Contribution to a public good with altruistic preferences

Anwesha Banerjee

November 29th, 2019

Abstract

This paper presents a general model of private provision of a public good where individuals in a group have altruistic preferences towards other members of their group. I show that at the unique Nash equilibrium, there exists a threshold level of income above which members start contributing to the public good. Moreover, I examine a second model where members can give private transfers of income to other members they care about, in addition to contributing to the public good. Under additive separability of the utility functions, I establish the uniqueness and existence of the Nash equilibrium of the model with transfers and find it to be closely connected to the equilibrium of the model without transfers. The threshold level of income of the model without transfers and the income of the poorest individual in the group together play a key role in determining the existence of private transfers of income.

Keywords: Public goods, Altruism, Private transfers.

JEL classification codes: C72, H41

1 Introduction

Altruistic behavior is an important aspect of interactions within social groups such as friends and family. It can also influence cooperative behavior, as demonstrated in public good experiments by Andreoni (1995) and Andreoni (1988b). Individuals who contribute to a public good may often be part of a community and care about the other members. This is, in particular, the case for many public goods such as neighbors contributing to the cleanliness of a neighborhood, or donating to a communal fund.

This paper presents a general model of private provision of a public good where individuals in a group have altruistic preferences towards all other members of their group, and care about the well-being of other members. The well-being of an individual depends not only on his own private utility from his contribution to the public good and the amount of the public good available, but also the private utilities of all other group members. I subsequently examine a second model that allows for private transfers between members of the group: members may transfer money directly to other agents they care about, as well as contribute to the public good.

The paper has three core contributions. Firstly, it presents the first analysis of a general model of private provision of a public good with altruistic preferences. It highlights the implications of this model both for the case when individuals can make private transfers to people they care about, and the case when such private transfers are not possible. Secondly, for the model without private transfers, the existence and uniqueness of a Nash equilibrium are established, and this equilibrium is then found to be closely connected to the unique equilibrium of the model with transfers under additive separability of preferences. The third main contribution of the paper lies in the implications of the connection between the two models. Only the richest individuals in the group contribute to the public good and /or make transfers. Private transfers between individuals are possible in equilibrium if and only if the richest members of the group, after contributing to the public good, are still relatively richer than the poorest member of the group. The relative difference in income between the contributors to the public good and the poorest individual is thus critical in determining the existence of private transfers of incomes between individuals.

The analysis of this paper is developed in three steps. First, I present a model of private provision of a public good à la Bergstrom, Blume, and Varian (1986) where every member of the group has altruistic preferences. Following Becker (1974) the social utility of a group member is modeled as a linear combination of his own private utility as well as the private utility of other group members. I first show that such a game always has a unique Nash equilibrium. At any equilibrium, every contributor to the public good enjoys a common level of consumption of a private good, and they consecrate the rest of their income to the public good. There thus exists a threshold level of income which determines the set of contributors to the public good. I also establish a comparative statics result wherein if everyone is contributing to the public good, an increase in altruism increases the public good contribution of every contributor. In the second step, I study the model where members can transfer money privately to other members of the group, in addition to contributing to the public good. I show that any equilibrium of this game has a structure similar to the charity game presented in Arrow (1981). Several properties of the equilibrium are established. No transfer intermediaries are possible in equilibrium: i.e. there can be no individual such that he both gives and receives transfers, implying that the set of receivers and donors of transfers are disjoint. At any equilibrium with transfers, all receivers of transfers enjoy the same level of private good consumption that is strictly lower than the identical level of private good consumption that is enjoyed by all givers of transfers. Receivers of transfers do not contribute to the public good in equilibrium. Further, the set of individuals who either give transfers or contribute to the public good (or do both) is determined by whether the individuals' incomes exceed a common threshold level of income. In other words, only the sufficiently rich members of the group donate to the poor, and/or contribute to the public good. Finally, using the properties of the equilibrium stated above, for the special case of additively separable utility functions, I establish existence and uniqueness of the equilibrium of the model with transfers. The equilibrium of the model with transfers is found to be closely related to the equilibrium of the model without transfers: I find a necessary and sufficient condition under which the equilibrium of the model without transfers also serves as the equilibrium of the model with transfers. The equilibrium of these two models coincides if and only if the threshold level of income required to become a donor of transfers is higher than the threshold level of income required to become a contributor in the model without transfers. This result establishes that even when we allow for private transfers, contributing to the public good is a priority for the group.

Existing literature on the private provision of a public good does not model preferences for altruism. As modeled in Bergstrom, Blume, and Varian (1986), and developed notably by Cornes and Sandler (1986), Andreoni (1988a), Andreoni (1989) and Andreoni (1990) amongst many others, an individual contributes to charity (which was modeled as the public good) because he derives some utility from the sum of all contributions. The altruism of individuals was implicit in the 'charity' interpretation of a public good: individuals donated to charity because they cared about the sum of contributions. Since altruistic preferences were not explicitly modeled, the consequences of caring about others on private provision of a public good were not examined by these papers. In contrast, the current paper explicitly models altruistic preferences and studies its implications for public good provision.

There is a second branch of literature which followed Becker (1974) and Becker (1981) wherein preferences for altruism were modeled explicitly: that is, the situation where an individual derives utility from an increase (or decrease) in the utility of someone that he cared about(of someone he dislikes) is examined. This form of modeling altruism has been widely used in a variety of applications, notably in models of household decision-making behavior such as Chiappori (1988, 1992). There is very limited research on models that attempt to bridge the literature on Beckerian altruism with an analysis of public good provision. Cherchye, Cosaert, Demuynck, and De Rock (2017) is a notable exception in this literature that combines the non-cooperative model of household behavior with caring preferences for the case of only two household members, and multiple public and private goods. They also allow for different caring preferences (that is, one member of the family could care more about the other member). However, their analysis is not extended to the n-person setting of a public goods game. To the best of my knowledge, the current paper presents the first general model of public good provision with altruistic preferences.

Finally, there is very sparse literature on the connections between public good provision and private transfers. The implications of altruistic preferences on private transfers were first modeled by Arrow (1981). More recently, Bourlès, Bramoullé, and Perez-Richet (2017) study a game of private transfers over a fixed social network of individuals with altruistic preferences. However, these papers do not include any analysis of public goods. Arora and Sanditov (2016) consider a model where individuals with altruistic preferences, embedded in a fixed network, contribute to a public good for the specific case of constant relative risk aversion utility functions. However, they do not examine the implications of a general model, and they do not include an analysis of private transfers, as considered by the current paper. Even more recently, Bommier, Goerger, Goussebaile, and Nicolai (2019) consider a model involving altruism, public good provision, and private transfers with one developed country and many developing countries. The developed nation has altruistic preferences towards other developing nations and may transfer money to developing nations. All countries derive utility from abatement of pollution, the public good. Unlike the setting of the current paper where both private transfers and public good provision are determined simultaneously, Bommier, Goerger, Goussebaile, and Nicolai (2019) model their game as a two stage process, where the amount of public good provided is determined by a simultaneous game in the first stage, and private transfers are decided only by the developed country in the second stage.

The rest of the paper is organized as follows: Section 2 presents a model of contribution to a public good with altruistic preferences towards a group. Section 3 presents the model with transfers, and Section 4 concludes.

