Stories from the Frontier

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Abstract

Should basic research be publicly or privately funded? This paper studies the impact of the shift in the U.S. patent system towards the patentability and commercialization of the basic R&D undertaken by universities during the early 1980s. We interpret this change as rendering universities responsive to "market" forces. Prior to 1980, universities undertook research using an exogenous stock of researchers motivated by "curiosity." After 1980, universities patent their research and behave as private firms. This move, in a context of two-stage inventions (basic and applied research) has an a priori ambiguous effect on innovation and welfare. We build a Schumpeterian model and match it to the data to assess this important turning point from an innovation and growth, as well from a welfare-enhancing perspective.

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1 Introduction

A recent influential literature on endogenous macroeconomic growth developed a novel approach to analyze a stylized fact known as the great divergence, namely the existence of great disparities in the observed growth rates for the different countries around the globe. In this way, the distance to frontier growth literature (see Aghion, Howitt and Mayer-Foulkes, 2005; Acemoglu, Aghion and Zilibotti, 2006; and more recently Chu, Cozzi and Galli, 2014 among others) proposed itself as the natural candidate to reconcile economic growth theory and the observed world-wide growth regularities. In particular, by emphasizing the role of appropriate institutions in promoting the advancement of a country towards the US technological target, the distance to frontier model proved a powerful tool to support the analysis of country-specific innovation policies. But how should one interpret the expression "appropriate institutions"?

A definition would include a broad set of factors both formal and informal which stimulate

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per-capita GDP growth: rules, jurisprudential precedents, procedures, habits and social norms which are sometimes deeply rooted in the country’s historical memory. Hence the attention focussed on the analysis of such factors and the causes which may have prevented (or may prevent) countries from adopting the most appropriate institutional growth-enhancing policy-mix and catching-up with the world technological frontier.

In this paper we choose a similar perspective, namely we develop a country-specific environment to depict countries located close to the world technological frontier, possibly on the same frontier, and we ask: what factors specifically affect growth in the most advanced economies?

In an international framework of advanced economies, data suggest the existence of a positive relation between basic research intensity and the multifactor-productivity growth. This can be seen by looking at figures 1.A and 1.B representing the relationship between countries’ basic research intensity relative to the US and the respective MFP growth (OECD MSTI data (OECD, 2014a and OECD Productivity Database (OECD, 2014b)).

Gersbach, Schneider and Schneller (2008 and 2013) argued that the impact of basic research on growth is intended to become more and more relevant as the country converges to
the world technological frontier. The same authors emphasize how US and Japan alone account for about half of the world basic research and highlight the circumstance that basic research is often publicly financed by governments. Figures 2.A and 2.B show the relationship between the government funded gross domestic expenditure on research and development (GERD) intensity and the distance to frontier (figure 2.A); and the public R&D expenditure and the distance to frontier (figure 2.B). The data are from the OECD Science, Technology, and Industry Outlook (OECD, 2012) and the OECD Productivity Database (OECD, 2014b). The first figure - GERD intensity - is a measure of R&D activities which are funded by the government, but non necessarily performed within public research institution. The second, instead, captures all R&D activities performed within higher education institutions and public R&D laboratories. In fact, for a relevant fraction basic research is performed by universities and public research institutions (see also Cozzi and Galli (2009): public basic research is essential for developing new scientific breakthroughs and creating the basis for developing subsequent technological advancements, as recently highlighted by the U.S. Congress’s Joint Economic Committee (see JEC 2010 and the insightful discussion provided by Akcigit, Hanley and Serrano-Velarde, 2014). Despite these considerations, very little attention has been devoted so far to a systematic study of the channels through which public basic research stimulate growth, innovation, and welfare.

To try to fill this gap and shed some light on this important issue, here we incorporate publicly provided basic research into a Schumpeterian multi-sector growth model. In particular, we assume that the government employs a share of researcher into basic research, and we make
further assumptions which specifically characterize the behavior of the public researchers and
distinguish them from the private sector researchers. We attempt to replicate the changes oc-
curred in the US intellectual property design by modelling the institutional framework provided
by different institutional scenarios. Hence, we discuss the consequences of publicly provided
basic research in terms of innovative capacity and its desirability in terms of welfare. To that
extent, we examine the evolution of intellectual property institutions in the U.S., with special
reference to their ability to protect basic research and to promote the technological advance-
ment of the frontier\textsuperscript{1}. By taking the R&D sequentiality into the Schumpeterian paradigm, this
paper investigates the relationship between the cumulative uncertainty involved in a two-stage
\textit{(basic-applied)} innovation process and the inefficiencies of the public research system, which is
an issue often left unmodelled so far.

In our view, this constitutes an important element of novelty for the literature on technology-
driven economic growth, which so far mostly concentrated on private R&D scenarios, where
typically profit-maximizing firms engage in R&D activity in order to secure the rents associated
with the introduction new markets or new technologies. There are a few exceptions which ex-
plicitly consider the role of the government as a research provider in the macroeconomic growth
literature. Most notably, Aghion, Dewatripont, and Stein (2008) and Spinesi (2013) analyzed
the effects of technological transfer institutions (intellectual property rights) between the acad-
emia and private research firms from different perspectives. A recent scientific contribution by
Akcigit et al. (2014) identified an important role for public basic research in promoting eco-
nomic growth in France. In an early contribution, Pelloni (1997) builds an endogenous growth
model with public research only, where the government faces a trade-off between financing
public research or public education.

Within this still thin literature, our model is the first which tries to endogenizes the public
sector inefficiency in basic research. In fact, public basic research is not as targeted as private
basic research, which is guided by the signalling device of future patent values. On the con-
trary public research is more career-motivated and less respondent to \textit{market stimuli}\textsuperscript{2}. This
circumstance determines that the amount of inefficiency created depends on the fraction of
industries where basic R&D can be effectively carried out, which is endogenous. Therefore, one
cannot unambiguously rank the two institutional scenarios: patentable or unpatentable basic
research? In some cases it would be best to keep basic research publicly driven, while in others
it would be best to facilitate privatizing institutions with basic research patents. In this paper,
depending on the parameters to be calibrated, the most innovation-fostering and the socially
optimal institution will be determined.

\textsuperscript{1}Curiously, the US National Institute of Health (NIH) recently introduced the concept of “appropriate
patenting”, according to which "patenting is one of the tools available to the NIH for transferring publicly
funded technology to the market" (see OECD, 2011).

\textsuperscript{2}For a complementary discussion on the role of relevant spillovers from the stock of academic basic knowledge
on industry, see also Spinesi (2012) and Akcigit et al. (2014).
1.1 Basic research patenting in the U.S.

Over the last 35 years, the U.S. patent system switched from the doctrine limiting the patentability of early-stage scientific findings - lacking in current commercial value - to the conception that also fundamental basic scientific discoveries - with no current tradeable application - fall in the general applicability of the patent system. This fundamental turning point marked the year 1980, when two important events characterized this new idea of the patentability requirements:

1. the United States Supreme Court’s decision on the Diamonds vs. Chakrabarty case established that microorganisms produced by genetic engineering could be patented;

2. the passing of the Patent and Trademark Act Amendments (P.L. 96-517, known as the Bayh-Dole Act) facilitated universities and public laboratories in patenting their innovations.

Such jurisprudential and juridical reforms opened the way to a flow of private funds into the academia, as well as facilitated professors in patenting their own research without incurring in legal obstacles linked to the public financing their research activities.

Recent studies focussed on the U.S. university licensing activity. In particular, Jensen and Thursby (2001) studied the licensing practices of 62 US universities and found that "Over 75 percent of the inventions licensed were no more than a proof of concept (48 percent with no prototype available) or lab scale prototype (29 percent) at the time of license!"

This process, which determined a cultural shift in the U.S. basic research culture, was reluctantly followed by Europe, where only in 1998 the European Directive on Biotechnologies aiming at extending patentability to most basic research patenting was adopted (see European Parliament and Council (1998)). Many observers commented on how such a Directive has always been implemented in contradictory ways, leading to a sort of an undefined situation for Europe. For this reason, we believe that an analysis of the U.S. turning point may give good insight to start a scientific debate rich of relevant policy implications also for Europe.

This paper is organized as follows. Section 2 explains the modifications in Schumpeterian theory needed to analyze the two-stage innovation process stylizing the innovation mechanism in the model. In order to facilitate readability, this section intentionally focusses only on the most original aspects of the model, leaving the most standard parts to the Appendix 1. Section 3 applies this new framework to a stylized pre-1980 US scenario: basic research findings are conceived in public institutions and put into the public domain, triggering patent races by freely entering perfectly competitive private R&D firms aiming at inventing a better quality product. Section 4 models a stylized post-1980 US scenario with targetable basic research, where basic R&D achievements are patented and, afterwards, developed into tradable applications within a completely privatized economy. Free entry patent races only occur in the basic research, whereas as soon as a research tool is discovered it will be developed by its patent holder. Section 5 matches the different scenarios developed by the model to the US data prevailing at
the time of the jurisprudence and legislative change. We estimate the relevant technological parameter and we undertake numerical simulations in order to assess under which underlying assumption (targetable or untargetable basic research) the reform has enhanced innovation. Section 6 concludes.

2 The Model

2.1 Overview

Consider an economy with a continuum of differentiated final good sectors with corresponding differentiated research and development (R&D) sectors, along the lines of Grossman and Helpman (1991). In each final good sector vertical innovation takes place, hence price-competition among firms determines - under the usual constant returns to scale assumption - the market monopolist, the owner of the patent on the highest quality product in its industry.

