	
Country trends	YES		YES		YES	
Controls	NO		YES		YES	
Pop. weights	NO		NO		YES	
Peak increase	3.426	after 6 y.	2.260	after 7 y.	3.548	after 7 y.
F-test p-value	0.006		0.083		0.000	
Decrease	3.221	(year 6 to 17)	1.247	(year 7 to 17)	1.343	(year 7 to 17)
F-test p-value	0.000		0.083		0.162	

Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

Table 4: Within country evolution of Gini coefficients. Controls include the real GDP per capita and its square, the real GDP per capita growth rate, inflation, the openness indes, the investment rate, the population growth and the share of secondary and high education completed in the population aged 15 and over. Sources: WDI, WIID, Barro and Lee (2010)

B Theory technical appendix

I assume complete insurance markets. The whole economy behaves as if there was a representative consumer. I assume that she only consumes the differentiated good and have constant elasticity of intertemporal substitution (IES). Hence total utility derived from consumption given the streams

$(Y_{t'}, P_{t'})_{t'=t..\infty}$ of revenues and aggregate price indexes is:

$$\mathcal{U}_t = \sum_{t'=t}^{\infty} \beta^t \frac{1 - C_t^{1-IES}}{IES - 1} \qquad \text{subject to} \sum_{t'=t}^{\infty} \frac{1}{R_{t,t'}} (P_t C_t - Y_t) = 0 \tag{26}$$

Revenues come from labor income and profits. I assume that each worker has an equal participation in a national fund as in Chaney (2008). Hence profits are equally rebated among workers of the two symmetric economies. The equilibrium interest rate is the given by a standard Euler equation $R_{t,t+1} = \frac{1}{\beta} \left(\frac{C_t}{C_{t+1}}\right)^{IES_{investors}} \frac{P_t}{P_{t+1}} = \frac{1}{\beta} \left(\frac{\phi_{t+1}}{\phi_t}\right)^{\sigma IES} \left(\frac{P_{t+1}}{P_t}\right)^{1-\sigma IES}$ making use of the definition of macro condition $\phi_t = C_t^{\frac{1}{\sigma}} P_t$. In any steady state $R = 1/\beta$.

B.1 Labor supply

This section provides more details on how to solve for the equilibrium labor supply relations of the main text. The values of being unemployed, employed in services and the differentiated sector at a job ω were defined by the following equations (dependence on ω is sometimes implicit for the sake of clarity):

$$\begin{cases} V_{r,t} = w_r - c_r + \frac{1}{R_{t,t+1}} V_{r,t+1} = (w_r - c_r) \left(\sum_{t'=t}^{\infty} \Pi_{s=t}^{t'} \frac{1}{R_{s,s+1}} \right) \\ V_{e,t}(\omega) = w_t(\omega) - c_e + \frac{1}{R_{t,t+1}} \left[(1 - \eta_{t+1}(\omega)) V_{e,t+1}(\omega) + \eta_{t+1}(\omega) V_{u,t+1} \right] \\ V_{u,t} = -c_u + \chi\mu \left(w_t - c_e \right) + (1 - \chi\mu) ui + \frac{1}{R_{t,t+1}} \left[\chi\mu(1 - \eta_{t+1}) V_{e,t+1}(\omega) + (1 - \chi\mu(1 - \eta_{t+1})) V_{u,t+1} \right] \end{cases}$$

Using the arbitrage condition $V_{u,t} = V_{r,t}$ and taking differences yields:

$$\begin{cases} V_{u,t} - V_{r,t} = ui - c_u - w_r + c_r + \chi\mu(w_t(\omega) - c_e - ui) + \frac{\chi\mu(1 - \eta_{t+1})}{R_{t,t+1}} \left[V_{e,t+1}(\omega) - V_{r,t+1} \right] \\ V_{e,t}(\omega) - V_{r,t} = (w_t(\omega) - c_e - w_r + c_r) + \frac{(1 - \eta_{t+1})}{R_{t,t+1}} \left[V_{e,t+1}(\omega) - V_{r,t+1} \right] \end{cases} \tag{27}$$

The second equation in (27) can be expanded by iteration to obtain:

$$V_{e,t}(\omega) - V_{r,t} \quad = (w_{t'}(\omega) - c_e - w_r + c_r) + \sum_{t'=t+1}^{\infty} \frac{\Pi_{s=t+1}^{t'}(1 - \eta_s(\omega))}{R_{t,t'}} (w_{t'}(\omega) - c_e - w_r + c_r)$$

while the combination of both equations yields:

$$V_{e,t}(\omega) - V_{r,t} \quad = (-w_r + c_r + ui) + \frac{1}{\chi(\omega)\mu(\omega)} \left[c_u - ui + w_r - c_r \right]$$

The above equations are not sufficient to fully characterize the wage profile of employed workers. The indeterminacy comes from the absence of attractive alternative options for workers and the corresponding absence of arbitrage since job-to-job transitions are ruled out of the model. Employers's only requirement is to ensure that the ex-ante present value of offered wage is maintained. Therefore two more assumptions are made to allow for a complete and unique solution.

In the case of a stationary firm in a steady state economy and under the assumption of a constant wage, equation (2) is obtained from the combination of the last two equations:

$$w(\omega) - c_e - w_r + c_r = (1-\beta(1-\eta_0))\left((-w_r + c_r + ui) + \tfrac{1}{\chi(\omega)\mu(\omega)}\left[c_u - ui + w_r - c_r\right]\right)$$

$$w(\omega) = \underbrace{\left(w_r - c_r + c_e - \frac{w_r - c_r - ui}{c_w}\right)}_{\equiv w_o} + \tfrac{1}{\chi(\omega)\mu(\omega)c_w}\left(w_r - c_r + c_u - ui\right) \tag{2}$$

In addition, I assumed that any one period deviation in the workers' environment from the full stationary case is offset by a permanent adjustment in the wage holding everything else equal. In other words, the permanent wage change from $w_{t-1}(\omega)$ to $w_t(\omega)$ is obtained by comparing expanded versions of the second equation in (27), one under the assumption of full stationarity and the other that only differs in the current period separation and interest rates.

$$\begin{cases} V_{e,t-1}(\omega) - V_{r,t-1} = (w_{t-1}(\omega) - c_e - w_r + c_r)... \\ \qquad + \beta(1-\eta_0)\left(w_{t-1}(\omega) - c_e - w_r + c_r + \sum_{s=1}^{\infty}\beta^s(1-\eta_0)^s(w_{t-1}(\omega) - c_e - w_r + c_r)\right) \\ V_{e,t-1}(\omega) - V_{r,t-1} = (w_{t-1}(\omega) - c_e - w_r + c_r)... \\ \qquad + \frac{(1-\eta_t)}{R_{t-1,t}}\left(w_t(\omega) - c_e - w_r + c_r + \sum_{s=1}^{\infty}\beta^s(1-\eta_0)^s(w_t(\omega) - c_e - w_r + c_r)\right) \end{cases}$$

Taking differences and using the fact $w_o = c_e + w_r - c_r$ once I assume $w_r - c_r - ui = 0$ yields:

$$w_t - w_o = \frac{\beta R_{t-1,t}(1-\eta_0)}{1-\eta_t}(w_{t-1} - w_o) \qquad ((3))$$

B.2 Labor demand

The probability of being matched depends on the ratio Λ of vacancies per job candidate and a matching technology $m(\lambda)$: $\mu = \frac{m(\Lambda)}{\Lambda}$.

The ability of workers is distributed according to a Pareto distribution with shape parameter $\kappa > 1$ and a lower bound α_{min}. Firms choose an admission cutoff α_c which is public information. The admission rate is then $\chi = \left(\frac{\alpha_c'}{\alpha_{min}}\right)^{-\kappa}$. When the cutoff is constant over time, workforce average ability follows from the distribution assumption: $\bar{\alpha} = \frac{\kappa}{\kappa-1}\alpha_c$.

Prime refers to end of period variables. I define Δ as the number of hires: $\Delta \equiv m(\lambda)V\left(\frac{\alpha_c'}{\alpha_{min}}\right)^{-\kappa}$.

Recruiting – Managing vacancies is costly: $C(V,\alpha_c',\alpha_c,l) = \frac{c_C}{1+\nu}V^{\nu+1}\left((1-s)\left(\frac{\alpha_c}{\alpha_c'}\right)^{\kappa} l\right)^{-\epsilon}$
With the new notation: $C(\Delta,\Lambda,\alpha_c',\alpha_c,l) = \frac{c_C}{1+\nu}\Delta^{\nu+1}\left((1-s)l\right)^{-\nu}\left(\frac{1}{m(\Lambda)}\right)^{1+\epsilon}\left(\frac{\alpha_c'}{\alpha_{min}}\right)^{\kappa(1+\nu)}\alpha_c^{-\nu\kappa}$

Screening – Employers can screen workers by having them take a test that determines whether their ability. The cost of employing workers with average ability $\overline{\alpha}$ takes the form $\frac{c_s}{\psi}\left(\frac{\kappa}{\kappa-1}\alpha_c\right)^{\psi}$ where I use the formula for the average of a Pareto distribution. In what follows and until I examine the transition path equilibrium, I will restrict firms to screen all workers (including matched job candidates) and to never decrease the screening cutoff. The match continuation rate is the probability of no firm death and no separation $(1-\eta) = (1-s)(1-\delta)\left(\frac{a_c}{a_c'}\right)^{\kappa}$ which becomes $(1-\eta_0) = (1-s_0)(1-\delta_0)$ for stationary firms.

Price, production and export decision – Given CES-demand, firms face demand curves of the form $\frac{c^M}{C^M} = \left(\frac{p^M}{P^M}\right)^{-\sigma}$, or equivalently $p^M = \left(c^M\right)^{-\frac{1}{\sigma}}\phi^M$ on the domestic ($M = D$) and foreign market ($M = F$). $\phi^M = C^{M\frac{1}{\sigma}}P^M$ stands for market conditions, and C^M aggregate consumption. Let $q = q^D + q^F$ be the total quantity produced by a firm. Firm potential revenues are $r^D = \left(q^D\right)^{\frac{\sigma-1}{\sigma}}\phi^D$ and $r^F = \left(\frac{q^F}{\tau}\right)^{\frac{\sigma-1}{\sigma}}\phi^F$. The optimal allocation for exporters is to allocate sales to equate marginal revenues across markets:

$$r_q^D = r_q^F \quad \Leftrightarrow \quad \frac{\sigma-1}{\sigma}\left(q^D\right)^{-\frac{1}{\sigma}}\phi^D = \left(\frac{1}{\tau}\right)^{\frac{\sigma-1}{\sigma}}\frac{\sigma-1}{\sigma}\left(q^F\right)^{-\frac{1}{\sigma}}\phi^F$$

From there it is possible to derive the quantity produced for each market and the associated revenues:

$$q^D = {}^{q}/_{\Upsilon}; \qquad q^F = {}^{q(\Upsilon-1)}/_{\Upsilon}, \qquad \text{with } \Upsilon = 1 + \left(\frac{1}{\tau}\right)^{\sigma-1}\left(\frac{\phi^F}{\phi^D}\right)^{\sigma} \tag{28}$$

$$r(q,I_X;\phi^D,\Upsilon) = r^D + r^F = q^{\frac{\sigma-1}{\sigma}}\phi^D\left(\Upsilon(I_X)\right)^{\frac{1}{\sigma}}, \text{ with } r^D = {}^{r}/_{\Upsilon}; \quad r^F = {}^{r(\Upsilon-1)}/_{\Upsilon} \tag{29}$$

Thus aggregate demand and competition faced by a firm are characterized by $\phi^M = C^{M\frac{1}{\sigma}}P^M, M \in \{D,F\}$ and $\Upsilon(1) = 1 + \left(\frac{1}{\tau}\right)^{\sigma-1}\left(\frac{\phi^F}{\phi^D}\right)^{\sigma}$ if the firm exports (when $I_X = 1$).

I now drop the superscript ^{D} for clarity of notation. Given the production function $q = x\bar{\alpha}l^{\rho}$, $(\rho < 1)$ with $\bar{\alpha} = \frac{\kappa}{\kappa-1}\alpha_c$, revenues can be written as follows:

$$r(\overline{\alpha}',l,x;\phi,\Upsilon) \quad = \quad \left(x\bar{\alpha}'l^{\rho}\right)^{\frac{\sigma-1}{\sigma}}\left(\Upsilon(I_X)\right)^{\frac{1}{\sigma}}\phi$$

Labor force accumulation – $l' = (1-s)\left(\frac{a_c}{a'_c}\right)^{\kappa} l + \Delta$

Accumulation of wage premia liabilities (in units of *numéraire*) – The dynamic nature of the model and the wage posting mechanism imply that wage payments depend on firms' past commitments. Let B be the beginning-of-period total amount of wage premium commitments accumulated by a firm. Every period, wage premium commitments vary in accordance with the wage equation (3), decrease with the number of separations and increase with new commitments. The end of period wage bill to be paid is:

$$B' = \underbrace{B\frac{(1-\eta_0)}{(1-\eta)}}_{\text{wage variation}} \underbrace{(1-s)\left(\frac{\alpha_c}{\alpha'_c}\right)^{\kappa}}_{\text{separations}} + \underbrace{\Delta(w-w_o)}_{\text{new commitments}} = B\beta R(1-s_0) + \Delta.(w-w_o)$$

$$\text{using } (1-\eta) = (1-\delta)(1-s)\left(\frac{\alpha_c}{\alpha'_c}\right)^{\kappa} \qquad (15)$$

The firm problem – Then firm value can be defined as:

$$G(\alpha_c, l, B, x; \phi, \Upsilon) = \max_{\delta,s,\Delta,\Lambda,w,\alpha'_c,I_X} (1-\delta)\Big\{r(\overline{\alpha}', l', x; \phi, \Upsilon) - C(\Delta, \Lambda, \alpha'_c, \alpha_c, l) - \tfrac{c_s}{\psi}\overline{\alpha}'^{\psi}_{c} ...$$
$$... - f_d - I_X f_X - B' - w_o l' + \beta G(\alpha'_c, l', B', x; \phi, \Upsilon)\}$$
$$\text{s.t.} \qquad l' = (1-s)\left(\tfrac{\alpha_c}{\alpha'_c}\right)^{\kappa} l + \Delta, \quad \Delta \geq 0, \; s \geq s_0, \; \delta \geq \delta_0, \; \alpha'_c \geq \alpha_c$$
$$B' = B\beta R(1-s_0) + \Delta.(w-w_o)$$
$$w = w_o + \tfrac{\Lambda}{m(\Lambda)}\left(\tfrac{\alpha'_c}{\alpha_{min}}\right)^{\kappa}\tfrac{c_u}{c_w}, \qquad \text{with } c_w = \tfrac{1}{1-\beta(1-\eta_0)}$$