2 The model

This section develops a model of contribution to a public good with altruism, but without private transfers. This model is a natural choice for situations where people care about others but do not have the means of transferring money privately. This may happen because people can care about other members of a society without knowing them personally, or because of high transfer costs. There is a group of $N = \{1, 2, ..., n\}$ individuals with $n \geq 2$. A member $i \in N$ has an income $y_i \in [0, \bar{y}]$, where $\bar{y}$ is some strictly positive number. He must choose how to divide his income between his consumption of a private good, x_i, and his contribution to a public good g_i. A given profile of contributions $(g_1, g_2, ...g_n)$ produces an amount of public good $G = \sum_i g_i$. An individual i's preferences have two components. The first is a private component: he derives utility from his consumption of the private good x_i, as well as the amount of the public good G. This private component is represented by the same utility function $U : [0, \bar{y}] \times [0, n\bar{y}] \rightarrow \mathbb{R}$ for all individuals in the group. In addition, individuals are altruistic and their preferences have a social component: they care *equally* about the levels of private utilities of *all* the other members in their group. Individual i's preferences are represented by the following function V:

$$V(x_1, ..., x_n, G, \alpha) = U(x_i, G) + \alpha \sum_{j \in N, j \neq i} U(x_j, G) \tag{1}$$

where $\alpha \in (0,1)$ is a parameter measuring the degree of altruism. We assume that α is positive, meaning that individuals benefit when the private utility of group members increases. We also assume $\alpha < 1$ meaning that the individual values his private utility strictly higher than his social utility, and that U is strictly concave [1]. We also impose the following assumption on U:

Assumption 1 *For any combination* $(x, G) \in [0, \bar{y}] \times [0, n\bar{y}]$

$$U_{xG}(x, G) \geq 0$$

This assumption means that private consumption and the public good are complementary. This assumption is satisfied for many commonly used utility functions: for instance, for Cobb-Douglas utility where $U(x, G)$ takes the form $U(x, G) = x^a G^(1-a)$.This assumption is a sufficient condition that guarantees the results of this paper. If it is reversed, then that the results of this paper may not hold. In addition, a second assumption is imposed:

Assumption 2 *For any* $G \in [0, n\bar{y}]$ *and* $\alpha \in (0,1)$

$$-U_x(0,0)+(1-\alpha)U_G(0,0)+\alpha\sum_{i\in N} U_G(0,0) > 0 > -U_x(\bar{y}, G)+(1-\alpha)U_G(\bar{y}, G)+\alpha\sum_{i\in N} U_G(\bar{y}, G)$$

An implication of this assumption is that there exists at least one contributor in any equilibrium. We impose this condition to rule out equilibria where nobody contributes. This assumption is useful in establishing the uniquess of the Nash equilibria of the model. Denote by $\mathcal{U}$ the class of utility functions that satisfy the assumptions above.

For a given distribution of incomes denoted by $\mathbf{y} = (y_1, y_2, ..., y_n)$ and a coefficient of caring α, denote the normal form game described above by $\mathcal{G}(\mathbf{y}, \alpha)$. The objective of

[1] Derivatives are denoted by subscript i.e. U_x denotes the first order (partial) derivative of U with respect to x

this paper is to examine the properties of the Nash equilibrium of this game. Since the social utility function V is strictly concave, finding the Nash equilibrium corresponds to solving the following maximization problem for every individual i, given the contributions of members other than i:

$$\max_{x_i, g_i} \; U(x_i, g_i + \sum_{j \in N, j \neq i} g_j) + \alpha \sum_{j \in N, j \neq i} U(x_j, g_i + \sum_{j \in N, j \neq i} g_j)$$

subject to

$$x_i + g_i \leq y_i \tag{2}$$

$$g_i \geq 0 \text{ and } x_i \geq 0$$

Since the constraint (2) must always hold with equality, it is possible to transform the maximization problem above to

$$\max_{g_i} \; U(y_i - g_i, g_i + \sum_{j \neq i}^{n} g_j) + \alpha \sum_{j \in N, j \neq i} U(x_j, g_i + \sum_{j \neq i}^{n} g_j)$$

subject to:

$$0 \leq g_i \leq y_i$$

The first-order condition (necessary and sufficient for a solution to the above program) for any $i \in N$ is:

$$-U_x(x_i, G) + U_G(x_i, G) + \alpha \sum_{j \in N, j \neq i} U_G(x_j, G) \leq 0 \, \forall \, i \in N \tag{3}$$

and

$$-U_x(x_i, G) + U_G(x_i, G) + \alpha \sum_{j \in N, j \neq i} U_G(x_j, G) = 0 \;\; if \; g_i > 0 \tag{4}$$

Using this setting, we start by noting a property of the Nash equilibrium (if it exists) which while interesting in itself, will also help us to prove the existence of a unique Nah equilibrium. The following lemma shows that contributions to the public good are ordered in increasing order of income.

Lemma 1 *Let* $\mathbf{g}^* = (g_1^*, ..., g_n^*)$ *denote a Nash equilibrium of the game* $\mathcal{G}(\mathbf{y}, \alpha)$ *defined with a function* $U \in \mathcal{U}$. *Then all contributors to the public good at a Nash equilibrium enjoy the same level of private consumption, denoted by* $\tilde{x}$, *which is also the threshold level of income above which a member starts contributing.*

Lemma 1 shows the amount of the public good provided at the equilibrium does not change with redistributions of income which keep the set of contributors unchanged. Since every contributor i donates $y_i - \tilde{x}$, if we redistribute income in a way that does not change the set of contributors the threshold $\tilde{x}$ does not change, and hence neither does the amount of the public good provided. This 'neutrality' of the equilibrium public good contribution to redistribution in incomes within an unchanged set of contributors was first established by Bergstrom, Blume, and Varian (1986) for the case where individuals are not altruistic, and this paper shows that it extends to the case of altruistic preferences in private provision of a public good. This is formally stated in the following proposition:

Proposition 1 *Any redistribution in incomes that keeps the set of contributors at an equilibrium unchanged for the game* $G(\mathbf{y}, \alpha)$ *leaves the equilibrium public good consumption unchanged.*

Lemma 1 is important in that it establishes that in *any* equilibrium, all contributors must have the same level of private consumption. However, this does not prove existence or uniqueness of the equilibrium since there could be two equilibria with two sets of contributors. In the following proposition, we prove that the equilibrium is indeed unique.

Proposition 2 *The game* $\mathcal{G}(\mathbf{y}, \alpha)$ *defined with a function* $U \in \mathcal{U}$ *admits a unique Nash equilibrium.*

Existence is established using general, fixed point arguments. Uniqueness, however, does not follow as straightforwardly. While the construction of the model may seem similar in concept to the model of public good provision as presented in Banerjee and Gravel (2018), the proof does not follow identically. In Banerjee and Gravel (2018), the changes in the levels of utility were captured by changes in one (or more) of only three variables: the individual's own contribution to the public good, the sum of contributions, or beliefs. However, in the case of this paper, an individual's utility can be changed by a change in any other member's consumption of the private good as well, because of the altruistic component of the utility function. The multiplicity of variables that affect an individual's utility implies that the proofs do not follow the same structure as Banerjee and Gravel (2018). As it turns out, Lemma 1 is key in proving the uniqueness of the equilibrium. The intuition of the proof by contradiction is the following: since there is only one threshold level of income determining the set of contributors at any equilibrium, if there are multiple equilibria, there must be multiple thresholds. We thus arrive at a contradiction by using the assumption that there must be one individual who always contributes to the public good.

While Lemma 1 is not specific to the model of public good provision with altruistic preferences (it holds true in particular for Bergstrom, Blume, and Varian (1986)) the threshold level of income it identifies also plays a role in the model with transfers analyzed in Section 3. Thanks to Proposition 2, we can now denote by $\mathbf{g}^*(\mathbf{y}, \alpha) = (g_1^*, ..., g_n^*)$ the unique equilibrium of the game $\mathcal{G}(\mathbf{y}, \alpha)$. In the following subsection, we turn to the effect of an increase in of altruism on the contributions to the public good.

2.1 Effect of an increase in altruism

We now examine how a small change in the degree of altruism affects contributions when the equilibrium is interior both before and after the change in α. The following proposition shows that an increase in altruism increases the contributions of all contributors to the public good at the Nash equilibrium.

Proposition 3 *Let $\mathbf{g}^*(\mathbf{y}, \alpha)$ and $\mathbf{g}^*(\mathbf{y}, \alpha^{'})$ denote two interior equilibria for the game of the form $G(\mathbf{y}, \alpha)$ defined with a function $U \in \mathcal{U}$, such that $\alpha^{'} > \alpha$. Then $g_i^*(\mathbf{y}, \alpha^{'}) > g_i^*(\mathbf{y}, \alpha)$ $\forall\, i \in N$.*

The intuition behind Proposition 3 is fairly straightforward: an increase in altruism reduces the threshold level of private good consumption enjoyed by the contributors at any equilibrium, and hence increases their public good contribution. While this result is intuitive for the case of an interior equilibrium, the implications for an extension for non-interior equilibria are not clear and merit future research. For the purposes of this paper, we now turn our attention to the model with transfers and its connections to the model without transfers.