2.2 The Mechanics of R&D

Product improvements occur in each final good industry, and, within each industry, firms are distinguished by the quality of the final good they can produce. When the state-of-the-art quality product in an industry $\omega \in [0,1]$ is $j_t(\omega)$, research efforts are necessary in order to achieve the $j_t(\omega) + 1/2$th inventive step, and then other researchers engage in a patent race to implement it in the $j_t(\omega) + 1$st quality product. So, in each industry, the R&D activity is a two-stage innovation process by which, first a new idea is invented through basic research activity and then it is used by applied researchers to find the way to introduce a higher quality product. Our definition of basic research output essentially coincides with a research-tool: "the full range of tools that scientists use in the laboratory" including "cell lines, monoclonal antibodies, reagents, animal models, growth factors, combinatorial chemistry libraries, drugs and drug targets, clones and cloning tools (...) methods, laboratory equipment and machines, databases and computer software", according to the definition provided by the US National Institute of Health (see NIH (1998) and OECD (2011)). Nearly all research tools became patentable in the US, thanks to the juridical innovations that took place in the last 30 years (see Cozzi and Galli (2014)).

The whole set of industries $\{\omega \in [0,1]\}$ gets partitioned into two subsets of industries: at each date $t$, there are industries $\omega \in A_0$ with (temporarily) no research tool and, therefore, with one quality leader (the final product patent holder), no applied research and a mass of basic researchers; and the industries $\omega \in A_1 = [0,1] \setminus A_0$, with one research tool and, therefore, one quality leader and a mass of applied researchers directly challenging the incumbent monopolist.

3Of course, upstream ideas could be as difficult to get as are Nobel prizes: see, for example, the Cohen-Boyer patents on the basic method and plasmids for gene cloning (granted in 1990).

4Note how this definition relies on the implicit assumption that basic research bears no utility increase for the consumers.
Let us define a perfectly targeted research economy when basic research focusses exclusively on industries $\omega \in A_0$, whose output can therefore be used by profit-motivated R&D firms engaging in applied R&D activity aimed at a final product innovation only in $A_1$ industries. When eventually a quality improvement occurs within an $A_1$ industry, the innovator becomes the new quality leader and the industry switches from $A_1$ to $A_0$. Similarly, when a discovery arises in an industry $\omega \in A_0$ this industry switches to $A_1$. This process can be better understood by considering the industry dynamics illustrated by the two-lakes representation of the economy in FIGURE 3: notice that in our multi-sector two-stage perpetual innovation process, basic R&D alternates with applied R&D in all sectors of the economy. The two sets $A_0$ and $A_1$ change over time, even if the economy will eventually tend to a steady state.

![FIGURE 3: Targeted research economy by flows of industries](image)

Suppose that at any instant one can measure the two sets $A_0$ and $A_1$. Let $m_0$ denote the measure of $A_0$; and $m_1$ respectively denote the measure of $A_1$. By construction, $m_1 = 1 - m_0$. In the steady-state equilibrium the two measures shall be constant, as the two-flows in and out of the lakes (the arrows denoted research tool and product innovation in FIGURE 3) will off-set each other. However, the endogenous nature of the steady-state distribution of sectors allows the model to analyze the effects of different institutional scenarios on the technology dynamics and the aggregate innovative performance.

Let index $i = B, A$ denote basic or applied research respectively. $n_i(\omega, t)$ indicates the mass of skilled workers employed in the two sages of the innovation process in sector $\omega \in [0, 1]$ at time $t$. We specify the per-unit time Poisson probability intensity of an innovative step (basic or applied) to occur in a generic sector $\omega$ as:

$$\theta_B(\omega, t) \equiv \lambda_0 n_B(\omega, t)^{1-a}, \omega \in A_0, \quad (1)$$

$$\theta_A(\omega, t) \equiv \lambda_1 n_A(\omega, t)^{1-a}, \omega \in A_1 \quad (2)$$
where \( \lambda_k > 0, k = 0, 1 \), are R&D productivity parameters\(^5\) and constant \( 0 < a < 1 \) is an intra-sectorial congestion parameter, capturing\(^6\) the risk of R&D duplication, knowledge theft, and other diseconomies of fragmentation, external to the single firm in competitive industries. Each Poisson process - with arrival rates described by (1)-(2) - is independent across researchers and across industries. Hence the probability per unit time of inventing a research tool in a sector \( \omega \in A_0 \) at date \( t \) is \( \theta_B(\omega, t) \), and the probability of completing a final blueprint in a sector \( \omega \in A_1 \) is \( \theta_A(\omega, t) \).

Moreover, in all our scenarios, symmetric equilibria exist, allowing simpler notation: \( n_B(\omega, t) \equiv n_B(t) \) and \( n_A(\omega, t) \equiv n_A(t) \).

### 2.2.1 Manufacturing

Adopting the unskilled wage as the numeraire, we will endogenously determine the skill premium, as summarized by the skilled labour (relative) wage \( w_s \).

In all our equilibria, the per-capita mass of skilled labour employed in manufacturing sector \( \omega \in [0, 1] \) at time \( t \), labeled \( x(\omega, t) \), will be constant across sectors and equal to \( x(\omega, t) = x(t) \). In fact, in the Appendix 1 we prove that the manufacturing employment of the skilled labour obeys the following decreasing function of the relative skilled wage \( w_s \):

\[
x(\omega, t) = \frac{1}{w_s(t)} \left( \frac{\alpha}{1 - \alpha} \right) M \equiv x(t),
\]

where \( 0 < \alpha < 1 \) is the skilled labour elasticity of output. Appendix 2 also show that profit flows are constant and equal to \( \pi = (\gamma - 1) \frac{1}{1 - \alpha} M \), where \( \gamma > 1 \) is the size of each product quality jump.

Since the total mass of sectors in the economy is normalized to 1, \( x(t) \) also denotes the aggregate employment of skilled in manufacturing. Hence, defining \( Y(t) \) the aggregate final good production, \( x(t)w_s(t) = \alpha Y(t) \) and \( M = Mw_u(t) = (1 - \alpha)Y(t) \).

In light of the previous discussion, and dropping time indexes for simplicity\(^7\), we can express the skilled labor market equilibrium as:

\[
L = \frac{1}{w_s} \left( \frac{\alpha}{1 - \alpha} \right) M + m_0n_B + m_1n_A.
\]

\(^5\)Eq.s (1)-(2) are built on the assumption of a stationary population. With increasing population, it is easy to recast our model, as done in Appendix 1, in terms of Dinopoulos and Segerstrom’s (1999) "permanent effects on growth" (PEG) framework, which captures the difficulty of improving a good in a way that renders a larger population happier. This eliminates the strong scale effect (Jones 2003) that plagued the early generation endogenous growth models, without leading to "semi-endogenous" growth (Jones 1995, Segerstrom 1998), as consistent with recent empirical evidence (e.g. Ha and Howitt, 2007, and Madsen, 2008). Despite its semplicity, this assumption is equivalent to eliminating the strong scale effect by means of an R&D "dilution effect" over an increasing range of varieties, as proved by Peretto (1998), Young (1998), Dinopoulos and Thompson (1998) and (1999), and Howitt (1999).

\(^6\)As in Jones and Williams’ (1998 and 2000) specification of the R&D technology.

\(^7\)Of course time dependence is implicit, as employment variables, wage, and the mass of sectors in which a half idea is present, respectively absent, keep changing over time, except in the steady state.
Eq. (4) states that, at each date, the aggregate supply of skilled labor, $L$, finds employment in manufacturing and in basic and applied R&D.

### 3 The Public Basic Research Economy

In order to depict a pre-1980 US normative environment, in this section we assume unpatentable basic scientific results. In fact, pre-1980 the prevailing practice in public basic research was granting open access to its scientific findings. Besides, public researchers were paid regardless of the development opportunities arising from their discoveries: their activity was "curiosity-driven" and totally indifferent to sectorial profitability, thus their efforts were potentially wrongly targeted from a social point of view. Therefore, we shall assume that public researchers are allocated across different industries according to a uniform distribution\(^8\). Please note that this assumption could be consistently incorporated in a microfoundation within an incomplete contract setting between the university (principal) and the public basic researchers (agents).

The university bureaucrats do not exert effective authority on the different research activities, which are carried out by the researchers employed in R&D. Hence, university bureaucrats have only very limited control. In particular, the bureaucrats do not have the authority to stipulate complete contracts, since they are unable to effectively specify in what sector basic research should be carried out by the academic researcher at each instant in time and to enforce the contract terms (see Aghion and Tirole, 1997).

We also make the assumption that the government exogenously sets the fraction, $\bar{L}_G \in [0, L]$, of the skilled workers to be allocated to basic research laboratories, funded by lump-sum taxes\(^9\). Given that the mass of sectors normalized to 1, $\bar{L}_G$ is also equal to the per-sector amount of R&D. Therefore the probability that in any sector $\omega \in A_0$ a basic research result appears is $\theta_B \equiv \bar{L}_G^{-1}\lambda_0$, whereas the probability that an existing research tool generates a new marketable product is $\theta_A = n_A^{-1-\alpha}\lambda_1$.

Let $v^0_L$ denote the value of a monopolistic firm producing the top quality product in a sector $\omega \in A_0$, and consistently let $v^1_L$ be the value of a monopolistic firm producing the top quality product in a sector $\omega \in A_1$. These two types of quality leaders earn the same profit flow, $\pi$, but the first type has a longer expected life, before being replaced by the new quality leader, i.e. by the patent holder of the next version of the product it is currently producing. In sectors that are currently of type $A_0$ no applied R&D firms enters because there is no research tool to develop: they shall wait until public researchers invent one, causing that sector to switch into $A_1$. Instead, in an $A_1$ sector, applied R&D firms hire skilled workers in order to complete the freely available basic research result. Since there is free entry into applied research, the R&D

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\(^8\)This assumption captures the idea of ivory-tower-oriented basic researchers, mainly concerned with academic advancement in an environment shaped by values such "universalism, disinterestedness, originality, skepticism, and communalism" (Davis, Larson and Lotz (2009)).