In a steady state environment with no other uncertainty for firms than the exit shock, firms never decide to exit except upon entry and $\delta = \delta_0$. Following Kaas and Kircher (2015), I guess that the value function is linear in wage commitment: $G(\alpha_c, l, B, x; \phi, \Upsilon) = J(\alpha_c, l, x; \phi, \Upsilon) - c_B B$. The guess is correct when $c_B = \frac{1-\eta_0}{1-\beta(1-\eta_0)}$. Indeed:

$$J(\alpha_c, l, x; \phi, \Upsilon) - c_B B = \max_{\Delta,\Lambda,w,\alpha'_c,I_X} (1-\delta_0)\Big\{r(\overline{\alpha}', l', x; \phi, \Upsilon) - C(\Delta, \Lambda, \alpha'_c, \alpha_c, l) - \frac{c_s}{\psi}\overline{\alpha}'^{\psi}_{c} ...$$
$$... - f_d - I_X f_X - w_o l' - (1+\beta c_B)\Delta.(w-w_o) + \beta J(a'_c, l', x; \phi, \Upsilon)\Big\}$$
$$... - \underbrace{(1-s_0)(1-\delta_0)(1+\beta c_B)}_{=c_B} B$$

From now on, I use the fact that policy variables are not affected by past wage commitments and I focus on the value function J:

$$
\begin{aligned}
J(\alpha_c, l, x; \phi, \Upsilon) \quad = \quad & \max_{\Delta,\Lambda,w,\alpha_c',I_X} (1-\delta_0)\left\{ r(\overline{\alpha}', l', x; \phi, \Upsilon) - f_d - I_X f_X - C(\Delta, \Lambda, \alpha_c', \alpha_c, l) - \frac{c_s}{\psi}\overline{\alpha}'^{\psi}_{c} \ldots \right. \\
& \left. \ldots - w_o l' - \frac{\Lambda}{m(\Lambda)}\left(\frac{\alpha_c'}{\alpha_{min}}\right)^{\kappa} c_u \Delta + \beta J(a_c', l', x; \phi, \Upsilon) \right\} \\
\text{s.t.} \qquad & l' = (1-s)\left(\frac{\alpha_c}{\alpha_c'}\right)^{\kappa} l + \Delta, \quad \Delta \geq 0, \;\; s \geq s_0, \;\; \alpha_c' \geq \alpha_c
\end{aligned}
$$

Assume the following regarding functional forms:

$$
\begin{aligned}
0 < \lambda < 1 \quad &: \quad m(\Lambda) = \frac{c_m}{1-\lambda}\Lambda^{1-\lambda} \quad \Rightarrow m_\Lambda(\Lambda) = c_m \Lambda^{-\lambda}; \\
0 < \nu \quad &: \quad C(\Delta, \Lambda, \alpha_c', \alpha_c, l) = \frac{c_C \Delta^{\nu+1}}{1+\nu}\left((1-s)\left(\frac{\alpha_c}{\alpha_c'}\right)^{\kappa} l\right)^{-\nu} m(\lambda)^{-1-\nu}\left(\frac{\alpha_c'}{\alpha_{min}}\right)^{\kappa(1+\nu)}
\end{aligned}
$$

I obtain:

$$
\begin{aligned}
\text{FOC wrt } \Lambda : \qquad & -C_\Lambda(\Delta, \Lambda, \alpha_c', \alpha_c, l) = c_u\left(\frac{m(\Lambda) - \lambda m_\Lambda(\Lambda)}{m^2(\Lambda)}\right)\left(\frac{\alpha_c'}{\alpha_{min}}\right)^{\kappa}\Delta \\
\Rightarrow \Lambda \quad = \quad & \left(\frac{c_C}{\lambda c_u}\right)^{\frac{1}{1+\nu(1-\lambda)}} (1-\lambda)^{\frac{1+\nu}{1+\nu(1-\lambda)}} \left(\frac{\Delta}{c_m(1-s)l}\left(\frac{\alpha_c'}{\alpha_c}\right)^{\kappa}\right)^{\frac{\nu}{1+\nu(1-\lambda)}} \left(\frac{\alpha_c'}{\alpha_{min}}\right)^{\frac{\kappa\nu}{1+\nu(1-\lambda)}} \\
\Lambda \quad = \quad & \Lambda_\Delta \left(\frac{\Delta}{(1-s)l}\left(\frac{\alpha_c'}{\alpha_c}\right)^{\kappa}\right)^{\frac{\gamma-1}{\lambda}} \left(\frac{\alpha_c}{\alpha_{min}}\right)^{\frac{\kappa(\gamma-1)}{\lambda}}, \\
& \text{with } \gamma \equiv 1 + \frac{\nu\lambda}{1+\nu(1-\lambda)} = \frac{1+\nu}{1+\nu(1-\lambda)}
\end{aligned}
$$

Then I define an adjustment function that represents vacancy costs and increases in the present value of the sum of discounted wage premia commitments:

$$
\begin{aligned}
A(\Delta, \Lambda, \alpha_c', \alpha_c, l) \quad \equiv \quad & C + c_w\Delta(w - w_o) = C(\Delta, \Lambda, \alpha_c', \alpha_c, l) + \frac{\Lambda}{m(\Lambda)}\left(\frac{\alpha_c'}{\alpha_{min}}\right)^{\kappa} c_u \Delta \\
A(\Delta, \alpha_c', \alpha_c, l) \quad = \quad & \frac{\Gamma}{\gamma} c_u^{1-\xi}\frac{c_C^{\xi}}{c_m^{\gamma}}\left(\frac{\Delta}{(1-s)l}\left(\frac{\alpha_c'}{\alpha_c}\right)^{\kappa}\right)^{\gamma-1}\left(\frac{\alpha_c'}{\alpha_{min}}\right)^{\kappa\gamma}\Delta \\
& \text{with } \Gamma \equiv \left(\frac{1-\xi}{\gamma}\right)^{\gamma-1}\left(\frac{1-\xi}{\gamma-1+\xi}\right)^{\xi} \qquad \text{and } c_A \equiv \Gamma\frac{c_u^{1-\xi}c_C^{\xi}}{c_m^{\gamma}} \\
& \text{with } \xi = \frac{\lambda}{1+\nu(1-\lambda)}; \quad 1-\xi = \frac{(1+\nu)(1-\lambda)}{1+\nu(1-\lambda)}; \\
& \nu = \frac{\gamma-1}{\xi}; \quad \lambda = \frac{\gamma-1+\xi}{\gamma} \\
\text{Then } C(\Delta, \alpha_c', \alpha_c, l) \quad = \quad & \xi\frac{c_A}{\gamma}\left(\frac{\Delta}{(1-s)l}\left(\frac{\alpha_c'}{\alpha_c}\right)^{\kappa}\right)^{\gamma-1}\left(\frac{\alpha_c'}{\alpha_{min}}\right)^{\kappa\gamma}\Delta \quad \text{with } c_w\Delta(w-w_o) = A - C
\end{aligned}
$$

ThusξA is used to pay vacancy costs while the rest represents the increase in the present value of the sum of discounted wage premia commitments. Also the number of workers searching for the firm vacancy is given by $\Lambda V = \Lambda\Delta\left[m(\Lambda)\left(\frac{\alpha'_c}{\alpha_{min}}\right)^{-\kappa}\right]^{-1} = \frac{\Lambda}{m(\Lambda)}\left(\frac{\alpha'_c}{\alpha_{min}}\right)^{\kappa}\Delta = \frac{(1-\xi)}{c_u}A$. The number of searching workers that do not get a job in the period is given by $\Lambda V - \Delta = \frac{1-\xi}{c_u}A - \Delta$.

B.3 Firm saddle path equilibrium properties

I restate the firm problem with the new adjustment function:

$$J(\alpha_c, l, x; \phi, \Upsilon) = \max_{\Delta, l', \alpha'_c, I_X} (1-\delta_0)\Big\{ r(\bar{\alpha}', l', x; \phi, \Upsilon) - w_o l' - A(\Delta, \alpha'_c, \alpha_c, l) - \frac{c_s}{\psi}\overline{\alpha}'^{\psi}_c - f_d ...$$
$$... - I_X f_X + \beta J(\alpha'_c, l', x; \phi, \Upsilon)\Big\}$$

$$\text{s.t.} \qquad l' = l(1-s)\left(\frac{a_c}{a'_c}\right)^{\kappa} + \Delta, \qquad \Delta \geq 0, \;\; s \geq s_0, \;\; \alpha'_c \geq \alpha_c$$
$$A(\Delta, \alpha'_c, \alpha_c, l) = \frac{c_A}{\gamma}\left(\frac{\Delta}{(1-s)l}\left(\frac{\alpha'_c}{\alpha_c}\right)^{\kappa}\right)^{\gamma-1}\left(\frac{\alpha'_c}{\alpha_{min}}\right)^{\kappa\gamma}\Delta$$
$$r(\alpha'_c, l', x; \phi, \Upsilon) = \left(x\bar{\alpha}l'^{\rho}\right)^{\frac{\sigma-1}{\sigma}}\left(\Upsilon(I_X(\alpha'_c, l', x; \phi, \Upsilon))\right)^{\frac{1}{\sigma}}\phi \quad \text{with } \overline{\alpha}' = \frac{\kappa}{\kappa-1}\alpha'_c$$

where actual wage rate for hires is $w = w_o + (1-\xi)(1-\beta(1-\eta_0))\frac{A}{\Delta}$.This makes clear that workers can extract a rent from their employer's profits and that their bargaining power varies with the firm adjustment cost.

A change of variable allows to reduce the state space: $h = l.\alpha_c^{\kappa}$ and $d = \frac{\Delta.\alpha'^{\kappa}_c}{(1-s)l\alpha_c^{\kappa}}$.

$$J(h, x; \phi, \Upsilon) = \max_{d, h', \alpha'_c, I_X} (1-\delta_0)\Big\{ r(\overline{\alpha}'_c, h', x; \phi, \Upsilon) - \frac{w_o h'}{\alpha'^{\kappa}_c} - \tilde{A}(d, \alpha'_c, h) - \frac{c_s}{\psi}\alpha'^{\psi}_c ... \tag{30}$$
$$... - f_d - I_X f_X + \beta J(h', x; \phi, \Upsilon)\Big\}$$

$$\text{s.t.} \qquad h' = h(1-s)(1+d), \qquad d \geq 0, \;\; s \geq s_0$$
$$\tilde{A}(d, \alpha'_c, h) = \frac{\tilde{c}_A}{\gamma}d^{\gamma}h(1-s)\alpha'^{\kappa(\gamma-1)}_c \qquad \text{with } \tilde{c}_A = c_A.\alpha_{min}^{-\kappa\gamma}$$
$$r(\alpha'_c, h', x; \phi, \Upsilon) = \left(x\frac{\kappa}{\kappa-1}\alpha'^{1-\kappa\rho}_c h'^{\rho}\right)^{\frac{\sigma-1}{\sigma}}\left(\Upsilon(I_X(\alpha'_c, h', x; \phi, \Upsilon))\right)^{\frac{1}{\sigma}}\phi$$

where I need to assume $0 < \kappa\rho < 1$

The I derive the first order conditions (FOC) and the envelop theorem (EVT):

$$\text{FOC wrt } \alpha'_c: \quad \frac{(\sigma-1)(1-\kappa\rho)}{\sigma}\alpha'^{\frac{\sigma-1}{\sigma}(1-\kappa\rho)-1}_c \left(x\frac{\kappa}{\kappa-1}h'^{\rho}\right)^{\frac{\sigma-1}{\sigma}} \Upsilon^{\frac{1}{\sigma}}\phi = ...$$

$$... + \frac{\tilde{c}_A}{\gamma}d^{\gamma}h(1-s)\kappa(\gamma-1)\alpha'^{\kappa(\gamma-1)-1}_c - w_o h'\kappa\alpha'^{-\kappa-1}_c + c_s\alpha'^{\psi-1}_c$$

FOC wrt d :

$$h(1-s)r_h(\alpha'_c, h', x; \phi, \Upsilon) + \beta h(1-s)J_h(h', x; \Phi) = \tilde{c}_A h(1-s)d^{\gamma-1}\alpha'^{\kappa(\gamma-1)}_c + w_o h(1-s)\alpha'^{-\kappa}_c$$

FOC wrt I_X :

$$I_X(\alpha'_c, h', x; \phi, \Upsilon) = \begin{cases} 1 & \text{if } r(\alpha'_c, h', x; \phi, \Upsilon)) - r(\alpha'_c, h', x; \phi, 0)) > f_X \\ 0 & \text{o.w.} \end{cases}$$

ECT wrt h' :

$$J_h(h', x; \Phi) + (1-\eta_0)\frac{\tilde{c}_A}{\gamma}d'^{\gamma}\alpha''^{\kappa(\gamma-1)}_c = (1-\delta_0)(1-s)(1+d')\left[r_h(\alpha''_c, h'', x) - \frac{w_o}{\alpha''^{\kappa}_c} + \beta J_h(h'', x; \Phi)\right]$$

Multiply by $-\beta$ and use the FOC with respect to d to substitute the J_h terms:

$$r_h(\alpha'_c, h', x) = \tilde{c}_A d^{\gamma-1}\alpha'^{\kappa(\gamma-1)}_c + w_o\alpha'^{-\kappa}_c - (1-\eta_0)\beta\left[\tilde{c}_A\frac{\gamma-1}{\gamma}d'^{\gamma}\alpha''^{\kappa(\gamma-1)}_c + \tilde{c}_A d'^{\gamma-1}\alpha''^{\kappa(\gamma-1)}_c\right]$$