3 Model with private transfers

In this section we consider the model of public goods contribution with altruistic preferences while allowing for private transfers between any two individuals in the group. That is, members of a group of n individuals have incomes $(y_1, ..., y_n)$ and the social utillity functions of the form V as defined in Section 3. We assume as before that the function V is defined using private utilities U belonging to the class of functions $\mathcal{U}$:

$$V(x_1, ...x_n, G, \alpha) = U(x_i, G) + \alpha \sum_{j \in N, j \neq i} U(x_j, G)$$

As before, every agent simultaneously chooses how to divide the income net of transfers between his consumption of a private good, x_i, and his contribution to a public good g_i. In addition, an individual i may now transfer an amount $t_{ij} \geq 0$ out of his income y_i to some individual j in his group. The individual's budget constraint gets modified from that in Section 3. It now reads:

$$x_i + g_i = y_i - \sum_j t_{ij} + \sum_k t_{ki}$$

The collection of all bilateral transfers is a matrix $\mathbf{T}$ whose elements t_{ij} are transfers from member i to member j. By convention, $t_{ii} = 0 \; \forall \, i$. The transfers are a means of redistribution of income within the group without changing the total income of the group, $Y = \sum_{i=1}^{n} y_i$.

Denote the game as described above by $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$ where we use as before $\mathbf{y} = (y_1, y_2, ..., y_n)$ to denote a given profile of income distributions. Our objective is to characterize the Nash equilibria of this game. Finding the Nash equilibrium corresponds to solving the following maximization problem for every individual i given the contributions to the public and private transfers by members others than i :

$$\begin{aligned} \max_{g_i, t_{i1}, ..., t_{ij}, ...t_{in}} & U(y_i - \sum_j t_{ij} + \sum_k t_{ki} - g_i, g_i + \sum_{j \in N, j \neq i} g_j) + \\ \alpha & \sum_{j \in N, j \neq i} U(y_j - g_j - \sum_h t_{jh} + \sum_k t_{kj}, g_i + \sum_{j \in N, j \neq i} g_j) \end{aligned} \tag{5}$$

subject to:

$$g_i \geq 0 \text{ and } x_i \geq 0 \; \forall \, i$$

$$t_{ij} \geq 0 \, \forall \, i, j, \;\; t_{ii} = 0 \, \forall \, i$$

The first order conditions for agent i are necessary and sufficient for the solution to the program above:

with respect to g_i:

$$-U_x(x_i, G) + U_G(x_i, G) + \alpha \sum_{j \in N, j \neq i} U_G(x_j, G) \leq 0 \;\; \forall \, i \in N \tag{6}$$

and

$$-U_x(x_i, G) + U_G(x_i, G) + \alpha \sum_{j \in N, j \neq i} U_G(x_j, G) = 0 \quad if \; g_i > 0 \tag{7}$$

with respect to t_{ij}:

$$-U_x(x_i, G) + \alpha U_x(x_j, G) \leq 0 \quad \forall \, i, j \in N \tag{8}$$

and

$$-U_x(x_i, G) + \alpha U_x(x_j, G) = 0 \quad if \;\; t_{ij} > 0 \tag{9}$$

While transfers are potentially unconstrained, we must have that the sum of all consumptions, net transfers made and the contributions to the public good cannot exceed the sum of all incomes available. This is because if individual i transfers a certain amount to individual j, individual j receives the same amount of money. Hence, transfers given, net of transfers received, must equal zero.

$$\sum_i (x_i + g_i) - \sum_i \sum_j t_{ij} + \sum_k \sum_i t_{ki} = \sum_i y_i$$

We will now establish some properties regarding the Nash equilibrium for this modified setting of the model with altruism. We have not yet proved the existence of this equilibrium. Denote by $\mathbf{x}^* = (x_1^*, ..., x_n^*)$, $\mathbf{T}^*$, and G^*, the levels of private good consumption, the matrix of transfers, and the level of the public good respectively at some Nash equilibrium.

Lemma 2 *If there exist at least one donor and a receiver of transfers in a Nash equilibrium of the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$ defined for a function $U \in \mathcal{U}$, then there exist threshold levels of consumption $\underline{x} \in \mathbb{R}$ and $\bar{x} \in \mathbb{R}$, $\underline{x} < \bar{x}$ such that,*

a. All receivers of transfers will have a private good consumption equal to the minimum of all private good consumptions in equilibrium, $\underline{x}$, and all donors will have the same private good consumption in equilibrium, denoted by $\bar{x}$.

b. There are no transfer intermediaries in equilibrium - no individual who both gives and receives transfers. The set of donors and the set of receivers of transfers are disjoint.

c. $\underline{x} \leq x_i^* \leq \bar{x} \, \forall \, i \in N$.

The thresholds in consumption $\underline{x}$ and $\bar{x}$, correspond to the thresholds in consumption established for the charity game by Arrow (1981). Indeed, the model of private transfers considered here is the extension of the charity game, to the case where individuals have two ways to give to individuals they care about - either transfer money privately, or contribute to the public good that everyone benefits from. Lemma 2 establishes that Arrow (1981)'s result establishing the thresholds in private consumption, still holds true for the extension of this model with private transfers. Another implication of Lemma 2 is that an individual in an equilibrium with transfers is going to be in one the following mutually cases: he will either give, or receive transfers, or do neither. This implication, together with the next result, will be very useful in further characterizing the equilibrium with transfers. It is important to note that for the purposes of this paper, neither existence, nor uniqueness of the equilibrium of the general model with transfers is established. The properties that we established in Lemma 2 and those that follow in the section will hold at any equilibrium, if it exists. The next lemma shows that a receiver cannot contribute to the public good at an equilibrium.

Lemma 3 *If there exist donors and receivers of transfers in a Nash equilibrium of the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$ defined for a function $U \in \mathcal{U}$, then at the Nash equilibrium a receiver of a transfer cannot contribute to the public good.*

Lemma 2 shows that at equilibrium there must be threshold levels of consumption for donors and receivers of transfers when there are transfers in equilibrium. However, it is not clear whether these thresholds in consumption are in fact thresholds in income. Lemma 3 will help establish the next result of this section: Lemma 4, which explores the implications of the thresholds in consumption established in Lemma 2 in terms of thresholds in income.

Lemma 4 *Let $\mathbf{g}^* = (g_1^*, ..., g_n^*)$ denote the profile of contributions to the public good at the Nash equilibrium of the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$ defined with a function $U \in \mathcal{U}$. If there exist donors and receivers in equilibrium, then for any agent $i \in N$*

i. If $y_i < \underline{x}$ then i is a receiver.

ii. If $\underline{x} \leq y_i \leq \bar{x}$ then i is neither a donor, nor a receiver.

iii. If $y_i > \bar{x}$ then i is in one of the mutually exclusive cases:

- *i is donor*
- *i is neither a donor nor a receiver and contributes exactly $g_i^* = y_i - \bar{x}$ to the public good.*

The thresholds in consumption $\underline{x}$ and $\bar{x}$, in Arrow (1981)'s classic paper on the charity game, translate to thresholds in income. Lemma 4 establishes that his result does not follow through quite exactly for the case of private transfers, since now contributions to the public good are possible. We next establish that there exists a common threshold level of consumption that all contributors to the public good enjoy. Lemma 5 is essentially a modification of Lemma 1, for the model with transfers. While it is still true that all contributors enjoy the same level of private consumption, this common level of consumption does not translate exactly to a threshold in income as in Lemma 1.

Lemma 5 *At any Nash equilibrium of the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$ defined with a function $U \in \mathcal{U}$, all active contributors to the public good at a Nash equilibrium enjoy a common level of private good consumption, denoted by $\hat{x}$. Further, the following is true for all $i \in N$:*

- $y_i \leq \hat{x}$ *then* i *is not a contributor*
- *If* $y_i > \hat{x}$ *then* i *is in one of the following mutually exclusive cases:*
  - i *is a contributor to the public good,*
  - i *does not contribute to the public good, and* i *is a donor and* $x_i^* = \hat{x} = \bar{x}$.