\(^9\)This guarantees that governmental R&D expenditure does not imply additional distortions on private decisions.
firm’s expected profits are dissipated. From a welfare perspective, entry into applied R&D could be excessive, thereby generating distortions.

Defining $r$ as the relevant real interest rate, the following equations hold:

1. \[ w_s = \lambda_1 n_A^{-a} v_L^0 \] (5a)
2. \[ rv_L^0 = \pi - \bar{L}^{-a} \lambda_0 (v_L^0 - v_L^1) + \frac{dv_L^0}{dt} \] (5b)
3. \[ rv_L^1 = \pi - n_A^{-a} \lambda_1 v_L^1 + \frac{dv_L^1}{dt} \] (5c)

Eq. (5a) is the free entry condition in applied research in each sector $\omega \in A_1$, equalizing the unit cost of R&D (the skilled wage) to the probability $\lambda_1 n_A^{-a}$ of inventing the next version of the final product times the value of its patent, $v_L^0$. Eq. (5b) is the financial arbitrage equation stating that $v_L^0$ is determined by equating the risk-free interest income attainable by realizing the stock market value of an industry leader in $A_0$, $rv_L^0$, to the flow of profit $\pi$ minus the expected capital loss from being challenged by subsequent basic research activity generating in a new research-tool, $\bar{L}^{-a} \lambda_0 (v_L^0 - v_L^1)$, plus the gradual appreciation in the case of such event not occurring, $\frac{dv_L^0}{dt}$. In a steady state $\frac{dv_L^0}{dt} = 0$.

Eq. (5c) equates the risk free income per unit time deriving from the liquidation of the stock market value of a leader in an $A_1$ industry, $rv_L^1$, with the relative flow of profit $\pi$ minus the expected capital loss, $n_A^{-a} \lambda_1 v_L^1$, due to the downstream applied researcher firms’ R&D, plus the gradual appreciation if replacement does not occur, $\frac{dv_L^1}{dt}$. In a steady state $\frac{dv_L^1}{dt} = 0$.

All jump processes are independent across industries. Hence, by the law of large numbers, the dynamics of the mass of industries is described by:

\[ \frac{dm_0}{dt} = (1 - m_0) n_A^{-a} \lambda_1 - m_0 \bar{L}^{-a} \lambda_0. \] (6)

The skilled labor market clearing condition imposes:

\[ x + \bar{L} + (1 - m_0) n_A = L. \] (7)

Recall the equilibrium value of $x$ derived by equation (3): $x = \frac{1}{w_s} \left( \frac{\alpha}{1-\alpha} \right) M$; by combining this expression with the skilled labor market clearing equilibrium, we get:

\[ n_A = \frac{L - \frac{1}{w_s} \left( \frac{\alpha}{1-\alpha} \right) M - \bar{L}}{(1 - m_0)}. \] (8)

Hence the dynamics of this economy is completely characterized by system (5a)-(5c), (6), and (8).
3.1 Balanced Growth Path

In a balanced growth path equilibrium all variables are constant except the average quality of consumer goods\footnote{Since we are following Grossman and Helpman’s (1991) framework, it is the geometric average $D(t) = \exp \int_0^1 \ln \left[ \gamma J^i(\omega)dJ^i(\omega) d\omega \right]$ that matters. Appendix 1 clarifies these aspects in detail.}, and therefore the instantaneous per-capita utility index, which grows at a constant rate\footnote{This is a usual property of quality ladder models (see e.g. Grossman and Helpman, 1991). Find more on this in the welfare calculations in Appendix 1.} $\ln(g_{PUBBL})$ proportional to the aggregate innovation rate $g_{PUBBL} = m_0 \tilde{L}_G^{-a} \lambda_0 = (1 - m_0) \lambda_1 n^{-a}_A$. Based on the previous characterization, we can state:

**Balanced Growth Path Equilibrium of the Public Basic Research Economy.** Definition 1: a balanced growth path equilibrium of the Public Basic Research economy is a vector $[m_0, n_A, v^0_L, v^1_L, w_s, x, g_{PUBBL}] \in R^7_+$, satisfying $m(A_0) \in [0, 1]$ and the following equations:

\[
\begin{align*}
    w_s & = \lambda_1 n^{-a}_A v^0_L & (9a) \\
    rv^0_L & = (\gamma - 1) \frac{1}{1-a} M - \tilde{L}_G^{-a} \lambda_0 (v^0_L - v^1_L) & (9b) \\
    rv^1_L & = (\gamma - 1) \frac{1}{1-a} M - n^{-a}_A \lambda_1 v^1_L & (9c) \\
    x & = \frac{1}{w_s} \left( \frac{\alpha}{1 - \alpha} \right) M & (9d) \\
    (1 - m_0) n^{-a}_A \lambda_1 & = m_0 \tilde{L}_G^{-a} \lambda_0 & (9e) \\
    x + \tilde{L}_G + (1 - m_0) n_A & = L & (9f) \\
    g_{PUBBL} & = \lambda_1 (1 - m_0) n^{-a}_A. & (9g)
\end{align*}
\]

Given the high non-linearity of system (9a)-(9g), we performed numerical simulations in Matlab\footnote{The Matlab and Dynare files used to simulate the model are available from the authors upon request.}. In all simulations a unique economically meaningful steady state equilibrium exists. Moreover, analyzing the eigenvalues of the Jacobian matrix of the fully dynamic (out of steady state) system shows that the steady-state equilibrium is saddle-point stable. Therefore the equilibrium is determinate.

Moreover, one can prove the uniqueness of the steady-state equilibrium. In fact, the following lemma holds:

**Lemma 1.** In the Public Basic Research economy there can exist no more than one balanced growth path equilibrium.

**Proof.** See Appendix 2.

4 The Privatized Basic Research Economy

4.1 Targetable Basic Research

In this section, stylizing a post-1980 US scenario, we assume that once a research tool is invented in an $A_0$ sector, it gets protected by a patent with infinite legal life. The presence of perfectly
enforced intellectual property rights on the research tools permits the existence of a market for basic research findings. We will here assume that the basic is perfectly efficient\textsuperscript{13}. Let us remark that this scenario does not preclude the existence of public universities, as long as their attitudes and internal incentive system is profit-seeking as well\textsuperscript{14}.

Let $v_A$, denote the value of a research-tool patent owned by an applied R&D firm. Such a firm will optimally choose to hire an amount $n_A$ of skilled research labour to maximize the difference between its expected gains from completing its own first stage - probability of inventing, $(n_A)^{1-a} \lambda_1$, times the net gain from inventing the final product, $(v_L^0 - v_A)$ - and the implied labour cost $w_s n_A$. The optimal applied R&D employment in an $A_1$ sector is

$$n^*_A = \left[ \frac{(1-a) \lambda_1 (v_L^0 - v_A)}{w_s} \right]^{\frac{1}{a}}. \tag{10}$$

Unlike the previous section, now the sole research-tool patent holder can undertake applied R&D in its industry\textsuperscript{15}, whereas free entry is relegated to the basic research stage, where researchers vie for inventing the research-tool that will render the winner the only owner of a research tool patent worth $v_A$. Hence their freely entering and exiting mass will dissipate any excess earning, by equalizing wage to the probability flow $\lambda_0 n_B^{-a}$ times the value of a research tool patent, $v_A$. Therefore excessive entry into basic research can determine welfare losses.

Costless arbitrage between risk free loans and firms’ equities implies:

$$w_s = \lambda_0 n_B^{-a} v_A \tag{11a}$$

$$r v_A = (n_A^*)^{1-a} \lambda_1 (v_L^0 - v_A) - w_s n_A^* + \frac{dv_A}{dt} \tag{11b}$$

$$r v_L^0 = \pi - (n_B)^{1-a} \lambda_0 (v_L^0 - v_L^1) + \frac{dv_L^0}{dt} \tag{11c}$$

$$r v_L^1 = \pi - (n_A^*)^{1-a} \lambda_1 v_L^1 + \frac{dv_L^1}{dt} \tag{11d}$$

The first equation, (11a), characterizes the free entry condition in basic research. The second equation equalizes the risk free income deriving liquidating the expected present value of the research tool patent in an $A_1$ industry, $r v_A$, and the expected increase in value from becoming a top quality leader, $(n_A^*)^{1-a} \lambda_1 (v_L^0 - v_A)$, minus the relative R&D cost, $w_s n_A^*$, plus the gradual appreciation in the case of R&D success not arriving, $\frac{dv_A}{dt}$.

The interpretation of the third and forth equation is like that of equations (5b) and (5c) in the previous section.

\textsuperscript{13}This means that basic researchers target their activity only in the $A_0$ sectors.

\textsuperscript{14}Belenzon and Schankerman’s (2009) empirical analysis shows that the private or public university ownership does not change their licensing performance, provided they adopt the same incentive pay. Also see Lach and Schankerman (2004).