To summarize, the equilibrium path is determined by the following optimality conditions:

$$\begin{cases} h' = h(1-s)(1+d) \\ \frac{1-\kappa\rho}{\kappa\rho}\alpha'^{\frac{\sigma-1}{\sigma}(1-\kappa\rho)}_c h'^{\rho\frac{\sigma-1}{\sigma}} = \hat{c}_A\frac{\gamma-1}{\gamma}(1-s)d^{\gamma}h\alpha'^{\kappa(\gamma-1)}_c - \hat{w}_o h'\alpha'^{-\kappa}_c + \hat{c}_s\alpha'^{\psi}_c \\ \alpha'^{\frac{\sigma-1}{\sigma}(1-\kappa\rho)}_c h'^{\rho\frac{\sigma-1}{\sigma}-1} = \hat{c}_A d^{\gamma-1}\alpha'^{\kappa(\gamma-1)}_c + \hat{w}_o\alpha'^{-\kappa}_c - (1-\eta_0)\beta\left[\frac{\gamma-1}{\gamma}d' + 1\right]\hat{c}_A d'^{\gamma-1}\alpha''^{\kappa(\gamma-1)}_c \\ \text{with } X \equiv \rho\frac{\sigma-1}{\sigma}x^{\frac{\sigma-1}{\sigma}}\Upsilon^{\frac{1}{\sigma}}\phi\left(\frac{\kappa}{\kappa-1}\right)^{\frac{\sigma-1}{\sigma}}, \quad \hat{c}_A \equiv \frac{\tilde{c}_A}{X}, \quad \hat{w}_o \equiv \frac{w_o}{X}, \quad \text{and } \hat{c}_S \equiv \frac{c_S}{\kappa X} \end{cases}$$

Stationary equilibrium: existence and uniqueness – Let $d_{SS}, \alpha_{SS}, l_{SS}$ be the stationary values of d, α_c, l. I use the optimality conditions and obtain: $d_{SS} = \frac{s}{1-s}$

$$(1-\kappa\rho)\frac{\sigma-1}{\sigma}\left(\frac{\kappa}{\kappa-1}\right)^{\frac{\sigma-1}{\sigma}}\alpha_{SS}^{\frac{\sigma-1}{\sigma}} l_{SS}^{\rho\frac{\sigma-1}{\sigma}} x^{\frac{\sigma-1}{\sigma}}\Upsilon^{\frac{1}{\sigma}}\phi = ... \tag{31}$$

$$...\frac{\tilde{c}_A}{\gamma}sl_{SS}\left(\frac{s}{1-s}\right)^{\gamma-1}\kappa(\gamma-1)\alpha_{SS}^{\kappa\gamma} - \kappa w_o l_{SS} + c_s\alpha_{SS}^{\psi}$$

$$\rho\frac{\sigma-1}{\sigma}\left(\frac{\kappa}{\kappa-1}\right)^{\frac{\sigma-1}{\sigma}} l_{SS}^{\rho\frac{\sigma-1}{\sigma}-1} x^{\frac{\sigma-1}{\sigma}}\Upsilon^{\frac{1}{\sigma}}\phi = ... \tag{32}$$

$$...w_o\alpha_{SS}^{-\frac{\sigma-1}{\sigma}} + \tilde{c}_A\left(\frac{s}{1-s}\right)^{\gamma-1}\alpha_{SS}{}^{\kappa\gamma-\frac{\sigma-1}{\sigma}}\left[1-(1-\delta_0)\beta\left(1-\frac{s}{\gamma}\right)\right]$$

Then solve for α_{SS} by substituting l_{SS} in (31) using (32):

$$\frac{(1-\kappa\rho)}{\rho}\left\{w_o+\tilde{c}_A\left(\frac{s}{1-s}\right)^{\gamma-1}\alpha_{SS}{}^{\kappa\gamma}\left[1-(1-\delta_0)\beta\left(1-\frac{s}{\gamma}\right)\right]\right\}=$$

$$...\kappa\frac{\gamma-1}{\gamma}\tilde{c}_A s\left(\frac{s}{1-s}\right)^{\gamma-1}\alpha_{SS}^{\kappa\gamma}-\kappa w_o+\frac{c_s}{l_{SS}}\alpha_{SS}^{\psi}$$

which becomes

$$\frac{w_o}{\rho}+\kappa\Omega_1\tilde{c}_A\left(\frac{s}{1-s}\right)^{\gamma-1}\alpha_{SS}^{\kappa\gamma}=\frac{c_s}{l_{SS}}\alpha_{SS}^{\psi}\qquad \text{with } \Omega_1=\frac{(1-\kappa\rho)}{\kappa\rho}\left[1-(1-\delta_0)\beta\left(1-\frac{s}{\gamma}\right)\right]-\frac{\gamma-1}{\gamma}s$$

note that

$$\Omega_1>(\gamma-1)\left[1-(1-\delta_0)\beta\left(1-\frac{s}{\gamma}\right)\right]-(\gamma-1)\frac{s}{\gamma}=(\gamma-1)\left(1-\frac{s}{\gamma}\right)(1-(1-\delta_0)\beta)>0$$

$$\text{as }\frac{1-\kappa\rho}{\kappa\rho}=\frac{1}{\kappa\rho}-1>\gamma-1\quad\text{when }\kappa\rho\gamma<1$$

Finally substitute l_{SS} using (32) again:

$$\left\{\frac{w_o}{\rho c_s}+\frac{\kappa}{c_s}\Omega_1\tilde{c}_A\left(\frac{s}{1-s}\right)^{\gamma-1}\alpha_{SS}^{\kappa\gamma}\right\}^{1-\rho\frac{\sigma-1}{\sigma}}\rho\frac{\sigma-1}{\sigma}\left(\frac{\kappa}{\kappa-1}\right)^{\frac{\sigma-1}{\sigma}}x^{\frac{\sigma-1}{\sigma}}\Upsilon^{\frac{1}{\sigma}}\phi=...$$

$$...\alpha_{SS}^{\psi(1-\rho\frac{\sigma-1}{\sigma})-\frac{\sigma-1}{\sigma}}\left\{w_o+\tilde{c}_A\left(\frac{s}{1-s}\right)^{\gamma-1}\alpha_{SS}{}^{\kappa\gamma}\left[1-(1-\delta_0)\beta\left(1-\frac{s}{\gamma}\right)\right]\right\}$$

$$\left\{\frac{w_o}{\rho c_s}\alpha_{SS}^{-\psi+\frac{\frac{\sigma-1}{\sigma}}{1-\rho\frac{\sigma-1}{\sigma}}}+\frac{\kappa}{c_s}\Omega_1\tilde{c}_A\left(\frac{s}{1-s}\right)^{\gamma-1}\alpha_{SS}^{\kappa\gamma-\psi+\frac{\frac{\sigma-1}{\sigma}}{1-\rho\frac{\sigma-1}{\sigma}}}\right\}^{1-\rho\frac{\sigma-1}{\sigma}}...$$

$$...\rho\frac{\sigma-1}{\sigma}\left(\frac{\kappa}{\kappa-1}\right)^{\frac{\sigma-1}{\sigma}}x^{\frac{\sigma-1}{\sigma}}\Upsilon^{\frac{1}{\sigma}}\phi=\left\{w_o+\tilde{c}_A\left(\frac{s}{1-s}\right)^{\gamma-1}\alpha_{SS}{}^{\kappa\gamma}\left[1-(1-\delta_0)\beta\left(1-\frac{s}{\gamma}\right)\right]\right\}$$

I obtain a sufficient condition for uniqueness when $\psi-\kappa\gamma\geq\frac{\frac{\sigma-1}{\sigma}}{1-\rho\frac{\sigma-1}{\sigma}}$ or equivalently $\psi\rho\geq\kappa\gamma\rho+\frac{\rho\frac{\sigma-1}{\sigma}}{1-\rho\frac{\sigma-1}{\sigma}}=\kappa\gamma\rho-1+\frac{1}{1-\rho\frac{\sigma-1}{\sigma}}$. In addition, this condition ensures that α_{SS} increases in $X=x^{\frac{\sigma-1}{\sigma}}\Upsilon^{\frac{1}{\sigma}}\phi$.

Then I obtain the other variables as functions of a_{SS}:

$$\begin{aligned}
l_{SS} &= \left\{ \frac{\rho^{\frac{\sigma-1}{\sigma}} \left(\frac{\kappa}{\kappa-1}x\right)^{\frac{\sigma-1}{\sigma}} \Upsilon^{\frac{1}{\sigma}}\phi}{w_o \alpha_{SS}^{-\frac{\sigma-1}{\sigma}} + \tilde{c}_A \left(\frac{s}{1-s}\right)^{\gamma-1} \alpha_{SS}{}^{\kappa\gamma-\frac{\sigma-1}{\sigma}} \left[1-(1-\delta_0)\beta\left(1-\frac{s}{\gamma}\right)\right]} \right\}^{\frac{1}{1-\rho\frac{\sigma-1}{\sigma}}} \\
r_{SS} &= \left(\alpha_{SS} l_{SS}^{\rho}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\kappa}{\kappa-1}x\right)^{\frac{\sigma-1}{\sigma}} \Upsilon^{\frac{1}{\sigma}}\phi \\
A_{SS} &= \frac{\tilde{c}_A}{\gamma}\left(\frac{s}{(1-s)}\right)^{\gamma-1} \alpha_{SS}^{\kappa\gamma} s.l_{SS} \\
S_{SS} &= \frac{c_s}{\psi}\alpha_{SS}^{\psi} \\
\pi_{SS} &= r_{SS} - A_{SS} - S_{SS} - f_d - I_{X,SS} f_X \\
\Delta_{SS} &= s.l_{SS} \\
w_{SS} &= w_o + (1-\xi)(1-\beta(1-\eta_0))\frac{A_{SS}}{\Delta_{SS}} = w_o + (1-\xi)(1-\beta(1-\eta_0))\frac{\tilde{c}_A}{\gamma}\left(\frac{s}{(1-s)}\right)^{\gamma-1} \alpha_{SS}^{\kappa\gamma} \\
u_{SS} &= (1-\xi)\frac{A_{SS}}{c_u} \\
J_{SS} &= \frac{(1-\delta_0)\pi_{SS}}{1-(1-\delta_0)\beta} \\
B_{SS} &= l_{SS}(w_{SS} - w_o) \\
G_{SS} &= J_{SS} - c_B B_{SS} = J_{SS} - \frac{1-\eta_0}{1-(1-\eta_0)\beta} B_{SS}
\end{aligned}$$

The saddle path with no screening – When ψ goes to infinity, the technology cost goes to infinity if there is any screening. Therefore firms do not screen, no associated costs, and $\alpha_c = \alpha_{min}$ at all times which I normalize to 1. In the steady state environment there is no firing $s = s_0$. The optimality conditions simplify to:

$$\begin{cases}
l' = l(1-s_0)(1+d) \\
l'^{\rho\frac{\sigma-1}{\sigma}-1} = \hat{c}_A d^{\gamma-1} + \hat{w}_o - (1-\eta_0)\beta\left[\frac{\gamma-1}{\gamma}d' + 1\right]\hat{c}_A d'^{\gamma-1} \\
\text{with } X \equiv \rho\frac{\sigma-1}{\sigma}x^{\frac{\sigma-1}{\sigma}}\Upsilon^{\frac{1}{\sigma}}\phi\left(\frac{\kappa}{\kappa-1}\right)^{\frac{\sigma-1}{\sigma}}, \qquad \hat{c}_A \equiv \frac{\tilde{c}_A}{X}, \qquad \hat{w}_o \equiv \frac{w_o}{X},
\end{cases}$$

For exporters, X jumps at the time at which firms start to export. First consider the case of a domestic firm. The two equations determine a phase diagram where the stationary curves are given by:

$$\begin{aligned}
&l\text{-curve}, \quad l' = l: \quad d = \frac{s_0}{1-s_0} \\
&d\text{-curve}, \quad d' = d: \quad l^{\rho\frac{\sigma-1}{\sigma}-1} = f(d) \quad \text{with } f(d) = \hat{c}_A d^{\gamma-1} + \hat{w}_o - (1-\eta_0)\beta\left[\frac{\gamma-1}{\gamma}d + 1\right]\hat{c}_A d^{\gamma-1}
\end{aligned}$$

that intersects only once. Note that the d-curve shifts up when increasing X. The dynamics are then given by:

$$
\begin{aligned}
l' > l \quad &\Rightarrow \quad d > \frac{s_0}{1-s_0} \\
d' > d \quad &\Rightarrow \quad \hat{c}_A d^{\gamma-1} + \hat{w}_o - (1-\eta_0)\beta\left[\frac{\gamma-1}{\gamma}d' + 1\right]\hat{c}_A d'^{\gamma-1} > \hat{c}_A d'^{\gamma-1} + \hat{w}_o - (1-\eta_0)\beta\left[\frac{\gamma-1}{\gamma}d' + 1\right]\hat{c}_A d'^{\gamma-1} \\
&\Rightarrow \quad l' > f(d')
\end{aligned}
$$

In conclusion, the hiring rate d decreases with l and increasing in X. Now consider the case of

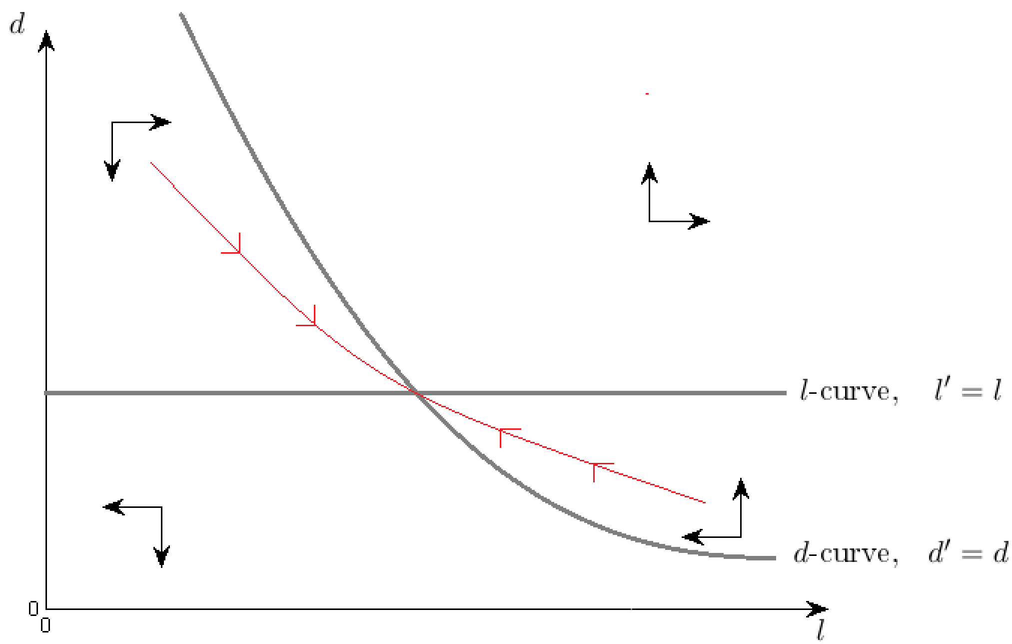

Figure 6: Phase diagram of the saddle path of a domestic firm in the no-screening case.

a firm that will export at some point. There is a change in the system of equations that govern the saddle path when it starts exporting. At that moment, the d-curve shifts up with X. The equilibrium path is as described in the graph below, the d-curve changes from the dashed line to the plain line when the firm starts exporting. Then the firm reaches the saddle path equilibrium under the export regime. Before, exporters speed growth and become because they anticipate the fact that they will need to serve the foreign market and reach a larger stationary size.