In the charity game presented by Arrow (1981), the thresholds in terms of consumption were thresholds in terms of incomes as well. Lemma 4 and Lemma 5 together lead to a useful result - indeed, that there is a common level of consumption that both givers of transfers and contributors to the public good enjoy, and this is the common threshold level of income required to become either a donor of transfers or a contributor to the public good, as we establish in Proposition 4, which also summarizes the common implications from Lemma 4 and Lemma 5.

Proposition 4 *At a Nash equilibrium of the game* $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$ *defined with a function* $U \in \mathcal{U}$*, the following is true for all* $i \in N$

- $y_i \leq \underline{x}$ *then* i *is not a contributor and* i *is a receiver, and* $x_i^* = \underline{x}$.
- *If* $\underline{x} \leq y_i \leq \bar{x}$ *then* i *is neither a contributor to the public good, nor a donor, nor a receiver of transfers, and* $x_i^* = y_i$.
- *If* $y_i > \bar{x}$ *then* i *is either a contributor to the public good, or a donor of transfers, or both, with* $x_i^* = \bar{x}$.

Proposition 4 shows that the minimum level of income required to contribute to a public good and the threshold income required to contribute to the public good, is the same.This of course does not imply that the set of contributors to the public good needs to be identical to the set of givers of transfers, just that their levels of private consumption must be the same at the equilibrium. Another implication is that the set of donors of transfers at an equilibrium, and contributors to a public good, may not be unique even if the thresholds $\underline{x}$ and $\bar{x}$ are uniquely defined. Proposition 4 provides an intuition for the following result that holds true at any equilibrium for the model with transfers:

Lemma 6 *At an equilibrium for the game* $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$ *defined with a function* $U \in \mathcal{U}$*, if* $g_i^* > 0$ *for all* $i \in N$*, then* $t_{ij}^* = 0$ *for all* $i, j \in N$.

Intuitively, Lemma 6 is an implication of Proposition 4. Since the threshold level of consumption required to be a donor or a contributor to the public good is in fact a threshold in terms of income, if everyone has an income which is higher than the threshold, then there is no need for transfers of income. In this sense, transfers of income are supplementing the role of the public good: even when there are no transfers, individuals would like to consume the public good. However, individuals only give transfers when incomes/consumptions of individuals that they care about are sufficiently low.

It is important to note that Proposition 4 holds true for an equilibrium with transfers, if it exists. It is also a counterpart to Arrow (1981)'s result establishing common thresholds in income for donors and receivers of transfers for the model without public good provision. Proving existence and uniqueness of the equilibrium however does not correspond in any way to the proof of existence of an equilibrium as established in Arrow (1981). Since Arrow (1981)'s proof relied on defining $\bar{x}$ as a explicit function of $\underline{x}$, the same approach cannot be used here because $\bar{x}$ in the case of this paper is a function of both G and $\underline{x}$. Defining $\bar{x}$ implicitly as a function of both these variables does not simplify the problem either because the consumption of an individual member at the equilibrium, x_i^*, is not a differentiable function due to Proposition 4, and hence the implicit function theorem cannot be used as an approach. Deriving the uniqueness of the thresholds $\underline{x}$ and $\bar{x}$ requires a more detailed analysis meriting further research. For the purposes of this paper, I provide the characterization, existence, and uniqueness of the equilibrium of the model with transfers for the case of additively separable utility functions.

3.1 Analysis for additively separable utility functions

This section provides the analysis for the case of additively separable utility functions. Since this is only a special case of the more general function form U examined in the rest of the paper, the results established in the paper still hold true. If U is additively separable, then the private utility function U can be expressed as $U(x_i, G) = Z^1(x_i) + Z^2(G)$. We assume that U belongs to the class of function $\mathcal{U}$ as before. The maximization problem as shown in (5) transforms into:

$$\max_{g_i,t_{i1},\ldots,t_{ij},\ldots t_{in}} Z^1(y_i - \sum_j t_{ij} + \sum_k t_{ki} - g_i) + Z^2(g_i + \sum_{j\in N, j\neq i} g_j) +$$
$$\alpha \sum_{j\in N, j\neq i} Z^1(y_j - g_j - \sum_h t_{jh} + \sum_k t_{kj}) + Z^2(g_i + \sum_{j\in N, j\neq i} g_j)$$

subject to:

$$g_i \geq 0 \text{ and } x_i \geq 0 \ \ \forall\, i$$

$$t_{ij} \geq 0 \,\forall\, i, j, \ \ t_{ii} = 0 \,\forall\, i$$

The first-order conditions for agent i must hold at a Nash equilibrium:

with respect to g_i:

$$-Z^{1'}(x_i) + (1 + \alpha(n-1))Z^{2'}(G) \leq 0 \ \ \forall\, i \in N \tag{10}$$

and

$$-Z^{1'}(x_i) + (1 + \alpha(n-1))Z^{2'}(G) = 0 \quad if\; g_i > 0 \tag{11}$$

with respect to t_{ij}:

$$-Z^{1'}(x_i) + \alpha Z^{1'}(x_j) \leq 0 \quad \forall\, i, j \in N \tag{12}$$

and

$$-Z^{1'}(x_i) + \alpha Z^{1'}(x_j) = 0 \quad if\;\; t_{ij} > 0 \tag{13}$$

Since the function $Z^{1'}$ is strictly increasing in 12, we can define it's inverse: $Z^{1'^{-1}}$. We can then define the function $\epsilon : \mathbb{R} \to \mathbb{R}$ as follows:

$$x_i \leq Z^{1'^{-1}}(\alpha Z^{1'}(x_j)) = \epsilon(x_j) \tag{14}$$

Since we must have at least one contributor to the public good, and by Proposition 4, this individual's level of private good consumption must equal $\bar{x}$, and equation 11 holds for this individual:

$$-Z^{1'}(\bar{x}) + (1+\alpha(n-1))Z^{2'}(G) = 0$$

$$\implies (1+\alpha(n-1))Z^{2'}(G) = Z^{1'}(\tilde{x})$$

$$\implies G = Z^{2'-1}(\frac{Z^{1'}(\tilde{x})}{1+\alpha(n-1)}) = \lambda(\bar{x}) \tag{15}$$

Similarly, at an equilibrium with transfers, there must be at least one donor and one receiver, and equation 13 holds for this pair of members:

$$\bar{x} = \epsilon(\underline{x}) \tag{16}$$

We can thus write $G = \gamma(\underline{x})$ where $\gamma(\underline{x}) = \lambda(\epsilon(\underline{x}))$. We add an additional assumption to ensure that the functions ϵ and λ are always defined over their respective domains:

Assumption 3 *For every $x \in [y_{(1)}, y_{(n)}]$ where $y_{(1)} = \min_i y_i$ and $y_{(n)} = \max_i y_i$, and every $\alpha \in (0,1)$*

$$\lim_{x\to\infty} Z^{1'}(x) = \lim_{x\to\infty} \alpha Z^{1'}(x)$$

and

$$\lim_{x\to\infty} Z^{2'}(x) = \lim_{x\to\infty} \frac{Z^{1'}(x)}{1+\alpha(n-1)}$$

If the above assumption does not hold, the functions ϵ and λ may not be defined for every $x \in [0, \bar{y}]$. Following Arrow (1981), define the function x^+ such that:

$$x^+ = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$$

The sum of all transfers given, net of contributions to the public good, and transfers received, must be zero. We can rewrite this constraint as

$$\sum_i (y_i - \epsilon(\underline{x}) - g_i)^+ - \sum_i (\underline{x} - y_i)^+ = 0$$

Alternatively,

$$\sum_i (y_i - \epsilon(\underline{x}))^+ - \sum_i (\underline{x} - y_i)^+ - \gamma(\underline{x}) = 0$$

We use the notations defined above to characterize the equilibrium of the model with transfers in the following proposition:

Proposition 5 *If the function $U \in \mathcal{U}$ is additively separable, an equilibrium for the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$ always exists. If $\tilde{x} \leq \epsilon(y_{(1)})$ where $y_{(1)} = \min_i y_i$ and $\tilde{x}$ is the unique threshold level of income required to become a contributor for the game $\mathcal{G}(\mathbf{y}, \alpha)$, the equilibrium for the model without transfers $\mathbf{g}^*(\mathbf{y}, \alpha)$ is also the equilibrium for the model with transfers. The equilibrium level of the public good G^* as well as individual levels of contributions to the public good g_i^* are uniquely determined. If $\tilde{x} > \epsilon(y_{(1)})$, there exists an equilibrium with transfers, where the threshold levels of private consumption for receivers ($\underline{x}$), for donors and contributors to the public good ($\bar{x}$), and the aggregate public good G^* are uniquely determined. Individual contributions to the public good and transfers may however not be unique.*

The proof of Proposition 5 involves three steps. It is first shown that for the case of additively separable utility functions, an equilibrium with transfers and an equilibrium without transfers are mutually exclusive. We derive a necessary and sufficient condition ($\tilde{x} \leq \epsilon(y_{(1)})$)under which there exists an equilibrium without transfers. When the model with transfers has an equilibrium without transfers, of course, the equilibrium of the model with transfers coincides with the equilibrium of the model without transfers. We have already shown in section 2 that the equilibrium of the game $\mathcal{G}(\mathbf{y}, \alpha)$ is unique. Secondly, we derive a necessary and sufficient condition under which $\underline{x}$, $\bar{x}$ and G^* are uniquely determined for the model with transfers. In our last step, we show that the latter necessary and sufficient condition condition holds if and only if $\tilde{x} \leq \epsilon(y_{(1)})$.

The structure of the equilibria in Arrow (1981) was simple: for transfers to exist, there had to be an individual in the society whose income crossed the threshold defined in terms of the income of the poorest individual ($\epsilon(y_{(1)})$). This was natural since there were no other goods in the society - the entirety of an individual's income was consumed. Proposition 5 shows that when the society also has to provide for a public good, the threshold level of income required for transfers to exist is in fact higher: there had to be an individual in the society whose *private good consumption* crosses the threshold defined in terms of the income of the poorest individual ($\epsilon(y_{(1)})$). In other words, even if there is an individual who is 'rich enough' to transfer income, if he isn't rich enough after contributing to the public good, he will not transfer money. The public good takes priority (in an implicit sense) for the society, even when private and public good consumption are perfect substitutes in the utility function.

Proposition 5 provides some other interesting insights. First, as in Arrow (1981), the relative differences in income between the poorest individual and the other members of the society determine the existence of transfers. Secondly, the equilibrium, when it exists, is unique. This allows for the possibility of examining comparative statics results which deserve further research.

4 Conclusion

This paper examines the implications of a model wherein individuals with altruistic preferences contribute to a public good. Two alternative versions of this model are studied and found to be closely related: in the first model, individuals cannot transfer money privately to other people they care about; in the second model, such private transfers are allowed. There are many examples of situations where both models may hold. People may care about other individuals in a society who benefit from the public good, while being unable to privately transfer money; at the same time, in circles such as family and friends, the possibility of transferring income privately may change the behavior of individuals with regard to public good contributions. While altruism in itself might positively influence contributions to a public good, the possibility of giving private transfers causes a conflict - is altruism best expressed by contributing to the public good, or by transferring money privately? The main contribution of this paper answers this question by establishing a link between the equilibria of the two models studied. The existence of private transfers depends on whether there exists an individual who, after contributing to the public good, has enough income left to transfer money to the poorest individual in the society. This simple result has important implications for redistribution of income in a society with altruism, when utility from the public and the private good are substitutable: if the relative difference in income between the poorest individual and the private consumption of the richest individual in society is not important enough, the public good takes priority, and redistribution of private consumption is secondary.

The conclusions of this paper also raise other questions. For instance, an increase in the level of altruism, even in the restricted setting of additively separable preferences, may have an effect on private good provision as well as private transfers, and may have other implications for income redistribution in a society. The analysis of the equilibrium/equilibria of the model with transfers for the general case of non-additively separable preferences is another context that may have interesting connections to the results of this paper. The case where individuals differ in the degree of their altruism (without or without including the possibility of private transfers) is also another notable avenue for further research.

A Appendix: Proofs

A.1 Proof of Lemma 1

Proof. We first show all agents who contribute to the public good have the same level of private good consumption, which will be denoted by $\tilde{x}$. Denote by $\mathbf{x}^* = (x_1^*, ..., x_n^*)$ the levels of private good consumption corresponding to $\mathbf{g}^*$ for the n group members at the equilibrium. For contradiction, assume that in an equilibrium there exist arbitrary agents a and b who contribute to the public good and without loss of generality, that $x_a^* < x_b^*$. Since condition 4 holds at the equilibrium for both agents:

$$-U_x(x_a^*, G^*) + U_G(x_a^*, G^*) + \alpha \sum_{j \in N, j \neq a} U_G(x_j^*, G^*)$$
$$= 0 = -U_x(x_b^*, G^*) + U_G(x_b^*, G^*) + \alpha \sum_{j \in N, j \neq b} U_G(x_j^*, G^*)$$

Cancelling the common terms out:

$$- U_x(x_a^*, G^*) + U_G(x_a^*, G^*) + \alpha U_G(x_b^*, G^*)$$
$$= -U_x(x_b^*, G^*) + U_G(x_b^*, G^*) + \alpha U_G(x_a^*, G^*)$$

Rearranging

$$U_x(x_b^*, G^*) - U_x(x_a^*, G^*) + (1 - \alpha)[U_G(x_a^*, G^*) - U_G(x_b^*, G^*)] = 0$$

Since we assumed that $x_a^* < x_b^*$,

$$U_x(x_b^*, G^*) - U_x(x_a^*, G^*) < 0$$

by strict concavity of U).

By assumption 1

$$U_G(x_a^*, G^*) - U_G(x_b^*, G^*) \leq 0$$

as well, leading to a contradiction.

We next show that $\tilde{x}$ is also the critical level of income above which an agent starts contributing. We show this in two parts: first, we show that an individual with an income $y_i < \tilde{x}$ cannot be a contributor. To show this, suppose for contradiction that $y_i \leq \tilde{x}$ for some i and i is still a contributor. Then the private good consumption for agent i, $x_i^* < y_i$, and hence $x_i^* < \tilde{x}$. Since all contributors at any equilibrium have the same private good consumption, this is violating what was just proved above, the first part of Lemma 1.

We next show that anyone with an income higher than $\tilde{x}$ must be a contributor. To show this, assume for contradiction that there exists some i such that $y_i > \tilde{x}$ and i does not contribute. Then $x_i^* = y_i > \tilde{x}$, again violating what was just proved above.

The final part of Lemma 1 follows easily from the arguments stated just above. If $y_a > y_b > \tilde{x}$, for any two agents a and b, it follows that $y_a - \tilde{x} = g_a^* > g_b^* = y_b - \tilde{x}$. Hence, the contributions of agents are ordered in increasing order of their income. ■

A.2 Proof of Proposition 2

Proof. We first show that a Nash equilibrium of the game $\mathcal{G}(\mathbf{y}, \alpha)$ always exists. For a given α, $\mathbf{y}$, the function

$$V(g_1, ..., g_n, \mathbf{y}, \alpha) = U(y_i - g_i, g_i + G_{-i}) + \alpha \sum_{j \in N, j \neq i} U(x_j, g_i + G_{-i})$$

is continuous, and given that $g_i \in [0, \bar{\mathbf{y}}]$ is a compact set for any agent i, then by Weierstrass theorem, the maximization problem 2 admits a solution. Define for every i, $G_{-i} = \sum_{j \neq i} g_j$ and $\mathbf{x_{-i}} = (x_1, ...x_{i-1}, x_{i+1}, ..., x_n)$ (the profile of private contributions of members other than i). And by Berge maximum theorem, the best response functions $g_i(G_{-i}, \mathbf{x_{-i}}, \mathbf{y}, \alpha)$ are continuous. Hence, by Brouwer's fixed point theorem, an equilibrium exists.