\textsuperscript{15}Here, perfect IPRs successfully restrict entry into applied R&D to only those (patent holder or \textit{ex ante} licensees) legally entitled to do so. For an alternative scenario, with weaker IPR protection, in which free entry into downstream research vanifies any attempt to impose to \textit{ex ante} licensing, see Cozzi and Galli (2014).
Plugging $w_s = \lambda_0 n_B^a v_A$ into the expression of the skilled labour wage ratio (eq. 39, in Appendix 1) and using per-capita notation, we obtain\textsuperscript{16}:

\[ x = \frac{1}{w_s} \left( \frac{\alpha}{1 - \alpha} \right) M = \min \left( \frac{n_B^a}{\lambda_0 v_A}, 1 \right) \left( \frac{\alpha}{1 - \alpha} \right) M. \quad (12) \]

The skilled labor market clearing condition states:

\[ x + m_0 n_B + (1 - m_0) n_A^* = L \quad (13) \]

Hence, since wages are pinned down by the optimal firm size and by the zero profit conditions in the perfectly competitive basic research labor markets, the unique equilibrium per-sector mass of entrant basic R&D firms consistent with skilled labor market clearing (13) is determined by solving equation (13) for $n_B$:

\[ n_B = \frac{1}{m_0} \left( L - x - (1 - m_0)n_A^* \right). \quad (14) \]

To complete our analysis, let us look more closely at the inter-industry dynamics depicted by Figure 3. In the set of basic research industries a given number of perfectly competitive (freely entered) basic researchers, $n_B^*$, have a flow probability of becoming applied researchers, while in the set of the applied R&D industries each of the $n_A^*$ per-industry applied researchers has a flow probability to succeed. By the law of large numbers, the industrial dynamics of this economy is described by the following first order ordinary differential equation:

\[ \frac{d m_0}{dt} = (1 - m_0) \lambda_1 (n_A^*)^{1-a} - m_0 (n_B^*)^{1-a} \lambda_0. \quad (15) \]

System (11b)-(11d) and eq. (15) - jointly with cross equation restrictions (12) and (14) - form a system of four first order ordinary differential equations, whose solution describes the dynamics of this economy for any admissible initial value of the unknown functions of time $v_0^L, v_0^L, v_A$, and System (11b)-(11d) and eq. (15) - jointly with cross equation restrictions (12) and (14) - form a system of four first order ordinary differential equations, whose solution describes the dynamics of this economy for any admissible initial value of the unknown functions of time $v_0^L, v_0^L, v_A$, and $m(A_0)$. In a steady state, $\frac{dv_0^L}{dt} = \frac{dv_0^L}{dt} = \frac{dv_A}{dt} = \frac{dm(A_0)}{dt} = 0$.

Let us remark that, unlike in the unpatentable research-tools case, here there is - potentially excessive - endogenous entry into basic research. Moreover, in this privatized scenario, congestion in applied research is internalized by the basic patent holder\textsuperscript{17}.

\textsuperscript{16}We have implicitly assumed that $w_s \geq 1$, because skilled workers always have the option to work as unskilled workers. Therefore skilled employment in manufacturing is inversely related to the market value of patented research tools.

\textsuperscript{17}Therefore both the growth and the welfare comparisons between the two regimes are not obvious, and the outcome could depend on the parameter values.
4.1.1 Balanced Growth Path

In the balanced growth path equilibrium all variables are constant except the average quality of consumer goods, and therefore the instantaneous per-capita utility index, which grows at a constant rate \( \ln(\gamma)g_{PRIV} \) proportional to the aggregate innovation rate \( g_{PRIV} = m_0 (n_B)^{1-a} \lambda_0 = (1 - m_0) \lambda_1 (n_A^*)^{1-a} \). Based on the previous characterization, we can state:

**Balanced Growth Path Equilibrium of the Privatized Basic Research Economy.**

\[ m_0 = (1 - m_0) \lambda_1 (n_A^*)^{1-a} \]

\[ g_{PRIV} = (1 - m_0) \lambda_1 (n_A^*)^{1-a} \]

**Definition 2:** a balanced growth path equilibrium of the Privatized Basic Research economy is a vector \([m_0, n_B, n_A^*, v_A, v_L^0, v_L^1, w_s, x, g_{PRIV}] \in R^9_+\) satisfying \( m_0 \in [0, 1] \) and the following equations:

\[
\begin{align*}
  w_s &= \lambda_0 n_B^{-a} v_A \\
  r v_A &= (n_A^*)^{1-a} \lambda_1 (v_L^0 - v_A) - w_s n_A^* \\
  n_A^* &= \left[ \frac{(1 - a) \lambda_1 (v_L^0 - v_A)^{\frac{1}{a}}}{w_s} \right] \\
  r v_L^0 &= \pi - (n_B)^{1-a} \lambda_0 (v_L^0 - v_L^1) \\
  r v_L^1 &= \pi - (n_A^*)^{1-a} \lambda_1 v_L^1 \\
  (1 - m_0) \lambda_1 (n_A^*)^{1-a} &= m_0 (n_B)^{1-a} \lambda_0 \\
  L &= x + m_0 n_B + (1 - m_0) n_A^* \\
  x &= \frac{1}{w_s} \left( \frac{\alpha}{1 - \alpha} \right) M \\
  g_{PRIV} &= (1 - m_0) \lambda_1 (n_A^*)^{1-a}.
\end{align*}
\]

In all numerical simulations of the fully dynamic system the steady state turned out to be saddle-point stable.

Also for the current scenario, the uniqueness of the steady-state equilibrium holds:

**Lemma 2.** In the Privatized Basic Research economy there can exist no more than one balanced growth path equilibrium.

**Proof.** See Appendix 2.

4.2 Untargetable Basic Research

So far we have assumed the ability of intellectual property rights, here represented by patents, to channel basic research efforts towards more profitable venues, thus implicitly assuming that market signals can be useful to direct research. In terms of our model, we assumed that granting basic researchers intellectual property is a viable option to increase the efficiency of the technological transfer aggregate mechanism.

However, a broad consensus in the literature on basic research share the a more pessimistic view about the possibility for the patent system to effectively operate in this dimension. In
particular, it is often observed how basic research cannot be targetable because its outcomes are hard to predict, and in many instances, the motivations behind its creation are pure intellectual curiosity and desire to achieve academic promotions. Given the importance of this issue, to gain a deeper understanding of the potential problems of a privatization of basic research has caused, one needs to explicitly study this case. For this purpose, this section modifies the privatized economy model to include untargetable basic research.

To define untargetable basic R&D technology, we re-specify the per-unit time Poisson probability intensity of innovation postulated in eq.s (1)-(2) as:

\begin{align}
\theta_B^u(\omega, t) & \equiv \lambda_0 n_B(t)^{1-a}, \omega \in A_0, \\
\theta_A(\omega, t) & \equiv \lambda_1 n_A(\omega, t)^{1-a}, \omega \in A_1.
\end{align}

Notice that, unlike the previous sections, here the probability per unit time of inventing a research tool in a sector \( \omega \in A_0 \) at date \( t \), that is \( \theta_B^u(\omega, t) \), only depends on aggregate basic R&D \( n_B(t) \): in fact, a basic researcher is not able ex-ante to predict the exact application or the potential impact of his/her discovery.

In a symmetric equilibrium applied R&D will satisfy \( n_A(\omega, t) \equiv n_A(t) \).

To replicate a post-1980 U.S. scenario with untargetable basic research within the framework provided by Definition 2, we need to modify the labour market equilibrium:

\[ L = x + n_B + (1 - m_0) n_A^* \tag{19} \]

Notice that, compared to eq. (16f), in (19) \( n_B \) does not multiply \( m_0 \) anymore, because the same amount of per-sector basic research is now spread uniformly over the unit interval.

Moreover, the basic research free entry condition (16a) now becomes

\[ w_s = m_0 \lambda n_B Bv_A. \tag{20} \]

The r.h.s. of eq. (20), unlike in (16a), multiplies \( m_0 \) because a basic researcher has an equal probability of discovering a research-tool in any sector \( \omega \in [0, 1] \), thereby risking to generate a duplicate of an already existing research-tool in a sector \( \omega \in A_1 \). Since the probability to end up in a sector \( \omega \in A_0 \) where research-tools are efficient is indeed \( m_0 \), it has to be included in the computation of the expected benefit of basic research. All other equations of Definition 2 remain unchanged. We can therefore define the steady-state equilibrium of this variant of the privatized basic research scenario as:

**Balanced Growth Path Equilibrium of the Privatized Untargetable Basic Research Economy.** Definition 3: a balanced growth path equilibrium of the Privatized Untargetable Basic Research economy is a vector \([m_0, n_B, n_A^*, v_A, v_L^1, v_L^1, w_s, x, g_{PRIV_u}] \in \mathbb{R}^9_+ \) satisfying \( m_0 \in [0, 1] \) and the following equations:
\[ w_s = m_0 \lambda_0 n_B^{-a} v_A \]  
\[ rv_A = (n_A^*)^{1-a} \lambda_1 (v_L^0 - v_A) - w_s n_A^* \]  
\[ n_A^* = \left[ \frac{(1-a) \lambda_1 (v_L^0 - v_A)}{w_s} \right]^{\frac{1}{a}} \]  
\[ rv_L^0 = \pi - (n_B)^{1-a} \lambda_0 (v_L^0 - v_L^1) \]  
\[ rv_L^1 = \pi - (n_A^*)^{1-a} \lambda_1 v_L^1 \]  
\[ (1 - m_0) \lambda_1 (n_A^*)^{1-a} = m_0 (n_B)^{1-a} \lambda_0 \]  
\[ L = x + n_B + (1 - m_0) n_A^* \]  
\[ x = \frac{1}{w_s} \left( \frac{\alpha}{1 - \alpha} \right) M \]  
\[ g_{PRIVu} = (1 - m_0) \lambda_1 (n_A^*)^{1-a} \].

In all numerical simulations of the fully dynamic system, the steady-state turned out to be saddle-point stable.

### 5 Quantitative Analysis

#### 5.1 Observed Regularities

In general, simulating our models\(^{18}\) suggests that an economy in which public basic research is conducted in a non-profit oriented manner can induce less or more innovations and/or welfare than an economy in which basic R&D is privately carried out. The untargetable privatized basic research economy outgrows the public basic research economy when the applied R&D productivity parameter, \(\lambda_1\), becomes very low: in such cases the equilibrium innovative performance of the private economy with patentable research tools becomes better than the equilibrium growth performance of the economy with a public R&D sector. In fact, if \(\lambda_1\) is very small or \(\lambda_0\) is high, the flow out of \(A_1\) will be scarce, whereas the flow out of \(A_0\) will be intense. Therefore in the steady state \(m(A_0)\) will be small, thereby exalting the wasteful nature of the public R&D activity uniformly diluted over \([0,1] - A_0\): in this case the social cost of a public R&D blind to the social needs signalled by the invisible hand would overwhelm the social costs of the restricted entry into the applied R&D sector induced by the patentability of research tools.