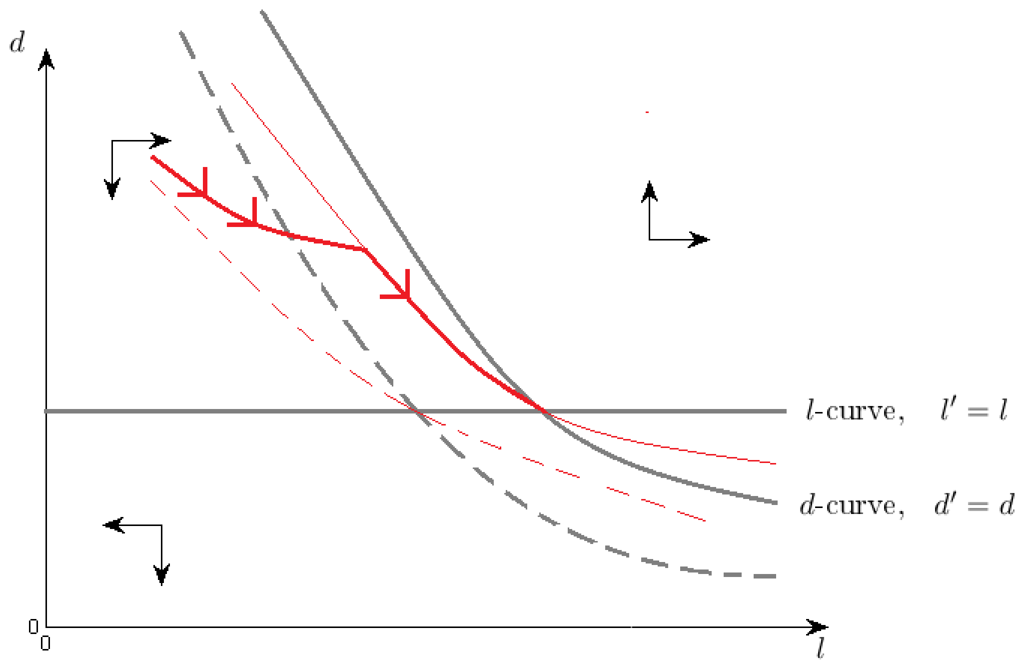

Figure 7: Phase diagram of the saddle path of an exporter in the no-screening case.

The local saddle path with screening – Intuitively the problem becomes similar to the case of no screening as ψ becomes larger. The formal proof requires to linearize the system of optimality conditions around the stationary state.

Lemma 1 (Properties of the value function)
$J(h, x; \phi, \Upsilon)$ is increasing in h, x and ϕ.

Proof: The Bellman equation (30) is equivalently defined on a compact state space $h \in [0, \overline{h}]$ where $\overline{h}$ is so large that it never binds. This is possible because of the dimishing returns of revenues and $\lim_{h\to\infty} r_h(h, x, \phi, \Upsilon) = 0$. The maximization in (30) defines an operator Θ which maps a function $J_0(h, x; \phi, \Upsilon)$ defined on $[0; \overline{h}] \times [x_{min}, x_{max}] \times [0, \overline{\phi}]$ into a function $J_1(h, x; \phi, \Upsilon) = \Theta(J_0)(h, x; \phi, \Upsilon)$ defined on the same domain. This operator is a contraction and it maps functions that are increasing in h, x and ϕ into functions with the same property. Therefore there exists a unique fixed point and it must be increasing in h, x and ϕ which follows from the differentiation of J with respect to h, x and ϕ.

B.4 Log linear cases

Case #1 – Consider the system of optimality conditions:

$$\begin{cases} l_{a+1} = l_a(1-s_0)(1+d_a) \\ l_{a+1}^{\rho\frac{\sigma-1}{\sigma}-1} = \hat{c}_A d_a^{\gamma-1} + \hat{w}_o - (1-\eta_0)\beta\left[\frac{\gamma-1}{\gamma}d_{a+1}+1\right]\hat{c}_A d_{a+1}^{\gamma-1} \\ \text{with } X \equiv \rho\frac{\sigma-1}{\sigma}x^{\frac{\sigma-1}{\sigma}}\Upsilon^{\frac{1}{\sigma}}\phi\left(\frac{\kappa}{\kappa-1}\right)^{\frac{\sigma-1}{\sigma}}, \qquad \hat{c}_A \equiv \frac{\tilde{c}_A}{X}, \qquad \hat{w}_o \equiv \frac{w_o}{X}, \end{cases}$$

If $(d_{a,*}, l_{a,*})_{a\geq 0}$ are solutions when $X=1$, then $(d_{a,*}, l_{a,*}.X^{\epsilon_l})_{a\geq 0}$ with $\epsilon_l = \frac{1}{1-\rho\frac{\sigma-1}{\sigma}}$ is a solution for any X.

Case #2 – Let $(\alpha_{a,*}, h_{a,*})_{a\geq 0}$ be the solution of the system:

$$\begin{cases} h_{a+1} = h_a(1-s)(1+d_a) \\ (1-\kappa\rho)\frac{\sigma-1}{\sigma}\alpha_{a+1}^{\frac{\sigma-1}{\sigma}(1-\kappa\rho)}\left(x\frac{\kappa}{\kappa-1}h_{a+1}^{\rho}\right)^{\frac{\sigma-1}{\sigma}}\Upsilon^{\frac{1}{\sigma}}\phi = \tilde{c}_A\kappa\frac{\gamma-1}{\gamma}\frac{d_a^{\gamma}h_{a+1}}{1+d_a}\alpha_{a+1}^{\kappa(\gamma-1)} + c_s\alpha_{a+1}^{\psi} \\ \rho\frac{\sigma-1}{\sigma}\alpha_{a+1}^{\frac{\sigma-1}{\sigma}(1-\kappa\rho)}\left(x\frac{\kappa}{\kappa-1}h_{a+1}{}^{\rho}\right)^{\frac{\sigma-1}{\sigma}-1}\Upsilon^{\frac{1}{\sigma}}\phi = \tilde{c}_A d_a^{\gamma-1}\alpha_{a+1}^{\kappa(\gamma-1)} - (1-\eta_0)\beta\tilde{c}_A d_{a+1}^{\gamma-1}\alpha_{a+2}^{\kappa(\gamma-1)}\left[1+\frac{\gamma-1}{\gamma}d_{a+1}\right] \end{cases}$$

when $X \equiv \left(x\frac{\kappa}{\kappa-1}\right)^{\frac{\sigma-1}{\sigma}}\Upsilon^{\frac{1}{\sigma}}\phi = 1$. It can be shown that there exists parameters ϵ_α and ϵ_h such that $(X^{\epsilon_\alpha}.\alpha_{c,a,*}, X^{\epsilon_h}.h_{a,*})_{a\geq 0}$ is a solution for any other $X>0$.

Proof. Suppose that $(X^{\epsilon_\alpha}.\alpha_{a,*}, X^{\epsilon_h}.h_{a,*})_{a\geq 0}$ is indeed a solution. It solves:

$$\begin{cases} X^{\epsilon_h}.h_{a+1} = X^{\epsilon_h}.h_a(1-s)(1+d_a) \\ \frac{(1-\kappa\rho)}{\kappa}\left(X^{\epsilon_\alpha}\alpha\right)_{a+1}^{\frac{\sigma-1}{\sigma}(1-\kappa\rho)}\left(X^{\epsilon_h}h\right)_{a+1}^{\rho\frac{\sigma-1}{\sigma}}X = \hat{c}_A\frac{\gamma-1}{\gamma}\frac{d_a^{\gamma}X^{\epsilon_h}h_{a+1}}{1+d_a}\left(X^{\epsilon_\alpha}\alpha_{a+1}\right)^{\kappa(\gamma-1)} + \hat{c}_s\left(X^{\epsilon_\alpha}\alpha_{a+1}\right)^{\psi} \\ \rho\left(X^{\epsilon_\alpha}\alpha_{a+1}\right)^{\frac{\sigma-1}{\sigma}(1-\kappa\rho)}\left(X^{\epsilon_h}h_{a+1}\right)^{\rho\frac{\sigma-1}{\sigma}-1}X = ... \\ \qquad ...\hat{c}_A d_a^{\gamma-1}\left(X^{\epsilon_\alpha}\alpha_{a+1}\right)^{\kappa(\gamma-1)} - (1-\eta_0)\beta\hat{c}_A d_{a+1}^{\gamma-1}\left(X^{\epsilon_\alpha}\alpha_{a+2}\right)^{\kappa(\gamma-1)}\left[1+\frac{\gamma-1}{\gamma}d_{a+1}\right] \\ \text{with } \hat{c}_A \equiv \tilde{c}_A\frac{\sigma}{\sigma-1}\left(\frac{\kappa}{\kappa-1}\right)^{-\frac{\sigma-1}{\sigma}} \text{ and } \hat{c}_S \equiv \frac{c_S}{\kappa}\frac{\sigma}{\sigma-1}\left(\frac{\kappa}{\kappa-1}\right)^{-\frac{\sigma-1}{\sigma}} \end{cases}$$

which is equivalent to the initial system when

$$\begin{cases} \epsilon_h = \epsilon_\alpha(\psi - \kappa(\gamma - 1)) \\ -\epsilon_h \left(1 - \rho\dfrac{\sigma - 1}{\sigma}\right) + 1 = \epsilon_\alpha \left(\kappa(\gamma - 1) - \dfrac{\sigma - 1}{\sigma}(1 - \kappa\rho)\right) \end{cases}$$

$$\Leftrightarrow \begin{cases} \epsilon_\alpha = \left[\psi\left(1 - \rho\dfrac{\sigma - 1}{\sigma}\right) - \dfrac{\sigma - 1}{\sigma}(1 - \kappa\gamma\rho)\right]^{-1} \\ \epsilon_h = \epsilon_\alpha(\psi - \kappa(\gamma - 1)) \end{cases}$$

End of Proof.

Then $(\Delta_{a,x}, l_{a,x}, \pi_{a,x}, w_{a,x})_{a\geq 0} = (X^{\epsilon_l}.\Delta_{a,*}, X^{\epsilon_l}.l_{a,*}, X^{\epsilon_\pi}.\pi a, *, X^{\epsilon_w}.wa, *)_{a\geq 0}$ with $\epsilon_l \equiv \epsilon_h - \kappa\epsilon_\alpha = \epsilon_\alpha(\psi - \kappa\gamma)$, $\epsilon_\pi \equiv 1 + \epsilon_\alpha\frac{\sigma-1}{\sigma}(1 - \kappa\rho) + \epsilon_h\rho\frac{\sigma-1}{\sigma} = \epsilon_\alpha\psi$, $\epsilon_w = \epsilon_\pi - \epsilon_\Delta = \epsilon_\alpha\kappa\gamma$ and where $l_{a,*} = \alpha_{a,*}^{-\kappa}h_{a,*}$, $\Delta_{a,*} = d_{a,*}h_{a,*}\alpha_{a+1,*}^{-\kappa}$, $\pi_{a,*} \equiv \left(\frac{\kappa}{\kappa-1}\alpha_{a+1,*}l_{a+1,*}^{\rho}\right)^{\frac{\sigma-1}{\sigma}} - A(\Delta_{a,*}, \alpha_{a+1,*}, \alpha_{a,*}, l_{a,*}) - \frac{c_s}{\psi}\alpha_{a+1,*}^{\psi}$, and $w_{a,*} = (1-\xi)(1-(1-\eta_0)\beta)^{A_{a,*}/\Delta_{a,*}}$. Note that employment at any given age $l_{a,x}$ is increasing in productivity if and only if $\psi > \kappa\gamma$. Moreover for any variable $z \in \{\alpha, h, l, \pi\}$ we can define firm the lifetime average $\overline{z}$ in a separable way:

$$\overline{z}(x, \Upsilon, \phi) = \sum_{a\geq 0}((1 - \delta_0)\beta)^a z_a = X^{\epsilon_z}.\overline{z}_* \qquad \text{with } \overline{z}_* \equiv \sum_{a\geq 0}((1 - \delta_0)\beta)^a z_{a,*}$$

B.5 Sectoral inequality in log linear cases

I formally define total, between-firm and within-firm wage inequality in the differentiated sector. I index cohorts of workers by c. The size $l'_{a,X,c}$ and wage $w_{a,X,c}$ of workers in every cohort are also log-linear in efficient productivity:

$$\begin{aligned} l'_{a,X,c} &= l'_{c,X,c}\left(\frac{\alpha'_{c,a,X,c}}{\alpha'_{c,a,X,a}}\right)(1 - \eta_0)^{a-c} = l'_{a,*,c}.X^{\epsilon_l} \quad \text{with } l'_{a,*,c} \equiv l'_{c,*,c}\left(\frac{\alpha'_{c,a,*,a}}{\alpha'_{c,a,*,c}}\right)(1 - \eta_0)^{a-c} \\ w_{a,X,c} &= w_{c,X,c}\left(\frac{\alpha'_{c,a,X,a}}{\alpha'_{c,a,X,c}}\right) = w_{a,*,c}.X^{\epsilon_w} \quad \text{with } w_{a,*,c} \equiv w_{c,*,c}\left(\frac{\alpha'_{c,a,*,a}}{\alpha'_{c,a,*,c}}\right) \end{aligned}$$

where I made use of equation (5) in order to derive the second line. We also have

$$l'_{a,X} = \sum_{c=1}^{a} l'_{a,X,c}, \quad \text{and define} \quad l'_a = \int_{\underline{x}}^{\infty} l'_{a,X}dF(x)$$