Next, we show uniqueness. We show in Lemma 1 that all contributors must have the same level of private consumption - the threshold level of income $\tilde{x}$ which must hold in any equilibrium. For contradiction, suppose that in fact there are two equilibria for a game $\mathcal{G}(\mathbf{y}, \alpha)$. Then in fact, there must exist two distinct thresholds, let us denote them by $\tilde{x}_1 \neq \tilde{x}_2$, with two sets of contributors, denoted by C_1 and C_2. Let us note the number of contributors by c_1 and c_2, and the amount of the public good G as G_1^* and G_2^* respectively as well. Without loss of generality, assume that $\tilde{x}_1 < \tilde{x}_2$.

We first argue that we must have $C_2 \subseteq C_1$. To see this, we use the fact that contributions are ordered in increasing order of income. Suppose we order individuals in increasing order of income i.e. $y_1 \leq y_2 \leq ... \leq y_n$. Then if agent n, the richest individual, is a contributor under the threshold $\tilde{x}_2$, he must be a contributor under $\tilde{x}_1$. Since the threshold $\tilde{x}_1$ is lower than $\tilde{x}_2$, the number of contributors under the threshold $\tilde{x}_1$ must be at least as large as under $\tilde{x}_2$.

We next argue that $G_1^* > G_2^*$. We use that $\tilde{x}_1 < \tilde{x}_2$, hence $y_i - \tilde{x}_1 > y_i - \tilde{x}_2$ for any $y_i \geq 0$. Summing these over the sets C_1 and C_2 and using that $C_2 \subseteq C_1$, we will have that $G_1^* = \sum_{i \in C_1}(y_i - \tilde{x}_1) > \sum_{i \in C_2}(y_i - \tilde{x}_2) = G_2^*$.

We next use that equation 4 must hold for any contributor in an equilibrium. We know from Assumption 2 that there must be at least one contributor in any equilibrium. Therefore, equation 4 will hold with equality for any contributor under the thresholds $\tilde{x}_1$ and $\tilde{x}_2$, giving us that:

$$\begin{aligned} &-U_x(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1)) + U_G(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1)) + \\ \alpha \sum_{j \neq i} U_G(x_j, &\sum_{i \in C_1}(y_i - \tilde{x}_1)) = 0 = -U_x(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2)) \\ &+U_G(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2)) + \alpha \sum_{j \neq i} U_G(x_j, \sum_{i \in C_2}(y_i - \tilde{x}_2)) \end{aligned}$$

We add and substract a term $\alpha U_G(\tilde{x}_1, \sum_{j \in C_1}(y_i - \tilde{x}_1))$ on the left hand side, and we do the same on the right hand side with the term $\alpha U_G(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2))$, giving us:

$$\begin{aligned} &-U_x(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1)) + (1-\alpha) U_G(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1)) + \sum_{j \in N} U_G(x_j, \sum_{i \in C_1}(y_i - \tilde{x}_1)) \\ &= -U_x(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2)) + (1-\alpha) U_G(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2)) + \sum_{j \in N} U_G(x_j, \sum_{i \in C_1}(y_i - \tilde{x}_2)) \end{aligned}$$

Rearranging, and collecting common terms:

$$\underbrace{U_x(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2)) - U_x(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1))}_{<0}$$

$$+(1-\alpha)\{\underbrace{U_G(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1)) - U_G(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2))}_{<0}\}$$

$$+\alpha \sum_{j \in N} \underbrace{U_G(x_j, \sum_{i \in C_1}(y_i - \tilde{x}_1)) - U_G(x_j, \sum_{i \in C_2}(y_i - \tilde{x}_2))}_{?} = 0$$

Given our assumptions on the increasingness and concavity of U and the cross-derivative $U_{xG} = U_{Gx} \geq 0$ (the assumption 1), the first two pair of terms of the above equation are clearly negative. For the first pair of terms, we use that

$$\tilde{x}_2 > \tilde{x}_1 \text{ and } G_1^* > G_2^*$$

since U_x is decreasing in x and increasing in G, we have $U_x(\tilde{x}_2, \sum_{i \in C_2}(y_i - \tilde{x}_2)) < U_x(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1))$. The same type of argument establishes the inequality for the second pair of terms. The only term whose sign is difficult to determine is the third term of the above equation. We already know that $C_2 \subseteq C_1$, or $c_1 \geq c_2$. Any $j \in N$ is in one of three categories:

- j is a contributor under both $\tilde{x}_1$ and $\tilde{x}_2$; in this case the sign of the third pair of terms is negative exactly by the same arguments by which the second pair of terms is negative for that agent j;
- j is a non-contributor under both equilibria; in which case, the third pair of terms is signed negative because $x_j = y_j$ under both thresholds and $G_1^* > G_2*$;
- j is a non-contributor under x_2 and a contributor under x_1. In this case, we use that

$$U_G(x_j, \sum_{i \in C_1}(y_i - \tilde{x}_1)) - U_G(x_j, \sum_{i \in C_1}(y_i - \tilde{x}_2)) = U_G(\tilde{x}_1, \sum_{i \in C_1}(y_i - \tilde{x}_1)) - U_G(y_j, \sum_{i \in C_1}(y_i - \tilde{x}_2)) < 0$$

  using $\tilde{x}_1 < y_j$ and similar arguments as before.

We will then have that the left hand side is strictly negative and the right hand side is 0, giving us a contradiction. Hence, the threshold must be unique for any set of contributors C. ■

A.3 Proof of Proposition 3

Proof. We consider only "small" changes in α. We will use the fact that if agents are at an interior equilibrium, they will all have the same private consumption, $\tilde{x}$ (the rest of the income goes to the public good). Equation 4 must hold for all agents, with equality. We consider the effect of a small change in α on G. Denote by $Y = \sum_{i \in N} y_i$ the sum of all incomes in the group. Writing out equation 4 with $x_i^* = y_i - g_i^* = \tilde{x}$ for any individual $i \in N$:

$$(1 + (n-1)\alpha)U_G(\tilde{x}, Y - n\tilde{x}) - U_x(\tilde{x}, Y - n\tilde{x}) = 0$$

This is an equation defined for the variable $\tilde{x}$. We will now take a derivative of the equation with respect to α.

$$\begin{aligned} -[U_{xx}(\tilde{x}, Y - n\tilde{x}) - nU_{xG}(\tilde{x}, Y - n\tilde{x})]\frac{d\tilde{x}}{d\alpha} + \\ (1 + (n-1)\alpha)[U_{xG}(\tilde{x}, Y - n\tilde{x}) - nU_{GG}(\tilde{x}, Y - n\tilde{x})]\frac{d\tilde{x}}{d\alpha} \\ + U_G(\tilde{x}, Y - n\tilde{x})(n-1) = 0 \end{aligned}$$

Rearranging and collecting the common terms, :

$$\frac{d\tilde{x}}{d\alpha} = \frac{-U_G(\tilde{x}, Y - n\tilde{x})(n-1)}{-U_{xx}(\tilde{x}, Y - n\tilde{x}) - n(1 + (n-1)\alpha)U_{GG}(\tilde{x}, Y - n\tilde{x}) + (1 + (n-1)\alpha + n)U_{xG}(\tilde{x}, Y - n\tilde{x})}$$

Given our assumptions on the utility functions in the class $\mathcal{U}$, the denominator is strictly positive. Since the numerator is negative, an increase in α leads to a strict decrease in the threshold consumption $\tilde{x}$. Since $g_i^* = y_i - \tilde{x}$ for every i, a decrease in the threshold consumption means that everyone strictly increases their public goods contribution. Hence, the sum of contributions also increases. ■

A.4 Proof of Lemma 2

Proof. Part [a.] First, we establish the existence of $\underline{x}$. Suppose an equilibrium with transfers exists. Then $t_{ij}^* > 0$ for some i and j and let k be any other player. Then we have from (9) and (8) that:

$$\alpha U_x(x_j^*, G^*) = U_x(x_i^*, G^*) \geq \alpha U_x(x_k^*, G^*)$$

Since k was any player, we will haveby the concavity of U that:

$x_j^* \leq \min_k \ x_k^* \quad \forall\, k$. Denote $\min_k \ x_k^* = \underline{x}$. Therefore, if j is a receiver, $x_j^* = \underline{x}$. All receivers will have a final private good consumption equal to the minimum of all private good consumptions.