While the discussion so far highlights the growth perspective, the aggregate consumer utility - welfare - is also affected negatively by the potentially excessive entry associated with patent races. Since in either regime there is free entry into one of the two types of research activities, this may lead to excessive entry into basic research in the private regime, and excessive entry

\(^{18}\) The codes we have used are available upon request.
into development in the public regime\textsuperscript{19}. While the lack of commercial focus in basic research can make publicly funded research worse, excessive entry into basic research in the private regime can potentially counter this handicap. Hence, it is not possible a priori to rank the two regimes.

In the next sections we will estimate the unknown parameters and use others taken from the literature, in order to evaluate the alternative patenting regimes. We will undertake our calibrations under the simplifying assumption that the US economy was in an unpatentable research tools balanced growth path before 1980. This will deliver the parameter values with which to simulate the alternative scenarios at the last year\textsuperscript{20} of the public basic R&D regime (1979).

5.2 Calibration

In this section we calibrate our model to a balanced growth path using U.S. data from 1973 to 1979, obtaining the values of these parameters as well as the endogenous variables in the unpatentable research tools case, which we believe prevailed during that period. Our exercise will obtain an estimation of the difficulty of R&D, summarized inversely by the basic and applied productivity parameters, \( \lambda_0 \) and \( \lambda_1 \). Consistently with our theoretical model, we use only skilled and unskilled labour as inputs and numbers of qualified innovations as R&D output, as represented by patents.

5.2.1 Description of the Procedure and the Data

1. Exact estimation of the values of the unobservable parameters \( \lambda_0, \lambda_1, \gamma, \alpha, \) and \( \alpha \) based on U.S. 1973-1979 data on the following moments: number of yearly patents/skilled labour employment ratio\textsuperscript{21}, equal to 0.000309692 (DATA); and skilled labour in manufacturing as a fraction of the labour force\textsuperscript{22}; applied R&D labour as a fraction of the labour force\textsuperscript{23}, equal to 0.00428941 (DATA); number of patents/basic research labour\textsuperscript{24}, equal to 0.197070187 (DATA); the skill premium\textsuperscript{25}, equal to 1.228 (DATA). The results are shown

\textsuperscript{19}However, in our stylized framework, research tool patentability should reduce applied research, as compared to the unpatentable basic research scenario. This is corroborated by the important evidence provided by Galasso and Schankerman’s (2013) careful identification strategy (based on judges propensity of invalidating patents), compellingly showing that following patent invalidation an idea gets more often cited in successive research.

\textsuperscript{20}Qualitative results would not change if we had chosen another year, or included an average of four years before 1979.

\textsuperscript{21}This proxies our economy’s innovation rate after accounting for the dilution effect. Therefore it is consistent with our assumption that the dilution effect (proportional to labour in the long term) neutralizes the strong scale effect (Peretto, 1998, Dinopoulos and Thompson, 1998, Young, 1998, and Howitt, 1999). This is called PEG by Dinopoulos and Segerstrom (1999) and Minnitti and Venturini (2014).

\textsuperscript{22}In our model economy, this underlies the macroeconomic trade-off in the allocation of skilled labour between manufacturing and R&D, as emphasized in the Schumpeterian growth literature (Aghion and Howitt, 1992, etc.).

\textsuperscript{23}Which pins down the allocation of R&D labour between basic and applied R&D, at the essence of our contribution. Normalization by labour force is a hallmark of the previously mentioned dilution effect.

\textsuperscript{24}R&D productivity measure of the successful interaction between basic and applied research.

\textsuperscript{25}Which responds to the allocation of incentives between basic and applied research (Cozzi and Galli, 2014).
2. Use of the estimated parameter values $\hat{\lambda}_0$ and $\hat{\lambda}_1$, $\hat{\gamma}$, $\hat{\alpha}$, and $\hat{\alpha}$, along with other parameters shown in Table 1 in the system of equations of the balanced growth path equilibrium of the Privatized Targetable Basic Research Economy.

3. Use of the estimated parameter values $\hat{\alpha}$, $\hat{\lambda}_0$ and $\hat{\lambda}_1$, along with other parameters shown in Table 1 in the system of equations of the balanced growth path equilibrium of the Privatized Untargetable Basic Research Economy.

4. Comparison of the steady state innovation rates and welfare levels of the two policy scenarios of steps 2 and 3 with the Public Basic Research Economy that has generated the data.

$L$ is the percentage of people who were 25 year old or more and who had completed at least 4 years of college, collected by the U.S. Census (2010a), Current Population Survey, Historical Tables\(^{26}\).

$\bar{L}_G$ is calculated by dividing the expenditure on basic research by the amount of wages paid to publicly employed scientist and engineers\(^{27}\). The relevant series of the expenditure on basic research in our estimations is the total basic R&D expenditure net of the industry performed basic R&D\(^{28}\).

$w_s$ is the skilled premium estimated by Krusell, Ohanian, Rios-Rull and Violante (2000).

The $g_{PUBL}$ data (according to our model, the measure of the actual U.S. innovation rate before 1980) are the number of utility patents granted to U.S. residents per million inhabitants\(^{29}\).

As for the real rate of return on consumer assets, we adopt the usual $r = \rho = 0.05$, consistently with Mehra and Prescott’s (1985) estimates for the pre-1980 period.

The following Table 1 reports the parameters we have outclassed and their sources:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>Skilled Labour (intensity 1979)</td>
<td>0.164</td>
<td>U.S. Census, Current Population Survey</td>
</tr>
<tr>
<td>$M$</td>
<td>Unskilled Labour (intensity 1979)</td>
<td>0.836</td>
<td>U.S. Census, Current Population Survey</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Subjective Rate of Time Preference</td>
<td>0.05</td>
<td>Mehra and Prescott (1985)</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Basic Research Productivity</td>
<td>0.00192</td>
<td>Estimation</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Applied Research Productivity</td>
<td>0.1607</td>
<td>Estimation</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Mark-up</td>
<td>1.082</td>
<td>Estimation</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Skilled Share in Manufacturing</td>
<td>0.189</td>
<td>Estimation</td>
</tr>
<tr>
<td>$a$</td>
<td>R&amp;D congestion</td>
<td>0.718</td>
<td>Estimation</td>
</tr>
</tbody>
</table>

---

\(^{26}\)Available at: www.census.gov/population/socdemo/education/tabA-2.xls


\(^{28}\)Both series are taken from the NSF Science & Engineering Indicators (2005).

\(^{29}\)Source: USPTO (2010).
Our estimated value of $\gamma$ is consistent with that estimated by Roeger (1995) and Martins et al. (1996).

Our estimate the intra-sectorial congestion parameter $a$ is consistent with Jones and Williams’ (1998) and (2000) calibrations.

The reason why we have also estimated parameter $\alpha$ - the high skilled labour elasticity in manufacturing production - instead of relying on available statistics on labour shares, is that they fail to single out the fraction of high skilled labour in production, consistently with our stylized economy.

## 5.3 Policy Comparisons

In this section we utilize the previously estimated values of the technological parameters, along with the previous exogenous variable to compute the hypothetical steady state equilibrium of the two scenarios - patentable research tools under targetable and untargetable basic research - for the year 1979, i.e. the last year of the non-patentable research tools regime. It is important to remark that the qualitative results do not change if instead we use any combinations of the data in the last 5 years time interval (from 1975 to 1979).

### 5.3.1 Targetable Basic Research

In our exercise, we compare the steady state equilibrium innovative performance of the patentable research tool scenario with a hypothetical public scenario constrained to employ the same number of basic researchers as in the privatized scenario.

We have also simulated the welfare levels associated with the different IPR and targetableness scenarios.

Table 2 lists the comparative innovation rates, skill premia, and steady state welfares - based on the 1979 data and estimated parameter values - of the public basic research regime, and the privatized targetable and untargetable, $g_{PRIVu}$, basic research regimes:

\[ Welf_s = \int_0^\infty e^{-\rho t} \left[ \log(\gamma) g_s t + \log(x_s^o M^{1-\alpha}) \right] dt = \]
\[ = \frac{\log(\gamma) g_s}{\rho^2} + \frac{\log(x_s^o M^{1-\alpha})}{\rho}, s = PUBBL, PRIV, and PRIVu. \quad (22) \]

\[ \text{associated with the different IPR and targetableness scenarios.} \]

\[ \text{Table 2 lists the comparative innovation rates, skill premia, and steady state welfares - based on the 1979 data and estimated parameter values - of the public basic research regime, and the privatized targetable and untargetable, } g_{PRIVu}, \text{ basic research regimes:} \]

\[ \text{30 In this paper’s restrictive interpretation as highly skilled workers with at least college education, and able to perform R&D activities competently.} \]

\[ \text{31 For example, the ratio of non-production workers in operating establishments to total employment in 1979 was 0.248 (Berman, Bound, and Griliches, 1994), but this would include a large fraction of not highly skilled workers, as well as people actually undertaking knowledge-related activities.} \]

\[ \text{32 See Appendix 1 for the derivation of this expression.} \]
Table 2

<table>
<thead>
<tr>
<th>Scenarios/Variables</th>
<th>Innovation Rate</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public</td>
<td>0.00030969</td>
<td>-9.85</td>
</tr>
<tr>
<td>Private Targetable</td>
<td>0.00001541</td>
<td>-9.31</td>
</tr>
<tr>
<td>Private Untargetable</td>
<td>0.00062476</td>
<td>-10.24</td>
</tr>
</tbody>
</table>

As the data in the table show, the privatized basic research scenario outgrows the public basic R&D regime if and only if the basic research is untargetable. This is quite surprising, because when basic research cannot be targeted to specific products the private sector cannot do better than the public sector in allocating it. The unpatentable R&D scenario was innovating more than would a privatized targetable basic research in the relevant period immediately before the US turning point. Contrary to our initial intuition about the invisible hand’s guidance provided by privatization, the government must have been relying on the untargetability of basic research: had basic research been targetable, the ensuing economic growth performance would have been quite disappointing, according to our Table 2 simulations.