Log-linearity also carries thtough to the firm average wage $\hat{w}_{a,X}$ and firm normalized standard deviation $nsd_{a,X}$:

$$\hat{w}_{a,X} = \frac{\sum_{c=1}^{a} l'_{a,X,c} w_{a,X,c}}{\sum_{c=1}^{a} l'_{a,Xc}} = \hat{w}_{a,*}.X^{\epsilon_w} \qquad \text{with } \hat{w}_{a,*} = \frac{\sum_{c=1}^{a} l'_{a,*,c} w_{a,*,c}}{l'_{a,*}}$$

$$nsd_{a,X} = \frac{1}{\hat{w}_{a,X}} \sqrt{\frac{\sum_{c=1}^{a} l'_{a,X,c} \left(w_{a,X,c} - \hat{w}_{a,X}\right)^2}{\sum_{c=1}^{a} l'_{a,X,c}}} = \frac{1}{\hat{w}_{a,*}} \sqrt{\frac{\sum_{c=1}^{a} l'_{a,*,c} \left(w_{a,*,c} - \hat{w}_{a,*}\right)^2}{\sum_{c=1}^{a} l'_{a,*,c}}} = nsd_{a,*}$$

The mass of firms of age a is M_a. The definitions of inequality in the main text correspond to:

$$S_T = \frac{\sqrt{\sum_a M_a \int_{\underline{x}}^{\infty} \sum_c l'_{a,X,c} \left(w_{a,X,c} - \hat{w}\right)^2 dF(x)}}{\hat{w} \sum_a M_a \int_{\underline{x}}^{\infty} \sum_c l'_{a,X,c} dF(x)} \qquad \text{with } \hat{w} = \frac{\sum_a M_a \int_{\underline{x}}^{\infty} \sum_c l'_{a,X,c} w_{a,X,c}}{\sum_a M_a \int_{\underline{x}}^{\infty} \sum_c l'_{a,X,c}}$$

$$S_B = \frac{\sum_a M_a l'_a \frac{\sqrt{\int_{\underline{x}}^{\infty} l'_{a,X} \left(\hat{w}_{a,X} - \hat{w}_a\right)^2 dF(x)}}{\hat{w}_a \sqrt{\int_{\underline{x}}^{\infty} l'_{a,X} dF(x)}}}{\sum_a M_a l'_a} \qquad \text{with } \hat{w}_a = \int_{\underline{x}}^{\infty} \hat{w}_{a,X} dF(x)$$

$$S_W = \frac{\sum_a M_a \int_{\underline{x}}^{\infty} l'_{a,X} \frac{\sqrt{\sum_c l'_{a,X,c} \left(w_{a,X,c} - \hat{w}_{a,X}\right)^2}}{\hat{w}_{a,X} \sqrt{\sum_c l'_{a,X,c}}} dF(x)}{\sum_a M_a \int_{\underline{x}}^{\infty} l'_{a,X} dF(x)} = \frac{\sum_a M_a \int_{\underline{x}}^{\infty} l'_{a,X} nsd_{a,X} dF(x)}{\sum_a M_a \int_{\underline{x}}^{\infty} l'_{a,X} dF(x)}$$

In the steady state, the mass of firm evolves with the exogenous exit shock. There is a a constant mass of entrants at anytime and therefore $M_{a'}/M_a = (1-\delta_0)^{a'-a}$. In the special cases and using the log-linearity property, the between- and within-firm wage dispersion collapses to

$$S_B = \frac{\sum_a (1-\delta_0)^a l'_a \frac{\sqrt{\int_{\underline{x}}^{\infty} l'_{a,X} \left(\hat{w}_{a,X} - \hat{w}_a\right)^2 dF(x)}}{\hat{w}_a \sqrt{\int_{\underline{x}}^{\infty} l'_{a,X} dF(x)}}}{\sum_a (1-\delta_0)^a l'_a}$$

$$S_W = \frac{\sum_a (1-\delta_0)^a l'_{a,*} nsd_{a,*} \int_{\underline{x}}^{\infty} X^{\epsilon_l} dF(x)}{\sum_a (1-\delta_0)^a l'_{a,*} \int_{\underline{x}}^{\infty} X^{\epsilon_l} dF(x)} = \frac{\sum_a (1-\delta_0)^a l'_{a,*} nsd_{a,*}}{\sum_a (1-\delta_0)^a l'_{a,*}}$$

This proves that the chosen measure of within-firm dispersion is independent of the level of trade costs.

Firm average wages $\hat{w}_{a,X}$ and firm employment $l'_{a,X}$ follow log-linear equations in X of the same type as equation (17) in Helpman, Itskhoki and Redding (2010) (see proposition 3 for the present model). Effective productivity can indeed be defined in the exact same way in their model. The slope and the intercept of the equation differ across models but both are independent from trade costs. Therefore the full effect of trade on the dispersion of firm average wage operates only through the effective productivity term and specifically through export participation (I_X), and the degree of openness (Υ). The dispersion of effective productivity and consequently sectoral inequality is larger when some but not all firms export. Technically their proposition 3 applies for every cohort of firms:

for a given cohort of firms, between firm inequality is higher under trade than under autarky. Since the relative weight of one cohort relative to another is invariant to the level of trade costs, their proposition 3 carries through to S_B, the average across cohorts of the dispersion of firm average wage.

Under the assumption $\gamma = 1$, the elasticity of firm average wage with respect to firm effective productivity (ϵ_w) depends on the same parameters and in the same way as in Helpman, Itskhoki and Redding (2010): the predicted changes in sectoral inequality resulting from the changes in effective productivity allowed by lower trade costs would be quantitatively equivalent in both models provided that changes in the effective productivity are the same. When $\gamma > 1$ however, the convexity in adjustment costs magnifies the dispersion in firm average wages.

B.6 General equilibrium in the steady state

The steady state equilibrium is the equilibrium under which aggregates are constant. The equilibrium is defined as a list of policy functions $(\alpha'_{c,a,x}, l_{a,x}, \Delta_{a,x}, w_{a,x}, I_{X,a,x})_{\{a\geq 1, x\sim F()\}}$, an entry cutoff $\underline{x}$ and the constant macroeconomic variables (ϕ, Υ, P, M_E) such that:

1. unemployed workers make optimal decisions taking vacancy characteristics as given: (7) holds for all $(\alpha'_{c,a,x}, l_{a,x}, \Delta_{a,x}, w_{a,x})$) for all age a and $x \sim F()$ with $x \geq \underline{x}$
2. incumbent firms comply with wage commitments and maximize their value at all times taking aggregate conditions, labor supply (7), product demand and initial workforce characteristics $(\alpha_{c,0,x}, l_{0,x})$ as given: $(\alpha_{c,a,x}, \Delta_{a,x}, l_{a,x}, I_{X,a,x})$ solve (18) for all age a and $x \sim F()$ with $x \geq \underline{x}$
3. there is positive free entry of new firms (FE) and the zero-cutoff condition is satisfied (ZCP)
4. the product and labor markets clear ((GMC) and (LMC))

When $\beta < 1$ investors get positive profits/returns because steady state profits are larger than the sunk costsf_E[47].

In general .

The export cutoffs depend on age as well as productivity. Nevertheless I define export cutoffs by age $\underline{x}_{X,a}$. Note that the least productive exporters will be less productive than in the constrained

[47]See footnote 12 and 16 in Melitz (2003) for a discussion of the implications of $\beta < 1$

case.

ZCP: $G(\alpha_{c,0}, l_0, 0, \underline{x}; \phi) = 0$ defines a decreasing relationship between $\underline{x}$ and ϕ

FE: $(1-\delta_0) \int_{\underline{x}} G(\alpha_{c,0}, l_0, 0, x; \phi, \Upsilon) dF(x) = f_E,$ or equivalently

$$\sum_{a \geq 0} \beta^a (1-\delta_0)^{a+1} \int_{\underline{x}} \Pi'(\alpha_{c,a,x}, l_{a,x}, B_{a,x}, x; \phi, \Upsilon) dF(x) = f_E$$

with $\Pi'_{a,x} = r_{a,x} - w_o l'_{a,x} - \frac{c_s}{\psi} \overline{\alpha}'^{\psi}_{c,a,x} - f_d - I_{X,a,x} f_X - C_{a,x} - B'_{a,x}$

or equivalently $\Pi'_{a,x} = r_{a,x} - w_o l'_{a,x} - \frac{c_s}{\psi} \overline{\alpha}'^{\psi}_{c,a,x} - f_d - I_{X,a,x} f_X - A_{a,x} - c_B B_{a,x}$

(where dependence on macro aggregates is implicit, and variables are null if the firm has exited)

The left hand side is monotonically increasing in ϕ once I substitute $\underline{x}$ using (**FE**).

Therefore, ϕ is obtained from (**FE**) and (**ZCP**).

LMC $POP = M_E f_E + ...$

$$...M_E \sum_{a \geq 0} (1-\delta_0)^{a+1} \int_{\underline{x}} u_{a,x} - \Delta_{a,x} + l'_{a,x} + f_d + I_{X,a,x} f_X + C_{a,x} + \frac{c_s}{\psi} \overline{\alpha}'^{\psi}_{a,x} dF(x)$$

Once ϕ is known, I obtain M_E with the above labor market clearing condition.

GMC $M_E \sum_{a \geq 0} (1-\delta_0)^{a+1} \int_{\underline{x}} r_{a,x}(\phi, \Upsilon) dF(x) = Y = P^{1-\sigma} \phi^{\sigma}$ allows for solving P.

Revenues from wages: $M_E \sum_{a \geq 0} (1-\delta_0)^{a+1} \int_{\underline{x}} B'_{a,x} + w_o l'_{a,x} + f_d + I_{X,a,x} f_X + C_{a,x} + \frac{c_s}{\psi} \overline{\alpha}'^{\psi}_{c,a,x} dF(x)$

from investments: $M_E \sum_{a \geq 0} (1-\delta_0)^{a+1} \int_{\underline{x}} \Pi'_{a,x} dF(x) \neq M_E f_E$ if $\beta < 1$

Welfare: $\Omega = \frac{(\phi/P)^{\sigma}}{POP}$

B.7 Solving for the general equilibrium with a representative firm

In the special cases, the log linearity property allows me to take averages over generations and get analytical results.

In the log-linear special cases #1 and #2

$$\overline{z}(x, \Upsilon, \phi) = \delta_0 \sum_{a \geq 0} (1-\delta_0)^a z_{a,x}(\Upsilon, \phi) = X^{\epsilon_z} . \overline{z}_* \qquad \text{with } \overline{z}_* \equiv \delta_0 \sum_{a \geq 0} (1-\delta_0)^a z_{a,*}, \tag{22}$$

$$\text{and } X = x^{\frac{\sigma-1}{\sigma}} \left(\Upsilon(I_X)\right)^{\frac{1}{\sigma}} \phi$$

In solving for the general equilibrium, I will follow the steps detailed in Melitz (2003) in order to leverage on the knowledge of this paper and highlight similarities and differences.

Define gross profits π:

$$\pi = r(x\overline{\alpha}l'^{\rho}, I_X; \phi, \Upsilon) - \frac{c_S}{\psi}\overline{\alpha}^{\psi} - w_o l' - \xi A(\Delta, \alpha'_c, \check{l}) - B'$$

and make use of the special cases assumptions to obtain :

$$\text{Special case \# 1:} \qquad \pi_{a,x} = \pi_{a,*}.X^{\epsilon_\pi}, \quad \text{with } \pi_{a,*} = r_{a,*} - w_o {l'}_{a,*} - \xi A(\Delta_{a,*}, \alpha_{min}, \check{l}_{a,*}) - B'_{a,*}$$

$$\text{Special case \# 2:} \qquad \pi_{a,x} = \pi_{a,*}.X^{\epsilon_\pi} \quad \text{with } \pi_{a,*} = r_{a,*} - \frac{c_S}{\psi}\overline{\alpha}^{\psi}_{a,*} - \xi A(\Delta_{a,*}, \alpha_{c,a,*}, \check{l}_{a,*}) - B'_{a,*}$$

where the associated elasticity is $\epsilon_\pi = \epsilon_r$ and where $\pi_{a,*}, \epsilon_\pi$ are actually different in the two cases.

I first solve for entry ($\underline{x}$) and export cutoffs ($\underline{x}_X$). Respectively the marginal entrant and the marginal exporter make no profits from entry and export:

$$\overline{\pi}_*.\left(\underline{x}^{\frac{\sigma-1}{\sigma}}\phi\right)^{\epsilon_\pi} - f_d = 0 \quad \text{and} \quad \left(\Upsilon^{\frac{\epsilon_\pi}{\sigma}} - 1\right)\overline{\pi}_*.\left(\underline{x}_X^{\frac{\sigma-1}{\sigma}}\phi\right)^{\epsilon_\pi} - f_X = 0$$

where the lifetime average gross profits $\overline{\pi}_*$ (gross of fixed costs) is defined as explained above. The second equation equates the lifetime average difference in profits from exporting or not to the lifetime average of the fixed export cost. Combining these equations, I get a relation for the relative thresholds:

$$\left(\frac{\underline{x}}{\underline{x}_X}\right)^{\epsilon_\pi\frac{\sigma-1}{\sigma}} = (\Upsilon^{\frac{\epsilon_\pi}{\sigma}} - 1)\frac{f_d}{f_X}$$

This equation turns out to be useful to characterize $p_X = \frac{1-F(\underline{x}_X)}{1-F(\underline{x})}$ the probability for a firm to export. Indeed, the properties of the Pareto distribution implies that $p_X = \left(\frac{\underline{x}}{\underline{x}_X}\right)^{\theta} = \min\left\{1, \left((\Upsilon^{\frac{\epsilon_\pi}{\sigma}} - 1)\frac{f_d}{f_X}\right)^{\frac{\theta}{\epsilon_\pi\frac{\sigma-1}{\sigma}}}\right.$

At this point it is useful to define the mass of all varieties in the economy given the constant mass M_E of entrants:

$$M_A \equiv (1-F(\underline{x}))\sum_{a\geq 0} M_E(1-\delta_0)^{a+1} + (1-F(\underline{x}_X))\sum_{a\geq 0} M_E(1-\delta_0)^{a+1} = \frac{1-\delta_0}{\delta_0}M_E(1-F(\underline{x})+1-F(\underline{x}_X))$$

where F is the cumulative distribution function of the productivity draws, $M_E(1-\delta_0)^{a+1}(1-F(\underline{x}))$ is the mass of firms in the cohort of age a and $(1-F(\underline{x}_X))M_E(1-\delta_0)^{a+1}$ is the mass of foreign varieties produced by firms of age a.