Establishing the existence of $\bar{x}$. Suppose there are two donors i and j who give to players k and l. Then by equation (9), and proposition 2 (a):

$$U_x(x_i^*, G^*) = \alpha U_x(\underline{x}, G^*) = \alpha U_x(\underline{x}, G^*) = U_x(x_j^*, G^*)$$

Hence $x_i^* = x_j^* = \bar{x}$.

It follows immediately from equation 9 that the equilibrium private good consumption of a donor must be higher than the equilibrium private good consumption of a receiver, hence $\underline{x} < \bar{x}$.

Part [b.] Suppose there is some player j such that he both gives and receives in equilibrium, i.e. $\exists\, i, k$ such that $t_{ij}^* > 0$ and $t_{jk}^* > 0$. Then (9) will hold for both players,

$$U_x(x_i^*, G^*) = \alpha U_x(x_j^*, G^*) < U_x(x_j^*, G^*) = \alpha U_x(x_k^*, G^*)$$

Violating equation (8) for the pair (i,k). This implies that the set of givers (donors) of transfers and receivers are disjoint in equilibrium.

Part [c.] In equilibrium, $\underline{x} \leq x_i^* \leq \bar{x}\, \forall\, i$.

We already know the result for the donors and the receivers. We only have to show it for a player k who is neither a donor, nor a receiver in equilibrium. We know that the minimum consumption possible is that of a receiver, so someone who neither gives, nor receives, cannot have a level of private good consumption lower than $\underline{x}$. We need to now show that his equilibrium consumption cannot exceed that of a donor. It suffices to show that from equation (8) that for any donor i and any individual k:

$$U_x(\bar{x}, G^*) \geq \alpha U_x(x_k^*, G^*)$$

From the concavity of U in x, $x_k^* \leq \bar{x}$.

■

A.5 Proof of Lemma 3

Proof. Suppose for contradiction that a receiver contributes to the public good i.e. for some receiver j, $\exists\, i$ such that $t_{ij} > 0$ and $g_j > 0$. Then equation 9 for individuals i and j and equation 7 for individual j will be the following:

$$U_x(\bar{x}, G^*) = \alpha U_x(\underline{x}, G^*)$$

$$-U_x(\underline{x}, G^*) + U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) = 0$$

Rearranging the above equation:

$$U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) = U_x(\underline{x}, G^*)$$

Using that $U_x(\underline{x}, G^*) > \alpha U_x(\underline{x}, G^*)$, since $\alpha < 1$ and equation (9):

$$U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) = U_x(\underline{x}, G^*) > \alpha U_x(\underline{x}, G^*) = U_x(\bar{x}, G^*)$$

Or,

$$-U_x(\bar{x}, G^*) + U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) > 0$$

Rewriting :

$$-U_x(\bar{x}, G^*) + U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) > 0$$

We now add and substract the terms $U_G(\bar{x}, G^*)$ and $\alpha U_G(\underline{x}, G^*)$:

$$-U_x(\bar{x}, G^*)+U_G(\underline{x}, G^*)+\alpha \sum_{k\neq j} U_G(x_k^*, G^*)+U_G(\bar{x}, G^*)-U_G(\bar{x}, G^*)+\alpha U_G(\underline{x}, G^*)-\alpha U_G(\underline{x}, G^*) > 0$$

Rearranging terms:

$$-U_x(\bar{x}, G^*)+U_G(\bar{x}, G^*)+\alpha \sum_{k\neq i} U_G(x_k^*, G^*) > U_G(\bar{x}, G^*)-U_G(\underline{x}, G^*)-\alpha U_G(\bar{x}, G^*)+\alpha U_G(\underline{x}, G^*)$$

$$-U_x(\bar{x}, G^*) + U_G(\bar{x}, G^*) + \alpha \sum_{k\neq i} U_G(x_k^*, G^*) > (1-\alpha)(U_G(\bar{x}, G^*) - U_G(\underline{x}, G^*))$$

We have that the right hand side is positive by Assumption 1, and hence the left hand side is positive as well, violating the first-order condition equation 6 for the donor i. Hence we cannot be at an equilibrium, giving us a contradiction.

■

A.6 Proof of Lemma 4

Proof. We know by Lemma 2 that an individual in an equilibrium with transfers has to be in one of the following three mutually cases: he will either give, or receive transfers, or do neither.

Lemma 4 therefore has three parts. We will first prove part(i): the case when $y_i < \underline{x}$. There are two possible cases, i either is, or is not, a contributor to the public good. In either case, $x_i^* \leq y_i - \sum_j t_{ij} + \sum_k t_{ki}$. If i is either a donor or in the case that he is neither a donor, nor a receiver, net transfers are negative or equal to zero. Hence, $x_i^* \leq y_i$. We must have that $x_i^* \leq \underline{x} < \bar{x}$. Then by Lemma 2 i cannot be a donor, nor an individual who neither gives nor receives. Hence, i has to be a receiver.

For part (ii), consider any individual with income y_i such that $\underline{x} \leq y_i \leq \bar{x}$. As before there are two possibilities: either i contributes, or he does not contribute, to the public good. In either case, as before $x_i^* \leq y_i - \sum_j t_{ij} + \sum_k t_{ki}$. If i is a donor, net transfers are negative, hence $x_i^* < y_i < \bar{x}$. Hence, i cannot be a donor, by Lemma 2. Next, we show that i cannot be a receiver. Assume for contradiction that i is a receiver. Then

$y_i - \sum_j t_{ij} + \sum_k t_{ki} > y_i$. There are two cases to consider here: in the first case, i does contributre to the public good, and thus, $x_i^* = y_i - \sum_j t_{ij} + \sum_k t_{ki} > y_i \geq \underline{x}$. Then i is not a receiver by Lemma 2. The other possibility is that i is a contributor to the public good. In this case, it may so happen that $x_i^* \leq \underline{x} < y_i - \sum_j t_{ij} + \sum_k t_{ki}$, while $g_i^* = y_i - \sum_j t_{ij} + \sum_k t_{ki} - x_i^*$. However, if $x_i^* < \underline{x}$, then i will be a receiver who contributes to the public good, and a receiver cannot contribute to the public good by Lemma 3, and we have a contradiction.

For the final part, consider an individual i such that $y_i > \bar{x}$. Suppose for contradiction first that such an individual is a receiver. Then $y_i - \sum_j t_{ij} + \sum_k t_{ki} > y_i > \bar{x}$. There are two possibilities as before - either i contributes or does not contribute to the public good. If i does not contribute to the public good then $y_i - \sum_j t_{ij} + \sum_k t_{ki} = x_i^* > \bar{x} > \underline{x}$ contradicting Lemma 2. If i does contribute to the public good, it contradicts Lemma 3. Hence, such an individual cannot be a receiver. Next, suppose again for contradiction that this individual neither receives nor gives transfers. Then $y_i - \sum_j t_{ij} + \sum_k t_{ki} = y_i$. As before there are two possibilities: i contributes, or does not contribute, to the public good. If i does not contribute, $x_i^* = y_i > \bar{x}$ contradicting Lemma 2. If i contributes, then x_i^* certainly cannot be less than $\underline{x}$ because of the arguments previously made. That leaves only two possibilities:

- i has a consumption x_i^* where $\underline{x} \leq x_i^* \leq \bar{x} < y_i$ with $g_i^* = y_i - x_i^*$.
- i has a consumption x_i^* where $\bar{x} < x_i^* < y_i$ with $g_i^* = y_i - x_i^*$.

The second case out of the above two is immediately ruled out by Lemma 2. In the first case, we show that $x_i^* < \bar{x}$ is impossible for the individual i. Since we are considering what happens in an equilibrium with transfers, there must exist at least one donor. By Lemma 2 we know that his level of private consumption will equal $\bar{x}$.

We will thus have that for this donor j that:

$$-U_x(\bar{x}, G^*) + U_G(\bar{x}, G^*) + \alpha \sum_{k \in N, j \neq j} U_G(x_k^*, G^*) \leq 0$$

Whereas for individual i it is the case that:

$$-U_x(x_i^*, G^*) + U_G(x_i^*, G^*) + \alpha \sum_{j \in N, j \neq i} U_G(x_j^*, G^*) = 0$$

for some $x_i^* \leq \bar{x}$.