Hence we can say that the 1980 US normative change was the growth-enhancing institutional response to the underlying technological modifications. In fact, the privatized steady state innovation rate cannot lead to more innovation if basic research is targetable and it has not advantage on the public sector if innovation is untargetable. Yet, the reform has delivered more innovation, as the 1980s and 1990s have seen a spur of innovations under the new regime. The reason suggested by our simulations seems to be that the US government wanted to privatize not in order to enhance a more efficient basic research system, but rather to incentivize market forces to attract more basic R&D investment than would have been acceptable with the public regime. This for two reasons: 1. The financing of more basic research would have required additional taxes; 2. It would be not in the interest of the welfare of the general public - as shown by Table 2’s welfare comparisons.

Similar results could have been obtained by merely increasing the amount of public basic research. In fact, we have simulated the public economy using the same parameters, but increasing the public basic research employment to the same steady state equilibrium level of the privatized untargetable basic research scenario. The results are as follows:

Table 3

<table>
<thead>
<tr>
<th>Scenarios/Variables</th>
<th>Innovation Rate</th>
<th>Skill Premium</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Simulated</td>
<td>0.0006263865998</td>
<td>1.607652</td>
<td>-10.85612778</td>
</tr>
</tbody>
</table>

Table 3 shows that the US government could have chosen to increase basic R&D employment in the same amount as the private sector would have done after the reform, with slightly better

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33Statistically speaking, we should qualify our argument because, given the absence of additional assumptions on error terms, we do not have confidence intervals. However, the 2000% drop in the innovation rate between the first two rows, as well as its 101% increase from the first to the third row seem quite striking.

34That exogenously setting a level of $L_G = 0.018902956$, as predicted by Table 4.
growth performance - due to the free availability of basic innovations - but with a major loss in consumer welfare. Consumers would not have been happy to support this growth-enhancing public expansion. Hence the government, needing more innovation to recover lost terrain in world technological leadership, must have decided to let the market accomplish this technological boom.\footnote{In terms of compensating variation of the steady consumption, the welfare loss associated with this increase in public basic research would have been equivalent to an 8.74\% drop in consumption. More details in the next footnote.}

This is further confirmed by the following Table 4, which shows the steady state amount of manufacturing, basic research, and applied research skilled employment - as a fraction of the skilled labour force - predicted by either regime:

| Table 4 |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| **Scenarios/Variables** | **Manufacturing Employment** | **Basic Research** | **Applied R&D** | **Skill Premium** |
| Public          | 0.1581          | 0.0015715       | 0.00428941     | 1.28            |
| Private Targetable | 0.1640          | 0.0000109       | 0.0000317      | 1.0874          |
| Private Untargetable | 0.1423          | 0.0189029       | 0.00268        | 1.3644          |

As the reader can see, the privatized scenario achieves faster growth by increasing total R&D employment and reducing manufacturing employment. This implies a negative level effect on consumption which, according to our welfare calculations, is not compensated by a positive growth effect on consumer good quality. Quite interesting, in our simulations basic research is responsible for the increase in total R&D labour, while applied R&D employment stays roughly constant. This is confirmed by the actual data on the early 1980s. Increasing the returns to basic research allowed the US to achieve a boost in innovation rate that would have been hard to finance publicly.\footnote{To quantify the welfare change in a cardinal manner, let us consider the consumption increase equivalent to the reform. Using the previous definition of welfare, we have to determine the level of consumption, $c_E$, such that the following hold:}

$$Welf_{PRIVu} = \int_0^\infty e^{-\rho t} [\log(\gamma)g_{PUBL} + \log c_E] \, dt =$$

$$= \frac{\log(\gamma)g_{PUBL}}{\rho^2} + \frac{\log c_E}{\rho},$$

$$c_E = \exp \left( \rho Welf_{PRIVu} - \frac{\log(\gamma)g_{PUBL}}{\rho} \right) =$$

$$\exp \left( -0.05 \times 10.2377 - \ln(1.082) \frac{0.00030969}{0.05} \right) = 0.599,$$

which imply an increase in consumption equal to $\frac{c_E - c_{PUBL}}{c_{PUBL}} = \frac{0.599 - 0.611}{0.611} = -1.91\%$. Therefore the 1980 US reform, according to our model, has generated a 101\% increase in the long-term innovation rate, at a relatively mild welfare cost.
6 Final Remarks

The debate on the effects of the patentability of research tools on the incentives to innovate is still very controversial, not only in the US but also in Europe and in other important areas of the world. This paper analyzed from a general equilibrium perspective the US policy shift towards the extension of patentability to research tools and basic scientific ideas that took place around 1980. These normative innovations have been modifying the industrial and academic lives in the last three decades, raising doubts on their desirability. The losses from the free entry into basic research and the monopolization of applied research induced by intellectual property of the research tools have been compared with the inefficacy of public research institutions to promptly react to downstream market opportunities and the potentially excessive entry into applied R&D.

Results were not a priory unambiguous, which forced us to use the available data and calibrate and simulate our model in order to check if the US did it right in changing their institutions around 1980. A broad consensus in economic literature, also confirmed by recent studies (see Lam 2009, OECD 2012, Howitt 2013), has been suggesting that the motivations for basic researcher goes beyond personal income and particularly include the opportunities to advance the scientist’s research agenda. Related with such contributions, a large literature pointed out the technological impossibility to direct or target basic research by granting the inventor property right on the basic research output. In this paper, we have robustly found that under the assumption of untargetable basic research, the U.S. assigning property rights to basic research findings and creating a market for research tools spurred innovation. However, the welfare maximizing scenario could be obtained with same IPR reform if the basic research would have been targetable: if that was the case a benevolent government would have still implemented the reform, with the aim of increasing welfare today. Since data generically witness an increase in the US innovativeness following 1980, we can confidently say that our model seems to falsify the hypothesis of targetable basic research and confirm the hypothesis of untargetable research.

The fact that the government was disregarding basic research targetability is also witnessed by the official statistics’ definitions of basic research, which have always highlighted its untargetable connotation.

As a final remark, we can say that, after carefully constructing three scenarios and simulating their calibrated economies, we conclude that the government has undertaken its 1980 patentability reform under the assumption that basic research is untargetable and to the aim of incentivizing the market to increase basic research and the innovativeness of the US economy over and above the politically tolerable level of those years.
7 Bibliography


Trademark Office.


Appendix 1
Model Details

This Appendix explains the details of the quality ladder model used in the main text. It may be skipped by readers familiar with this literature.

Time $t \geq 0$ population $P(t)$ is assumed growing at rate $g_{\text{Pop}} \geq 0$ and its initial level is normalized to 1. The representative household preferences are represented by the following intertemporally additive utility functional\(^{37}\):

$$U = \int_0^\infty e^{-rt} \ln D(t) dt, \quad (23)$$

where $r > 0$ is the subjective rate of time preference, and $D(t)$ is an infra-household per capita consumption index reflecting the household's taste for variety and for product quality. Per-family member instantaneous utility is given by:

$$\ln D(t) = \int_0^1 \ln \left( \sum_j \gamma^j d_{jt}(\omega) \right) d\omega, \quad (24)$$

where $d_{jt}(\omega)$ is the individual consumption of a good of quality $j = 1, 2, \ldots$ (that is, a product that underwent up to $j$ quality jumps) and produced in industry $\omega$ at time $t$. Parameter $\gamma > 1$ measures the size of the quality upgrades. This formulation, common to Grossman and Helpman (1991) and Segerstrom (1998), assumes that each consumer prefers higher quality products of different varieties\(^{38}\). Since we are not incorporating horizontal innovation, the set of varieties is bounded and normalized to the unit interval.

The representative consumer is endowed with $L > 0$ units of skilled labor and $M > 0$ units of unskilled labor summing to 1. Since initial population is normalized to 1, $L$ and $M$ will also equal, in equilibrium, the per capita supply of skilled, respectively, unskilled labour. Unskilled

\(^{37}\)We skip starting with an expectational operator in order to save notation. A more general setting of the consumer problem would not change results, as in our framework, due to perfectly diversifiable risks, law of large numbers, and perfect financial markets, the consumer's wealth evolves deterministically in equilibrium.

\(^{38}\)Of course, $D(t) = \exp \left[ \int_0^1 \ln \left( \sum_j \gamma^j d_{jt}(\omega) \right) d\omega \right]$ could alternatively be interpreted as a CRS production function of a homogenous final product, produced with a range of different intermediate goods of different qualities. Hence, in this model the growth rate of the consumption index $D(t)$ has an immediate interpretation as the growth rate of final production per capita.
labor can only be employed in the final goods production. Skilled labour is able to perform R&D activities.