I define different average productivities depending that can be adapted to the variable z being considered:

$$\tilde{x}(\underline{x}; \epsilon_z) \equiv \left[\frac{1}{1-F(\underline{x})}\int_{\underline{x}}^{\infty} x^{\epsilon_z\frac{\sigma-1}{\sigma}}dF(x)\right]^{\frac{1}{\epsilon_z}\frac{\sigma}{\sigma-1}}$$

which helps simplify the formulation of the average productivity of domestic and foreign firms competing in each market:

$$\begin{aligned}
\tilde{x}_A(\epsilon_z) \quad & \equiv \left[\frac{\sum_{a\geq 0} M_E(1-\delta_0)^{a+1}}{M_A}\left(\int_{\underline{x}}^{\infty} x^{\epsilon_z\frac{\sigma-1}{\sigma}} dF(x) + \int_{\underline{x}_X}^{\infty} (x)^{\epsilon_z\frac{\sigma-1}{\sigma}} (\Upsilon^{\frac{\epsilon_z}{\sigma}} - 1) dF(x)\right)\right]^{\frac{1}{\epsilon_z}\frac{\sigma}{\sigma-1}} \\
& = \left[\frac{M_E(1-\delta_0)}{\delta_0 M_A}(1-F(\underline{x}))\left(\tilde{x}(\underline{x};\epsilon_z)^{\epsilon_z\frac{\sigma-1}{\sigma}} + p_X(\Upsilon^{\frac{\epsilon_z}{\sigma}}-1)\tilde{x}(\underline{x}_X;\epsilon_z)^{\epsilon_z\frac{\sigma-1}{\sigma}}\right)\right]^{\frac{1}{\epsilon_z}\frac{\sigma}{\sigma-1}}
\end{aligned}$$

The aggregate price index can be related to the average productivity as follows:

$$\begin{aligned}
P \quad & \equiv \left[\sum_{a\geq 0} M_E(1-\delta_0)^{a+1}\int_{\underline{x}}^{\infty}(p_a^D(x))^{1-\sigma}dF(x) + \sum_{a\geq 0} M_E(1-\delta_0)^{a+1}\int_{\underline{x}_X}^{\infty}(p_a^F(x))^{1-\sigma}dF(x)\right]^{\frac{1}{1-\sigma}} \\
& = \left[\sum_{a\geq 0} M_E(1-\delta_0)^{a+1}\left(\int_{\underline{x}}^{\infty} r_a(x)\phi^{-\sigma}\Upsilon(I_X(x))^{-1}dF(x) + \int_{\underline{x}_X}^{\infty} r_a(x)\phi^{-\sigma}\Upsilon^{-1}\tau^{1-\sigma}\right)dF(x)\right]^{\frac{1}{1-\sigma}} \\
& = \left[\sum_{a\geq 0} M_E(1-\delta_0)^{a+1}\left(\int_{\underline{x}}^{\infty} r_a(x)\phi^{-\sigma}dF(x) + \int_{\underline{x}_X}^{\infty} r_a(x)\phi^{-\sigma}\underbrace{(\Upsilon^{-1}\tau^{1-\sigma}+\Upsilon^{-1}-1)}_{=0}dF(x)\right)\right]^{\frac{1}{1-\sigma}} \\
& = \left[M_E\frac{1-\delta_0}{\delta_0}\phi^{-\sigma}\left(\int_{\underline{x}}^{\infty}\overline{r}_* x^{\epsilon_\pi\frac{\sigma-1}{\sigma}}\phi^{\epsilon_\pi}dF(x) + \int_{\underline{x}_X}^{\infty}\overline{r}_* x^{\epsilon_\pi\frac{\sigma-1}{\sigma}}\phi^{\epsilon_\pi}(\Upsilon^{\frac{\epsilon_\pi}{\sigma}}-1)dF(x)\right)\right]^{\frac{1}{1-\sigma}} \\
& = \left[\overline{r}_* M_A\tilde{x}_A(\epsilon_\pi)^{\epsilon_\pi\frac{\sigma-1}{\sigma}}\phi^{\epsilon_\pi}\right]^{\frac{1}{1-\sigma}} \Leftrightarrow Y = P^{1-\sigma}\phi^{\sigma} = \overline{r}_* M_A\tilde{x}_A(\epsilon_\pi)^{\epsilon_\pi\frac{\sigma-1}{\sigma}}\phi^{\epsilon_\pi}
\end{aligned}$$

The zero-profit curve makes use of the two cutoff equations in order to compute net average profits $\widetilde{\Pi}$ in each country:

$$\widetilde{\Pi} = \overline{\pi}_*\tilde{x}(\underline{x};\epsilon_\pi)^{\epsilon_\pi\frac{\sigma-1}{\sigma}}\phi^{\epsilon_\pi} - f_d + p_X(\Upsilon^{\frac{\epsilon_\pi}{\sigma}}-1)\overline{\pi}_*\tilde{x}(\underline{x}_X;\epsilon_\pi)^{\epsilon_\pi\frac{\sigma-1}{\sigma}}\phi^{\epsilon_\pi} - p_X f_X \qquad (\mathbf{ZCP})$$

$$\widetilde{\Pi} = f_d k(\underline{x};\epsilon_\pi) + p_X f_X k(\underline{x}_X;\epsilon_\pi) \qquad \overset{\text{Pareto}}{=} k f_d.\,(1+\mathcal{F}), \quad \text{with } \mathcal{F} \equiv \left(\frac{f_X}{f_d}\right)^{\frac{\epsilon_\pi\frac{\sigma-1}{\sigma}-\theta}{\epsilon_\pi\frac{\sigma-1}{\sigma}}} (\Upsilon^{\frac{\epsilon_\pi}{\sigma}}-1)^{\frac{\theta}{\epsilon_\pi\frac{\sigma-1}{\sigma}}}$$

where the function k is defined as $k(\underline{x};\epsilon_\pi) = \left(\frac{\tilde{x}(\underline{x};\epsilon_\pi)}{\underline{x}}\right)^{\epsilon_\pi\frac{\sigma-1}{\sigma}} - 1$ and simplifies to $k(\epsilon_\pi) = \frac{\theta}{\theta-\epsilon_z\frac{\sigma-1}{\sigma}} - 1$ under the assumption of a Pareto distribution. The free entry equation can be simplified from the general case ($\beta = 1$ in special cases) and becomes:

$$\frac{(1-\delta_0)}{\delta_0}(1-F(\underline{x}))\widetilde{\Pi} - f_E = 0 \Leftrightarrow \widetilde{\Pi} = \frac{\delta_0/(1-\delta_0) f_E}{1-F(\underline{x})} \qquad \overset{\text{Pareto}}{=} \delta_0/(1-\delta_0) f_E \underline{x}^{\theta} \qquad (\mathbf{FE})$$

As in the original Melitz (2003) model, combining the (**ZCP**) and (**FE**) equations solves for the entry cutoff $\underline{x}$. Moreover, if the variable trade costs, all fixed costs, the productivity distribution and the elasticity of profits with respect to productivity are the same ($\epsilon_\pi = \sigma$), then the models generate the exact same cutoffs.

From there the market conditions can be recovered from the (**ZCP**) alone: $\phi = \underline{x}^{-\frac{\sigma-1}{\sigma}} \left(\frac{f_d}{\overline{\pi}_*}\right)^{\frac{1}{\epsilon_\pi}}$. Then the mass of entrants is obtained from the labor market clearing condition:

$$M_A \left((\overline{u}_* + \xi \overline{A}_* + \frac{c_S}{\psi} \overline{\alpha}_*^\psi) \tilde{x}_A(\epsilon_\pi)^{\epsilon_\pi \frac{\sigma-1}{\sigma}} \phi^{\epsilon_\pi} + (\overline{l'}_* - \overline{\Delta}_*) \tilde{x}_A(\epsilon_l)^{\epsilon_l \frac{\sigma-1}{\sigma}} \phi^{\epsilon_l} \right) + M_F(f_d + p_X f_X) + M_E f_E = POP$$

where $M_F = \dfrac{(1-\delta_0)}{\delta_0}(1 - F(\underline{x}))$ is the mass of firms.

Then use the (**FE**) and the special case assumption $\beta = 1$ to obtain:

$$M_A \overline{\pi}_* \tilde{x}_A(\epsilon_\pi)^{\epsilon_\pi \frac{\sigma-1}{\sigma}} \phi^{\epsilon_\pi} - M_E \frac{(1-\delta_0)}{\delta_0} (1 - F(\underline{x}))\,(f_d + p_X f_X) = M_E f_E$$

and

$$M_A \left(\left(\overline{\pi}_* + \overline{u}_* + \xi \overline{A}_* + \frac{c_S}{\psi} \overline{\alpha}_*^\psi \right) \tilde{x}_A(\epsilon_\pi)^{\epsilon_\pi \frac{\sigma-1}{\sigma}} \phi^{\epsilon_\pi} + (\overline{l'}_* - \overline{\Delta}_*) \tilde{x}_A(\epsilon_l)^{\epsilon_l \frac{\sigma-1}{\sigma}} \phi^{\epsilon_l} \right) = POP$$

and specifically

case #1: $M_A b_{1,*} \tilde{x}_A(\epsilon_\pi)^{\epsilon_\pi \frac{\sigma-1}{\sigma}} \phi^{\epsilon_\pi} = POP \qquad \text{with } b_{1,*} \equiv \overline{\pi}_* + \overline{u}_* + \xi \overline{A}_* + \overline{l'}_* - \overline{\Delta}_*$

case #2: $M_A b_{2a,*} \tilde{x}_A(\epsilon_\pi)^{\epsilon_\pi \frac{\sigma-1}{\sigma}} \phi^{\epsilon_\pi} \left(1 + \dfrac{b_{2b,*} \tilde{x}_A(\epsilon_l)^{\epsilon_l \frac{\sigma-1}{\sigma}}}{b_{2a,*} \tilde{x}_A(\epsilon_\pi)^{\epsilon_\pi \frac{\sigma-1}{\sigma}}} \phi^{\epsilon_l - \epsilon_\pi} \right) = POP$

with $b_{2a,*} \equiv \overline{\pi}_* + \overline{u}_* + \xi \overline{A}_* + \dfrac{c_S}{\psi} \overline{\alpha}_*^\psi; \;\; b_{2b,*} = \overline{l'}_* - \overline{\Delta}_*$

In the second special case, few more steps are necessary to simplify the labor market clearing condition. Thanks to the property of the Pareto distribution, and in particular the fact that $p_X = \left(\frac{\underline{x}}{\underline{x}_X}\right)^\theta = \min\left\{1, \left((\Upsilon^{\frac{\epsilon_\pi}{\sigma}} - 1)\frac{f_d}{f_X}\right)^{\frac{\theta}{\epsilon_\pi \frac{\sigma-1}{\sigma}}}\right\}$, I obtain and define:

$$\begin{aligned}
\frac{b_{2b,*} \tilde{x}_A(\epsilon_l)^{\epsilon_l \frac{\sigma-1}{\sigma}}}{b_{2a,*} \tilde{x}_A(\epsilon_\pi)^{\epsilon_\pi \frac{\sigma-1}{\sigma}}} \phi^{\epsilon_l - \epsilon_\pi} &= \frac{b_{2b,*} \left(\underline{x}^{\frac{\sigma-1}{\sigma}} \phi\right)^{\epsilon_l} (k(\epsilon_l)+1) \left(1 + p_X (\Upsilon^{\frac{\epsilon_l}{\sigma}} - 1) \left(\frac{\underline{x}_X}{\underline{x}}\right)^{\epsilon_l \frac{\sigma-1}{\sigma}}\right)}{b_{2a,*} \left(\underline{x}^{\frac{\sigma-1}{\sigma}} \phi\right)^{\epsilon_\pi} (k(\epsilon_\pi)+1) \left(1 + p_X (\Upsilon^{\frac{\epsilon_\pi}{\sigma}} - 1) \left(\frac{\underline{x}_X}{\underline{x}}\right)^{\epsilon_\pi \frac{\sigma-1}{\sigma}}\right)} \\
&= \frac{b_{2b,*} \left(\frac{f_d}{\overline{\pi}_*}\right)^{\frac{\epsilon_l}{\epsilon_\pi}} (k(\epsilon_l)+1) \left(1 + (\Upsilon^{\frac{\epsilon_l}{\sigma}} - 1) \left((\Upsilon^{\frac{\epsilon_\pi}{\sigma}} - 1)\frac{f_d}{f_X}\right)^{\frac{\theta}{\epsilon_\pi \frac{\sigma-1}{\sigma}} - \frac{\epsilon_l}{\epsilon_\pi}}\right)}{b_{2a,*} \left(\frac{f_d}{\overline{\pi}_*}\right) (k(\epsilon_\pi)+1) \left(1 + (\Upsilon^{\frac{\epsilon_\pi}{\sigma}} - 1)^{\frac{\theta}{\epsilon_\pi \frac{\sigma-1}{\sigma}}} \left(\frac{f_d}{f_X}\right)^{\frac{\theta}{\epsilon_\pi \frac{\sigma-1}{\sigma}} - 1}\right)} \\
&= b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)
\end{aligned}$$