Focussing on the set $J_t(\omega)$ of the existing quality levels with the lowest quality-adjusted prices, the household, at each instant, allocates maximizes the instantaneous utility (24) according to the following static constraint

$$E(t) = \int_0^1 \sum_{j \in J_t(\omega)} p_{jt}(\omega)d_{jt}(\omega) d\omega,$$

where $E(t)$ denotes a given per capita consumption expenditure and $p_{jt}(\omega)$ is the price of a product of quality $j$ produced in industry $\omega$ at time $t$. Let us define $j_t^*(\omega) \equiv \max \{j : j \in J_t(\omega)\}$. Using the instantaneous optimization results, we can re-write (24) as

$$u(t) = \int_0^1 \ln \left[ \gamma^{j_t^*(\omega)} E(t)/p_{jt}(\omega) \right] d\omega = \ln[E(t)] + \ln(\gamma) \int_0^1 j_t^*(\omega) d\omega - \int_0^1 \ln[p_{jt}(\omega)] d\omega$$

The solution of this maximization problem yields the static demand function:

$$d_{jt}(\omega) = \begin{cases} E(t)/p_{jt}(\omega) \text{ for } j = j_t^*(\omega) \\ 0 \text{ otherwise.} \end{cases}$$

where we posit that if two products have the same quality-adjusted price, consumers buy the higher quality product.

Therefore the consumer chooses the piecewise continuous per-family member expenditure trajectory, $E(\cdot)$, that maximizes:

$$U = \int_0^\infty e^{-rt} \ln[E(t)] dt.$$  

Households possess equal shares of all the firms at time $t = 0$, hence later. Letting $A(0)$ denote the present value of human capital plus the present value of asset holdings at $t = 0$, each household’s intertemporal budget constraint is:

$$\int_0^\infty e^{-I(t)} e^{g_{Pop}t} E(t) dt \leq A(0)$$

where $I(t) = \int_0^t i(s) ds$ represents the equilibrium cumulative real interest rate up to time $t$.

Finally, the representative consumer chooses the time pattern of consumption expenditure to maximize (29) subject to the intertemporal budget constraint (30). The equilibrium expenditure trajectory satisfies the Euler equation:

$$\dot{E}(t)/E(t) = i(t) - (r + g_{Pop})$$
where $i(t) = I(t)$ is the instantaneous market interest rate at time $t$ - along with the usual transversality condition and the no-Ponzi game condition.

Since preferences are homothetic, in each industry aggregate demand is proportional to the representative consumer. $E$ denotes the aggregate consumption spending and $d$ denotes the aggregate demand.

As for the production side, we assume constant returns to scale technologies in the (differentiated) manufacturing sectors represented by the following production functions:

$$y(\omega) = X^\alpha(\omega) M^{1-\alpha}(\omega), \text{ for all } \omega \in [0, 1],$$

(32)

where $\alpha \in (0, 1)$, $y(\omega)$ is the output flow per unit time, $X(\omega)$ and $M(\omega)$ are, respectively, the skilled and unskilled labour input flows in industry $\omega \in [0, 1]$. Letting $w_s$ and $w_u$ denote the skilled and unskilled wage rates, in each industry the quality leader seeks to minimize its total cost flow $C = w_s X(\omega) + w_u M(\omega)$ subject to constraint (32). For $y(\omega) = 1$, the conditional unskilled (33) and skilled (34) labour demand per-unit of output are:

$$M(\omega) = \left(\frac{1-\alpha}{\alpha}\right)^\alpha \left(\frac{w_s}{w_u}\right)^\alpha,$$

(33)

$$X(\omega) = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{w_u}{w_s}\right)^{1-\alpha}.$$

(34)

Thus cost is:

$$C(w_s, w_u, y) = c(w_s, w_u)y$$

(35)

where $c(w_s, w_u)$ is the per-unit cost function:

$$c(w_s, w_u) = \left[\left(\frac{1-\alpha}{\alpha}\right)^{-\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\right] w_s^\alpha w_u^{1-\alpha}.$$  

(36)

Since unskilled labour is uniquely employed in the final good sectors and all price variables (including wages) are assumed to instantaneously adjust to their market clearing values, unskilled labour aggregate demand $\int_0^1 M(\omega) d\omega$ is equal to its aggregate supply, $MP(t)$, at any date. Since industries are symmetric and their number is normalized to 1, in equilibrium\(^{39}\) $M(\omega) = MP(t)$.

Unskilled labour as numeraire implies $w_u = 1$. From equations (33) and (34) we get the firm’s skilled labour demand function:

$$X(\omega) = \frac{1}{w_s} \left(\frac{\alpha}{1-\alpha}\right) MP(t)$$

(37)

In percapita terms,

$$x(\omega) \equiv \frac{X(\omega)}{P(t)} = \frac{1}{w_s} \left(\frac{\alpha}{1-\alpha}\right) M.$$  

(38)

\(^{39}\)More generally, with mass $N > 0$ of final good industries, in equilibrium $M(\omega) = \frac{MP(t)}{N}$.  

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In each industry, at each instant, firms compete in prices. Given demand function (28), within each industry product innovation is non-drastic, hence the quality leader will fix its (limit) price by charging a mark-up $\gamma$ over the unit cost:

$$p = \gamma c(w_s, 1) \Rightarrow d = \frac{E}{\gamma c(w_s, 1)}.$$  \hfill (39)

Hence each monopolist earns a flow of profit, in percapita terms, equal to

$$\pi = \frac{\gamma - 1}{\gamma} E = (\gamma - 1) \frac{w_s x}{\alpha}$$

$$\pi = (\gamma - 1) \frac{1}{1 - \alpha} M.$$  \hfill (40)

From eq.s (40) follows:

$$\frac{\gamma - 1}{\gamma} E = (\gamma - 1) \frac{1}{1 - \alpha} M \Rightarrow E = \frac{\gamma}{1 - \alpha} M.$$  \hfill (41)

Interestingly, eq. (41) implies that in equilibrium total expenditure is always constant. Therefore, eq. (31) implies a constant real interest rate:

$$i(t) = r + g_{Pop}.$$  \hfill (42)

### Population Growth and Scale Effects

In the main text, we assume constant population. However, introducing growing population would not alter neither our model nor its main empirical results, if we stationarize the growing variables in percapita terms. In particular, we define $n_B(\omega, t) \equiv \frac{N_B(\omega, t)}{P(t)}$ and $n_A(\omega, t) \equiv \frac{N_A(\omega, t)}{P(t)}$ - where $P(t)$ denotes total population at time $t$ - as the skilled labor employment in each basic and, respectively, applied R&D sector.

Notice that as the economies analyzed in the two models of this paper tend to their balanced growth path, the corresponding firm profits and stock market values will tend to evolve at the population growth rate $g_{Pop}$. To stationarize them we normalize firm values by dividing them by population. Therefore, for example, $v^1_L(\omega, t) \equiv \frac{v^1_L(\omega, t)}{P(t)}$, and similarly for all other firm values. Based on this, the reader can easility re-obtain the equations involving firm values in the main text, because the terms that explicitly contain the growth rate of population cancel out, as for example in

$$iv^1_L = \pi - (n^*_A)^{1-a} \lambda_1 v^1_L + v^1_L g_{Pop} + \frac{dv^1_L}{dt}.$$  

---

40 We are following Aghion and Howitt’s (1992) and (1998) definition of drastic innovation as generating a sufficiently large quality jump to allow the new monopolist to maximize profits without risking the re-entry of the previous monopoly. Given the unit elastic demand, here the unconstrained profit maximizing price would be infinitely high: that would induce the previous incumbent to re-enter.

which, based on eq. (42), becomes
\[
(r + g_{Pop})v^1_L = \pi - (n^*_A)^{1-a}\lambda_1 v^1_L + v^1_L g_{Pop} + \frac{dv^1_L}{dt}
\]
and hence
\[
r v^1_L = \pi - (n^*_A)^{1-a}\lambda_1 v^1_L + \frac{dv^1_L}{dt},
\]
as it appears in eq.s (5c) and (11d). For the same reason, \(g_{Pop}\) disappears from the other financial market arbitrage equations.

**Steady State Welfare**

We here derive the equation used in our simulations to assess the steady state welfare associated with each scenario. In equilibrium the instantaneous utility function (24), after reminding that \(d_{j(t)}(\omega) = x^\alpha M^{1-\alpha}\), becomes
\[
\ln D(t) = \int_0^1 \ln [\gamma j^* t(\omega) d_{j(t)}(\omega)] d\omega = \log(\gamma) \int_0^1 j^*_s(\omega) d\omega + \log(x^\alpha M^{1-\alpha}). \tag{43}
\]
In equilibrium \(j^*_t(\omega) = j_t(\omega)\) in all industries. Focussing on balanced growth paths, we can assume that the economy starts from the steady state value of all variables (including \(m(A_0)\)). Hence:
\[
\ln D(t) = \log(\gamma) g_s t + \log(x^\alpha M^{1-\alpha}) + \log(\gamma) \int_0^1 j^*_0(\omega) d\omega, \tag{44}
\]
with index \(s = PUBBL, PRIV,\) and \(RExem,\) depending on the institutional scenario chosen. In fact, \(\int_0^1 j^*_t(\omega) d\omega = g_s t + \int_0^1 j^*_0(\omega) d\omega.\) To understand this, it is important to remember that all processes are independent, all sectors are symmetric within \(A_0\) and \(A_1,\) and there is an infinite number of them. Define \(\phi(t) \equiv \int_0^1 j^*_t(\omega) d\omega.\) Consider a positive and small time increment \(\Delta t,\) and the increment \(\phi(t + \Delta t) - \phi(t) = \int_0^1 [j^*_{t+\Delta t}(\omega) - j^*_t(\omega)] d\omega.\) Notice that, by the properties of Poisson processes, \(j^*_{t+\Delta t}(\omega) - j^*_t(\omega) = 0 \text{ or } 1,\) except for events with probability of a zero of higher order than \(\Delta t,\) which we write \(o(\Delta t).\) By the law of large numbers the average number of jumps is equal to its expected value. Hence:
\[
\phi(t + \Delta t) - \phi(t) = \int_{A_1(t)} [0 * (1 - (n^*_A)^{1-a}\lambda_1\Delta t) + 1 * (n^*_A)^{1-a}\lambda_1\Delta t] d\omega + o(\Delta t) = (1 - m(A_0))(n^*_A)^{1-a}\lambda_1\Delta t + o(\Delta t).
\]
Dividing both sides by \(\Delta t\) and taking the limit \(\Delta t \to 0,\) and remembering that \(\lim_{\Delta t \to 0} o(\Delta t)/\Delta t = 0,\) gives \(\phi'(t) = (1 - m(A_0))(n^*_A)^{1-a}\lambda_1 \equiv g_s.\) Along a steady state \(g_s\) is constant, and hence \(\phi(t) = g_s t + \phi(0) = g_s t + \int_0^1 j^*_0(\omega) d\omega.\) Assuming that the initial value of \(\int_0^1 j^*_0(\omega) d\omega\) is the same under each scenario \(s = PUBBL, PRIV,\) and \(PRIVu,\) we can normalise it at zero. Therefore, with no loss of generality, we can use the following simpler expression:
\[
Welf_s = \int_0^\infty e^{-\rho t} \left[\log(\gamma) g_s t + \log(x^\alpha M^{1-\alpha})\right] dt = \frac{\log(\gamma) g_s}{\rho^2} + \frac{\log(x^\alpha M^{1-\alpha})}{\rho}, \tag{45}
\]
\[
\text{Steady State Welfare}
\]

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This is the expression we have used in all our numerical welfare comparisons.

As a by-product of our analysis, notice that taking the derivative of both sides of eq. (44) with respect to time gives:

\[
\frac{\dot{D}(t)}{D(t)} = \log(\gamma)g_s,
\]

which clarifies the link between the aggregate innovation rate \(g_s\) and the percapita consumption\(^\text{42}\) index growth rate.

Appendix 2

**Lemma 1.** In the Public Basic Research economy there can exist no more than one balanced growth path equilibrium.

**Proof.** At the steady state, \(\frac{dm_0}{dt} = 0\), and hence eq. (6) can be rewritten as:

\[
(1 - m_0)n_A^{1-a}{\lambda_1} = m_0\bar{L}_G^{1-a}{\lambda_0},
\]

which defines \(m_0\) as an increasing function of \(n_A\):

\[
m_0 = \frac{n_A^{1-a}{\lambda_1}}{\bar{L}_G^{1-a}{\lambda_0} + n_A^{1-a}{\lambda_1}}.
\]

From (48) it is easily seen that \((1 - m_0)n_A\) is an increasing function of \(n_A\).

Eq. (5b) implies that \(v_0^0\) is an increasing function of \(v_1^1\); in turn, (5c) implies that \(v_1^1\) is a decreasing function of \(n_A\). Therefore, also \(v_0^0\) is a decreasing function of \(n_A\). But then, eq. (5a) implies that \(w_s\) too will be a decreasing function of \(n_A\).

Let us then rewrite the labour market equilibrium condition (8) as

\[
(1 - m_0)n_A = L - \frac{1}{w_s} \left( \frac{\alpha}{1 - \alpha} \right) M - \bar{L}_G.
\]

In light of the preceding discussion, the left side of equation (49) is an increasing function of \(n_A\), while the right side is a decreasing function of \(n_A\). The steady state equilibrium value of \(n_A\) will be associated with the unique intersection between the curves defined by the two sides of this equation. Since the real values of all the other endogenous variables at the steady state are pinned down by \(n_A\), they will be uniquely determined. Therefore, if a steady state equilibrium exists it will be unique. QED.

**Lemma 2.** In the Privatized Basic Research economy there can exist no more than one balanced growth path equilibrium.

\(^{42}\)Or of actual percapita consumption, in the production function interpretation of \(D(t)\).
Proof. Use eq. (11a) to obtain \( w_s \), and plug into (10) to obtain the steady state version of eq. (11b), which, solved for \( v_A \) gives:

\[
v_A = \left( \frac{a}{r} \right)^a \left( \frac{1 - a}{\lambda_0} \right)^{1-a} (n_B)^{1-a} \lambda_1 (v^0_L - v_A). \tag{50}\]

Plugging (11a) and (50) into (11b) and solving for \( v^1_L \) gives:

\[
v^1_L = \frac{\pi}{r + (n_B)^{1-a} \frac{r(1-a)}{a\lambda_0} (n_B)^{a(1-a)} \lambda_1},
\]

which can be plugged into eq. (11c) to solve for \( v^0_L \) as:

\[
v^0_L = \frac{\pi}{r + (n_B)^{1-a} \lambda_0} \left[ 1 + \frac{(n_B)^{1-a} \lambda_0}{r + (n_B)^{a(1-a)} \lambda_1} \right]. \tag{51}\]

Plugging (51) into eq. (50) and solving for \( v_A \) yields:

\[
v_A = \frac{\pi}{r + (n_B)^{1-a} \lambda_0} \left[ 1 + \frac{(n_B)^{1-a} \lambda_0}{r + (n_B)^{a(1-a)} \lambda_1} \right] \frac{1 + \frac{r^a \lambda_0^{1-a}}{a^a(1-a)^{(1-a)} n_B^{a(1-a)} \lambda_1}}{1 + \frac{r^a \lambda_0^{1-a}}{a^a(1-a)^{(1-a)} n_B^{a(1-a)} \lambda_1}} \tag{52}\]

As will soon be clear, it is important to study how \( \frac{v_A}{n_B} \) changes with \( n_B \). Based on eq. (50), we can write:

\[
\frac{d}{dn_B} \left( \frac{v_A}{n_B} \right) = \frac{d}{dn_B} \left[ \frac{\pi}{r n_B^{a(1-a)} + n_B \lambda_0} \left( \frac{n_B^{1-a} \lambda_0}{r + n_B^{a(1-a)} \lambda_1 \left( \frac{-1}{a \lambda_0} (a - 1) \right)^{1-a}} \right) \right] = \frac{d}{dn_B} \left[ \frac{a^a(1-a)^{(1-a)} n_B^{a(1-a)} \lambda_1}{r^a \lambda_0^{1-a} + a^a(1-a)^{(1-a)} n_B^{a(1-a)} \lambda_1} \right]
\]
\[
\begin{align*}
2a^2 & r_3 \lambda_0 n_B^{2a^2-a-1} (1-a)^{1-a} + a^2 r_4 \lambda_0 n_B^{3a^2-2} (1-a)^{1-a} + \\
a^2 & r_3 \lambda_1 n_B^{2a^2+a-2} (1-a)^{2-2a} + a^2 r_4 \lambda_0 n_B^{a(3a-2)} (1-a)^{1-a} + \\
& r_2 \lambda_0 n_B^{2a^2-1} (1-a)^{1-a} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{1-a} + \\
& a \lambda_0 \lambda_1 n_B^{2a-1} (1-a)^{2-2a} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{2-2a} + \\
& 2a^2 r_2 \lambda_0 n_B^{2a^2-2} (1-a)^{2-2a} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{1-a} + \\
2a^2 & r_2 \lambda_0 \lambda_1 n_B^{2a^2-1} (1-a)^{2-2a} + 2a^2 r_3 \lambda_0 n_B^{2a^2+a-2} (1-a)^{1-a} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{1-a} + \\
& a^2 r_2 \lambda_0 \lambda_1^{a(2a-1)} (1-a)^{1-a} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{1-a} + \\
& a^2 r_2 \lambda_0 \lambda_1^{a^2+a-1} (1-a)^{2-2a} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{1-a} + \\
(1-a) & r \lambda_0 \lambda_1 n_B^{2a^2+a-1} (1-a)^{1-a} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{2-2a} + \\
(3-a) & a^2 r \lambda_0 \lambda_1^{a^2+a-1} n_B^{2a^2+a-1} (1-a)^{2-2a} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{1-a} + \\
& \frac{\pi a^2 \lambda_1}{n_B^{4a^2+2a+1}} \left( \lambda_0 n_B + r n_B^a \right)^2 \left( r^a \lambda_0^{1-a} + a^2 \lambda_0 \frac{\lambda_1}{n_B^{1-a}} (1-a)^{1-a} \right)^2 \left( r + \frac{\lambda_1}{n_B^{1-a}} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{1-a} \right)^2 \left( r + \frac{\lambda_1}{n_B^{1-a}} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{1-a} \right)^2 \left( r + \frac{\lambda_1}{n_B^{1-a}} \left( -\frac{1}{a} \frac{r}{\lambda_0} (a-1) \right)^{1-a} \right)^2
\end{align*}
\]

which is certainly negative because \(0 < a < 1\), that is:

\[
\frac{d}{dn_B} (n_B^{1-a} v_A) < 0.
\]

Plugging (10) into (15), setting \(\frac{dm_0}{dt} = 0\), and solving for \(m_0\) gives:

\[
m_0 = \frac{1}{1 + \frac{\lambda_1}{\lambda_2} \left( \frac{a}{r(1-a)} \right)^{1-a} n_B^{(1-a)^2}}.
\]  

Eq. (54) shows that \(m_0\) is a decreasing function of \(n_B\), and therefore \(1 - m_0\) is an increasing function of \(n_B\). However, notice also that \(m_0 n_B\) is an increasing function of \(n_B\).

Obtaining skilled wage from (11a) and plugging it into (12), and in light of eq's (10) and (50), we can rewrite the skilled labour market condition (14) as:

\[
m_0 n_B = L - \frac{\alpha M}{(1-a) \lambda_0 n_B^{1-a} v_A} - \frac{(1-m_0) n_B^a (1-a) r}{\lambda_0 a}.
\]
Recalling the discussion after eq. (54), the left side of equation (55) is an increasing function of $n_B$. From (53) and (54), the right side of (55) is instead a decreasing function of $n_B$. Therefore there will exist only one intersection between the corresponding curves, and therefore a unique real value of $n_B$ that solves equation (55). Since the real values of all other endogenous variables are uniquely pinned down by $n_B$, there can exist only a unique steady state equilibrium. QED