Under autarky, $b = \frac{b_{2b,*}\left(\frac{f_d}{\overline{\pi}_*}\right)^{\frac{\epsilon_l}{\epsilon_\pi}} (k(\epsilon_l)+1)}{b_{2a,*}\left(\frac{f_d}{\overline{\pi}_*}\right)(k(\epsilon_\pi)+1)}$. I then prove that b is smaller under trade than under autarky by showing that:

$$(\Upsilon^{\frac{\epsilon_l}{\sigma}}-1)\left((\Upsilon^{\frac{\epsilon_\pi}{\sigma}}-1)\frac{f_d}{f_X}\right)^{\frac{\theta}{\epsilon_\pi\frac{\sigma-1}{\sigma}}-\frac{\epsilon_l}{\epsilon_\pi}} < (\Upsilon^{\frac{\epsilon_\pi}{\sigma}}-1)^{\frac{\theta}{\epsilon_\pi\frac{\sigma-1}{\sigma}}}\left(\frac{f_d}{f_X}\right)^{\frac{\theta}{\epsilon_\pi\frac{\sigma-1}{\sigma}}-1}$$

because $\epsilon_l < \epsilon_\pi$ and

$$(\Upsilon^{\frac{\epsilon_l}{\sigma}}-1)\frac{f_d}{f_X} < (\Upsilon^{\frac{\epsilon_\pi}{\sigma}}-1)\frac{f_d}{f_X} < \left((\Upsilon^{\frac{\epsilon_\pi}{\sigma}}-1)\frac{f_d}{f_X}\right)^{\frac{\epsilon_l}{\epsilon_\pi}} < 1$$

With a slight abuse of notation, I re-write the labor market clearing condition in both cases as:

$$M_A\tilde{x}_A(\epsilon_\pi)^{\epsilon_\pi\frac{\sigma-1}{\sigma}}\phi^{\epsilon_\pi}b_*\left(1+b\left(\Upsilon,\frac{f_d}{f_X};\epsilon_\pi,\epsilon_l\right)\right) = POP$$

where $\left(b_*, b\left(\Upsilon,\frac{f_d}{f_X};\epsilon_\pi,\epsilon_l\right)\right)$ are $(b_{1,*},0)$ in the first special case, and $\left(b_{2a,*}, b\left(\Upsilon,\frac{f_d}{f_X};\epsilon_\pi,\epsilon_l\right)\right)$ in the second.

I use the above equation, the good market clearing condition together with the (ZCP) condition to eliminate ϕ and solve for the price index:

$$M_A\overline{r}_*\tilde{x}_A^{\epsilon_\pi\frac{\sigma-1}{\sigma}}\phi^{\epsilon_\pi} = P.C = P^{1-\sigma}\phi^\sigma \quad \Rightarrow P = \left(\frac{b_*\left(1+b\left(\Upsilon,\frac{f_d}{f_X};\epsilon_\pi,\epsilon_l\right)\right)}{\overline{r}_*POP}\right)^{\frac{1}{\sigma-1}}\frac{1}{\underline{x}}\left(\frac{f_d}{\overline{\pi}_*}\right)^{\frac{1}{\epsilon_\pi}\frac{\sigma}{\sigma-1}}$$

Finally welfare measured as annual consumption per capita is

$$\Omega = \frac{C}{POP} = (\tfrac{\phi}{P})^\sigma/POP = \left(\frac{1}{POP}\right)^{\frac{1}{\sigma-1}}\left(\frac{\overline{r}_*}{b_*\left(1+b\left(\Upsilon,\frac{f_d}{f_X};\epsilon_\pi,\epsilon_l\right)\right)}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\overline{\pi}_*}{f_d}\right)^{\frac{1}{\epsilon_\pi}\frac{\sigma}{\sigma-1}}\underline{x} \tag{23}$$

In the **special case #1**, the trade costs only affect welfare through the entry cutoff. Because lowering trade costs causes an increase in the latter, welfare is increasing in the degree of openness. In the **special case #2**, I get the weaker result that welfare in higher under trade. On the one hand it rises because of the increase in average productivity operating though stricter selection at entry. on the other hand, $b()$ is lower under trade, reflecting productivity gains from more screening.

Unemployment – The sectoral unemployment rate at the end of each period is the ratio of unmatched job-searchers to the sum of job-searchers and employed workers:

$$\begin{aligned} U &= \frac{M_A\tilde{x}_A^{\epsilon_\pi\frac{\sigma-1}{\sigma}}\phi^{\epsilon_\pi}\left(\overline{u}_*-\overline{\Delta}_*\left(1+b\left(\Upsilon,\frac{f_d}{f_X};\epsilon_\pi,\epsilon_l\right)\right)\right)}{M_A\tilde{x}_A^{\epsilon_\pi\frac{\sigma-1}{\sigma}}\phi^{\epsilon_\pi}\left(\overline{u}_*+(\overline{l'}_*-\overline{\Delta}_*)\left(1+b\left(\Upsilon,\frac{f_d}{f_X};\epsilon_\pi,\epsilon_l\right)\right)\right)} \\ &= 1-\frac{\overline{l'}\left(1+b\left(\Upsilon,\frac{f_d}{f_X};\epsilon_\pi,\epsilon_l\right)\right)}{\overline{u}_*+(\overline{l'}_*-\overline{\Delta}_*)\left(1+b\left(\Upsilon,\frac{f_d}{f_X};\epsilon_\pi,\epsilon_l\right)\right)} \end{aligned}$$

Because $b()$ is larger under autarky, the above equation implies that the sectoral unemployment rate is higher in the steady state open economy. This long run effect results from two effects. First there is a reallocation towards the more productive firms with a higher level of screening and vacancies with longer queues. Second, the new market opportunities provide incentives for exporters to screen even more and post vacancies with even longer queues.

Alternatively, one could have defined the unemployment rate $\widetilde{U}$ as the ratio of end of period unemployed workers over total population. In this case, I show that the same result applies: unemployment is higher under trade than under autarky:

$$\begin{aligned}
\widetilde{U} &= \frac{\overline{u}_* - \overline{\Delta}_* \left(1 + b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)}{\overline{\pi}_* + \xi\overline{A}_* + \frac{c_S}{\psi}\overline{\alpha}_*^\psi + \overline{u}_* + (\overline{l'}_* - \overline{\Delta}_*)\left(1 + b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)} \\
\frac{\partial \widetilde{U}}{\partial b} &= \frac{-\overline{\Delta}_* \left(\overline{\pi}_* + \xi\overline{A}_* + \frac{c_S}{\psi}\overline{\alpha}_*^\psi + \overline{u}_*\right) - (\overline{l'}_* - \overline{\Delta}_*)\overline{u}_*}{\left[\overline{\pi}_* + \xi\overline{A}_* + \frac{c_S}{\psi}\overline{\alpha}_*^\psi + \overline{u}_* + (\overline{l'}_* - \overline{\Delta}_*)\left(1 + b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)\right]^2} < 0
\end{aligned}$$

Labor allocation across sectors – Let LF_p be the labor share of production workers, including unemployed job-seekers:

$$\begin{aligned}
LF_p &= \frac{M_A \tilde{x}_A^{\epsilon_\pi \frac{\sigma-1}{\sigma}} \phi^{\epsilon_\pi} (\overline{l'}_* - \overline{\Delta}_*)\left(1 + b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)}{M_A \tilde{x}_A^{\epsilon_\pi \frac{\sigma-1}{\sigma}} \phi^{\epsilon_\pi} \left(\overline{\pi}_* + \xi\overline{A}_* + \frac{c_S}{\psi}\overline{\alpha}_*^\psi + \overline{u}_* + (\overline{l'}_* - \overline{\Delta}_*)\left(1 + b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)\right)} \\
&= 1 - \frac{\overline{\pi}_* + \xi\overline{A}_* + \frac{c_S}{\psi}\overline{\alpha}_*^\psi}{\overline{\pi}_* + \xi\overline{A}_* + \frac{c_S}{\psi}\overline{\alpha}_*^\psi + \overline{u}_* + (\overline{l'}_* - \overline{\Delta}_*)\left(1 + b\left(\Upsilon, \frac{f_d}{f_X}; \epsilon_\pi, \epsilon_l\right)\right)}
\end{aligned}$$

Because $b()$ is larger under autarky, the above equation implies that the share of workers searching or working in the differentiated sector is lower in the steady state open economy. This long run effect results from two effects. First there is a reallocation towards the more productive firms with a higher level of screening and a higher workforce average productivity that use relatively less workers and more services. Second, the new market opportunities provide incentives for exporters to screen even more and use even less production workers.

B.8 The transition path equilibrium following a reduction in trade costs

Assumption 1 – Declining firms are constrained to choose a new screening threshold equal to the new stationary screening level $\alpha'_{c,\infty}$ and keep it constant until they exit: $\alpha'_{c,\infty} \geq \alpha'_c \geq min(\alpha_c, \alpha'_{c,\infty})$

The new firm optimization problem is the generalization of equation (17) to a richer environment:

$$\begin{aligned} G(\alpha_c, l, B, x; \Phi) \quad &= \quad \max_{\delta, l', \alpha_c', I_X} (1-\delta) \Big\{ r - w_o l' - B' - \xi A - \frac{c_s}{\psi} \overline{\alpha}'^{\psi} - f_d - I_X f_X ... \\ &\qquad\qquad ... + \frac{1}{R'} G\left(\alpha_c', l', \beta R' B', x; \Phi\right) \Big\} \\ \text{s.t.} \qquad & \Delta \geq 0, \;\; \delta \geq \delta_0, \;\; s \geq s_0, \quad \alpha_{c,\infty}' \geq \alpha_c' \geq min(\alpha_c, \alpha_{c,\infty}') \\ & A = \frac{c_A}{\gamma} \Delta \left(\frac{\Delta}{\check{l}}\right)^{\gamma-1} \left(\frac{\alpha_c'}{\alpha_{min}}\right)^{\kappa\gamma} \text{ with } \check{l} = (1-s) l \left(\frac{\alpha_c}{\alpha_c'}\right)^{\kappa} \\ & w = w_o + (1-\xi)(1-\beta(1-\eta_0)) \frac{A}{\Delta} \\ & r = \left(x \overline{\alpha} l'^{\rho}\right)^{\frac{\sigma-1}{\sigma}} \phi \left(\Upsilon(I_X)\right)^{\frac{1}{\sigma}} \qquad\qquad \text{with } \overline{\alpha} = \frac{\kappa}{\kappa-1} \alpha_c' \\ & l' = (1-s) \left(\frac{\alpha_c}{\alpha_c'}\right)^{\kappa} l + \Delta, \qquad\qquad B' = B(1-s_0) + \Delta.(w - w_o) \\ & R' = \frac{1}{\beta} \left(\frac{\phi'}{\phi}\right)^{\sigma IES} \left(\frac{P'}{P}\right)^{1-\sigma IES} \end{aligned} \tag{33}$$

The value function depends on the entire history and future path of aggregate conditions as represented by $\Phi = (\phi, R)_{t=-\infty..+\infty}$. Firms may decide to reduce the number of workers they employ through exogenous attrition ($\Delta < s_0 l$) or firing ($s > s_0$). At this point, I still assume that firms still pay their incumbent workers in accordance with initial commitments at all times.

As in the steady state, I prove that the above problem is separable when firms don't chose to exit $\delta = \delta_0$. I guess that the value function is linear: $G(\alpha_c, l, B, x; \Phi) = J(\alpha_c, l, , x; \Phi) - c_B B$ and show that there exist a constant $c_B = \frac{1-\eta_0}{1-\beta(1-\eta_0)}$ such that this is the case:

$$\begin{aligned} J(\alpha_c, l, x; \phi, \Upsilon) - c_B B \quad &= \quad \max_{\Delta, \Lambda, w, \alpha_c', I_X} (1-\delta_0) \Big\{ r - w_o l' - \xi A - \frac{c_s}{\psi} \overline{\alpha}'^{\psi} - f_d - I_X f_X ... \\ &\qquad ... - (1 + \frac{1}{R'} c_B \beta R') \Delta.(w - w_o) + \frac{1}{R'} J(a_c', l', x; \phi, \Upsilon) \Big\} \\ &\qquad ... - \underbrace{(1-s_0)(1-\delta_0)(1 + \frac{1}{R'} c_B \beta R')}_{=c_B} B \end{aligned}$$

Assumption 2 – All firms with values that are negative after having set wage premia commitments to zero exit: $\delta'_{a,x,t_0} = \mathbb{I}_{G_{t_0}(\alpha_c', l', 0, x; \Phi) < 0}$. Otherwise firms that are such that $G_{t_0}(\alpha_c', l', B', x; \Phi) < 0 \leq G_{t_0}(\alpha_c', l', 0, x; \Phi)$ renegotiate wages with their incumbent workers. Specifically, wage cuts $cut'_{t_0,a,x}$ amount to the minimum decreases ensuring that the value of renegotiating firms is non-negative: $G_{t_0}(\alpha_c', l', B' - cut'_{t_0,a,x}, x; \Phi) = 0$. The workers at a renegotiating firm (a, x) are all assumed to get the same percentage cut.

Separability allows me to derive the per-period wage bill cut:

$$0 = G_{t_0}(\alpha'_c, l', B' - cut'_{t_0,a,x}, x; \Phi) = J_{t_0}(\alpha_c, l, x; \phi, \Upsilon) - c_B(B' - cut'_{t_0,a,x})$$
$$\Rightarrow cut'_{t_0,a,x} = \frac{1}{c_B}\left(G_{t_0}(\alpha'_c, l', B', x; \Phi) - J_{t_0}(\alpha_c, l, x; \phi, \Upsilon)\right)$$

Definition of the transition path equilibrium –An equilibrium path following a reduction of $(\tau_0, f_{X,0})$ to $(\tau_\infty, f_{X,\infty})$ at $t = t_0$ is a list of exit and renegotiation decisions at the time of the shock $(\delta'_{a,x,t_0}, cut'_{a,x,t_0})_{\{a>0, x\sim F()\}}$, policy functions $(\delta_{a,x,t}, \alpha'_{c,a,x,t}, l_{a,x,t}, \Delta_{a,x,t}, w_{a,x,t}, I_{X,a,x,t})_{\{a\geq 1, x\sim F(), t>t_0\}}$, a sequence of entry cutoffs $\underline{x}_{t>t_0}$ and a sequence of variables $(\phi_t, P_t, M_{E,t})_{\{t\geq t_0\}}$ such that:

1. $(\alpha'_{c,a,x,t_0}, l_{a,x,t_0}, \Delta_{a,x,t_0}, w_{a,x,t_0}, I_{X,a,x,t_0})_{\{a\geq 1, x\sim F()\}}$ and $(\phi_{t_0}, P_{t_0}, M_{E,t_0})$ is a steady state equilibrium with $(\tau_0, f_{X,0})$.
2. At $t = t_0$, firms with negative value negotiate wage cuts cut'_{a,x,t_0} or exit δ'_{a,x,t_0} in accordance with assumption 2
3. unemployed workers make optimal decisions taking vacancy characteristics as given: (7) holds for all $(\Delta_{a,x,t}, l_{a,x,t}, w_{a,x,t})$ for all a, all $x \sim F()$ and all $t > t_0$ such that $\delta_{a,x,t} = 0$
4. incumbent firms comply with wage commitments and maximize their value at all $t > t_0$ taking aggregate conditions, labor supply (7), product demand and initial workforce characteristics $(\alpha'_{c,a,x,t_0}, l_{a,x,t_0}, \Delta_{a,x,t_0}, w_{a,x,t_0}, I_{X,a,x,t_0})$ and $(\alpha_{c,0,x}, l_{0,x})$ as given: $(\delta_{a,x,t}, \alpha_{c,a,x}, \Delta_{a,x}, l_{a,x}, I_{X,a,x})$ solve (33) for all a, all $x \sim F()$ and all $t > t_0$ such that $\delta_{a,x,t} = 0$
5. there is positive free entry of new firms (FE) at the zero-cutoff condition (ZCP) is satisfied for all $t > t_0$
6. the product and labor markets clear for all $t > t_0$ ((GMC) and (LMC))

In order to obtain analytical predictions for the transition path, I consider the special case in which the inverse elasticity of intertemporal substitution takes a particular value:

Special case #3 – I assume $IES\sigma = 1$ and that the reduction in trade costs is "not too big".

In the special case #3, firm values are independent of the evolution of the price index and only depend on the sequence of macro-conditions $\Phi = (\phi_t)_{t=-\infty..+\infty}$. The zero-cutoff and the free entry conditions ((ZCP), (FE)) are purely forward looking conditions. It is straightforward to verify that the sequence of conditions are satisfied by $\phi_t = \phi_\infty$ and $\underline{x}_t = \underline{x}_\infty$ for all $t > t_0$ where $(\phi_\infty, \underline{x}_\infty)$ are the new steady state values. At every period and given $\phi_t = \phi_\infty$, there is always a price index that can clear the good market. Therefore the only condition that remains to be checked is the labor market clearing condition. This is important because, a positive mass of entrants is needed to ensure that the free entry condition holds with equality. While it is not guaranteed that M_E is never null in general, this could be obtained with a sufficiently small reduction in trade costs.

Lemma 2 (Effective productivity comparison)

In the new steady state with lower trade costs, the effective productivity of exporters must be larger than in the initial steady state: $\phi_\infty \Upsilon_\infty^{\frac{1}{\sigma}} \geq \phi_0 \Upsilon_0^{\frac{1}{\sigma}}$.

Proof by contradiction: If exporters experienced a drop in effective productivities, then the expected value of creating a new firm would be strictly lower than in the initial steady state for all productivity x. In that case the free entry condition would be violated:

$$(1-\delta_0)\int_{\underline{x}} G(\alpha_{c,0}, l_0, 0, x; \phi_\infty, \Upsilon_\infty) dF(x) < (1-\delta_0)\int_{\underline{x}} G(\alpha_{c,0}, l_0, 0, x; \phi_0, \Upsilon_0) dF(x) = f_E$$

Lemma 3 (Exit, growth, export cutoff functions)

*(A).[**The case of no-screening**] Following the reduction in trade costs:*

1. *Firms with age a and productivity* $x < x_{exi}(a)$ *exit*
2. *Firms with age a and productivity* $\geq x_f(a) \geq x \geq x_{exi}(a)$ *fire workers, and* $x_f(a)$ *is increasing in a in the space above* $x_{exi}(a)$
3. *Firms with age a and productivity* $x_{gth}(a) \geq x \geq x_f(a)$ *stay stable or shrink by attrition while firms with age a and productivity* $x > x_{gth}(a)$ *grow , and* $x_{gth}(a)$ *is increasing in a in the space above* $x_{exi}(a)$
4. *Firms with age a and productivity* $x > x_{exp}(a)$ *export,* $x_{exp}(a)$ *is decreasing in a*
5. *Let* $\underline{x}_{X,a=\infty,t=\infty}$ *be the export cutoff for firms in their stationary state. Domestic-only firms with productivity* $x < \underline{x}_{X,a=\infty,t=\infty}$ *grow more slowly than in the previous period.*

*(B).[**The general case of screening**] The above results hold locally for firms that are close enough to their new stationary state.*

Proofs: While I state the above predictions in terms of productivity and age, it is equivalent to replace age with size because of the one-to-one mapping between age and size when conditioning on productivity.

1. In accordance with assumption 2, firms exit if and only if
$G_{t_0}(\alpha'_c, l', 0, x; \phi_\infty, \Upsilon_\infty) = J_{t_0}(\alpha'_c, l', x; \phi_\infty, \Upsilon_\infty) < 0$. Applying lemma B.3, I obtain that:
$J_{t_0}(\alpha'_{c,a}, l'_a, x; \phi_\infty, \Upsilon_\infty) \geq J_{t_0}(\alpha'_{c,a+1}, l'_{a+1}, x; \phi_\infty, \Upsilon_\infty)$ because $\alpha'_{c,a} \leq \alpha'_{c,a+1}$ and $l'_a \leq l'_{a+1}$, meaning that younger firms are more likely to exit conditional on x, and that:
$J_{t_0}(\alpha'_{c,a,x}, l'_{a,x}, x; \phi_\infty, \Upsilon_\infty) \leq J_{t_0}(\alpha'_{c,a,x'}, l'_{a,x'}, x; \phi_\infty, \Upsilon_\infty) \leq J_{t_0}(\alpha'_{c,a,x'}, l'_{a,x'}, x'; \phi_\infty, \Upsilon_\infty)$ when $x < x'$

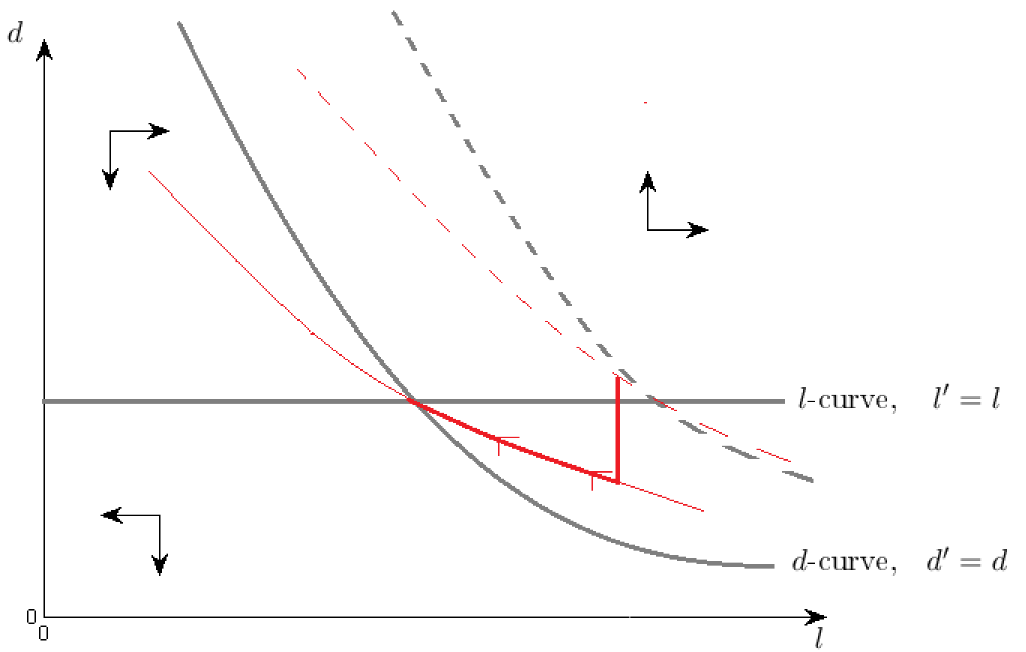

Figure 8: Phase diagram of the transition equilibrium for a domestic firm. The dash grey line correspong to the d-curve in the intial steady state. The dash red line correspond to the saddle path in the initial steady state. The plain lines correspond to the new evironment with lower ϕ. The thick red line illustrates the evolution for a domestic firms that had not reached its stationary equilibrium at the time of the shock.

meaning that less productive firms are more likely to exit conditional on age.

2 and 3. These predictions are a direct consequence of the fact that firm growth rates are decreasing in size and increasing in productivity as stated in proposition 2.

4. The export cutoff function is determined by the first order condition stated in equation (11).

5. Firms with productivity $x < \underline{x}_{X,a=\infty,t=\infty}$ do not export in their stationary state. Their new saddle path equilibrium as described in figure 8 shows that they grow more slowly: for these firms, the saddle path equilibirum is shifted to the left and their stationary state features a smaller size and a a lower average worker productivity compared to the pre-reform stationary state.

Lemma 4 (Evolution of future stationary exporters)

Consider the firms that were in their stationary state at t_0 and that will export in their new stationary state ($x \geq \underline{x}_{X,a=\infty,t=\infty}$): these firms expand, separate from some workers as they raise *their screening cutoff, increase the wages of incumbent workers and experience an increase in the dispersion of wages among their workers.*

Proof: Lemma 3 The stationary "effective productivity" of firms that export in the new stationary state is higher after the reduction in trade costs whether they used to export or not: $x^{\frac{\sigma-1}{\sigma}}\Upsilon_{\infty}^{\frac{1}{\sigma}}\phi_{\infty} \geq x^{\frac{\sigma-1}{\sigma}}\Upsilon_{0}^{\frac{1}{\sigma}}\phi_{0} \geq x^{\frac{\sigma-1}{\sigma}}\phi_{0}$. Therefore proposition 1 shows that they have now less workers with a lower average worker ability than in their new stationary state. As a result, they grow in size and in average worker productivity by hiring better workers and by separating from the least productive of their workers as they raise their screening cutoff.

Firms in their stationary equilibrium have zero wage dispersion because all their workers were hired under the same firm growth rate, namely the replacement rate s_0, and all workers have the following wage:

$$w_{SS} = w_o + (1-\xi)(1-\beta(1-\eta_0))\frac{\tilde{c}_A}{\gamma}\left(\frac{s_0}{(1-s_0)}\right)^{\gamma-1}\alpha_{SS}^{\kappa\gamma}$$

Firms that export in their new stationary state raise the screening cutoff from α_{SS} to $\alpha_c' > \alpha_{SS}$ (see figure 9) and raise the wage of incumbent workers accordingly:

$$w_{\text{incumbent}} = w_o + (1-\xi)(1-\beta(1-\eta_0))\frac{\tilde{c}_A}{\gamma}\left(\frac{s_0}{(1-s_0)}\right)^{\gamma-1}{\alpha'}_c^{\kappa\gamma}$$

In the meantime, they hire new workers at a premium since their growth rate is higher than when they were in a stationary state: $\left(\frac{\Delta}{\tilde{l}}\right) > \frac{s_0}{(1-s_0)}$ and

$$w_{\text{new hire}} = w_o + (1-\xi)(1-\beta(1-\eta_0))\frac{\tilde{c}_A}{\gamma}\left(\frac{\Delta}{\tilde{l}}\right)^{\gamma-1}{\alpha'}_c^{\kappa\gamma}$$

Because $w_{\text{incumbent}} < w_{\text{new hire}}$, the wage dispersion is now positive.

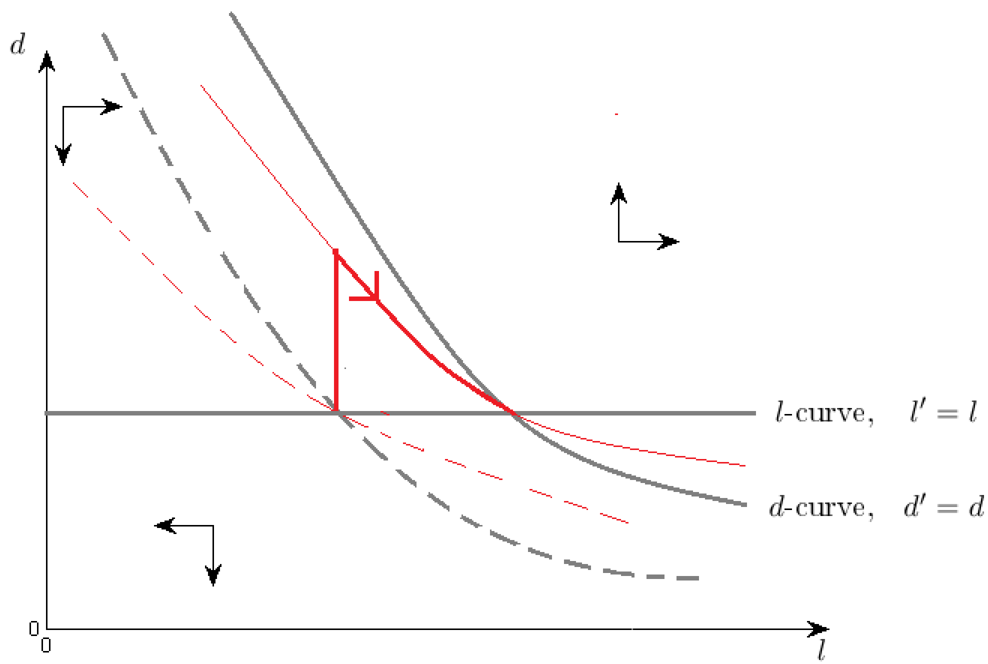

Figure 9: Phase diagram of the transition equilibrium for current or future exporters. The dash grey line correspong to the d-curve in the intial steady state. The dash red line correspond to the saddle path in the initial steady state. The plain lines correspond to the new d-curve with trade. The thick red line illustrates the evolution of a firm that was initially in its stationary state.