We thus have that:

$$-U_x(x_i^*, G^*)+U_G(x_i^*, G^*)+\alpha \sum_{j\in N, j\neq i} U_G(x_j^*, G^*) \geq -U_x(\bar{x}, G^*)+U_G(\bar{x}, G^*)+\alpha \sum_{k\in N, j\neq j} U_G(x_k^*, G^*)$$

Cancelling the common terms and rearranging:

$$U_x(\bar{x}, G^*) - U_x(x_i^*, G^*) + (1-\alpha)(U_G(x_i^*, G^*) - U_G(\bar{x}, G^*)) \geq 0$$

This gives us an immediate contradiction for any $x_i^* < \bar{x}$ by Assumption 1 and the concavity of U. The above equation is only true if $x_i^* = \bar{x}$. When i is neither a donor nor a receiver, this is only possible when he is contributing to the public good to an amount $g_i^* = y_i - \bar{x}$.

Since we have eliminated all other possibilities, the only case that remains is that when $y_i > \bar{x}$, i is a donor. This completes the final part of Lemma 4.

■

A.7 Proof of Lemma 5

Proof. We prove this in three parts. First, we show that: every person who contributes to the public good must have the same private good consumption. Denote this level of consumption by $\hat{x}$. Suppose for contradiction that there are two contributors to the public good with private consumptions x_i^* and x_j^*, $x_i^* \neq x_j^*$.

Without loss of generality assume $x_i^* < x_j^*$. Since both of them contribute to the public good, (9) holds for both the agents:

$$-U_x(x_i^*, G^*)+U_G(x_i^*, G^*)+\alpha \sum_{k\neq i} U_G(x_k^*, G^*) = -U_x(x_j^*, G^*)+U_G(x_j^*, G^*)+\alpha \sum_{k\neq j} U_G(x_k^*, G^*)$$

Which can equivalently be written as

$$-U_x(x_i^*, G^*) + (1-\alpha)U_G(x_i^*, G^*) + \alpha \sum_{k} U_G(x_k^*, G^*) = -U_x(x_j^*, G^*)$$
$$+(1-\alpha)U_x(x_j^*, G^*) + \alpha \sum_{k} U_G(x_k^*, G^*)$$

Cancelling the common terms and rearranging:

$$\underbrace{U_x(x_j^*, G^*) - U_x(x_i^*, G^*)}_{<0} + (1-\alpha)\underbrace{\{U_G(x_i^*, G^*) - U_G(x_j^*, G^*)\}}_{\leq 0} = 0$$

Which is a contradiction given our assumptions on the class of functions $\mathcal{U}$.

For the next part of the lemma, we need to establish that if $y_i \leq \hat{x}$, then i cannot be a contributor to the public good. Suppose for contradiction that i still is a contributor to the public good. Thanks to Lemma 2, there are three mutually exclusive possibilities in an equilibrium with transfers: an individual is either a donor, a receiver, or neither.

- If i is neither a donor nor a receiver, his income after transfers $y_i - \sum_j t_{ij} + \sum_k t_{ki}$ is the same as his income before transfers: y_i, and hence $x_i^* < y_i = y_i - \sum_j t_{ij} + \sum_k t_{ki} \leq \hat{x}$ violating the first part of lemma 5 where we show that all contributors to the public good have the same equilibrium private good consumption $x_i^* = \hat{x}$. Thus, an individual who neither gives nor receives transfers cannot contribute when $y_i \leq \hat{x}$.

- if i is a donor. Then, $y_i - \sum_j t_{ij} + \sum_k t_{ki} < y_i \leq \hat{x}$. If i is a contributor to the public good, then i's equilibrium private good consumption x_i^* must be strictly lower than $y_i - \sum_j t_{ij} + \sum_k t_{ki}$ and hence lower than $\hat{x}$, violating the first part of Lemma 5 where we showed above when all contributors have the same private good consumption.

- i is a receiver. We know from Lemma 3 that in this case i cannot contribute to the public good.

For the final part of the lemma, we need to establish that if $y_i > \hat{x}$, then i is in one of the following mutually exclusive cases: either i is a contributor to the public good, or i does not contribute to the public good, is a donor, and $x_i^* = \hat{x}$.

Suppose for contradiction that i is not a contributor to the public good despite his income $y_i > \hat{x}$. As before, there are three cases to consider:

- If i is a receiver. If i is a receiver who doesn't contribute to the public good, then i consumes his entire income after transfers: $\underline{x} = y_i - \sum_j t_{ij} + \sum_k t_{ki}$. Since

$y_i - \sum_j t_{ij} + \sum_k t_{ki} > y_i > \hat{x}$ by the assumptions for this particular case, it must be that $\underline{x} > \hat{x}$.

By the assumptions on the class of functions U, there must be at least one contributor to the public good, with a level of private consumption $\hat{x}$, denote such an individual as individual j. Since j contributes but i doesn't, it holds that:

$$-U_x(\underline{x}, G^*) + U_G(\underline{x}, G^*) + \alpha \sum_{k \neq i} U_G(x_k^*, G^*) < 0 = -U_x(\hat{x}, G^*) \\ + U_G(\hat{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*)$$

Rearranging and cancelling the common terms as in previous proofs in this paper:

$$U_x(\hat{x}, G^*) - U_x(\underline{x}, G^*) + (1 - \alpha)(U_G(\underline{x}, G^*) - U_G(\hat{x}, G^*)) < 0$$

The expression on the left hand side is strictly positive when $\underline{x} > \hat{x}$, giving us a contradiction. Hence, i cannot be a receiver if $y_i > \hat{x}$.

- If i is neither a donor nor a receiver, his income after transfers $y_i - \sum_j t_{ij} + \sum_k t_{ki}$ is the same as his income before transfers: y_i. By arguments similar to the one made above for this case, this implies that $x_i^* = y_i = y_i - \sum_j t_{ij} + \sum_k t_{ki} > \hat{x}$, giving us a contradiction exactly in the same way as the case of receivers. Hence, i cannot be an individual who is neither a donor nor a receiver if $y_i > \hat{x}$.

- This leaves the last case where i is a donor, and does not contribute. We next need to show that in this case i's level of private consumption equals exactly $\hat{x}$. There are two possibilities for i's level of private consumption:

  - $\hat{x} < x_i^*$, and this is the case where $\hat{x} < x_i^* = y_i - \sum_j t_{ij} + \sum_k t_{ki} < y_i$. In this case, if i does not contribute to the public good, then by exactly the argument that we made for the receiver and the individual who neither gives nor receives, we arrive at a contradiction.

– $x_i^* \leq \hat{x}$. This is the case where $y_i - \sum_j t_{ij} + \sum_k t_{ki} = x_i^* \leq \hat{x} < y_i$: where perhaps, after making the transfers, the donor doesn't have a high enough level of income left to have the level of consumption enjoyed by contributors to the public good. It remains to be shown that $x_i^* < \hat{x}$ is impossible. There will always be at least one contributor to the public good, and since by the arguments above such a person cannot be a receiver or someone who neither gives nor receives, this person must be a donor. Since all donors enjoy the same level of private consumption $\bar{x}$, it must be the case that $\hat{x} = \bar{x}$, and the level of private consumption enjoyed by a donor who does not contribute is $\hat{x}$.

■

A.8 Proof of Proposition 4

Proof. The first part and second part of Proposition 4 are direct implications of combining results established in Lemma 4 and Lemma 5. For the final part, it must be established that $\bar{x} = \hat{x}$.

By Lemma 5 we know that this is true when $y_i > \bar{x}$ and i does not contribute to the public good. By combining Lemma 5 with Lemma 4 we know that this also holds for the case when $y_i > \bar{x}$ and i is not a donor. We also know from Lemma 4 that it is not possible that when $y_i > \bar{x}$, i neither contributes, nor donates. The only case left is when $y_i > \bar{x}$ and i both contributes and donates. In this case, if $\hat{x} \neq \bar{x}$, i has two different levels of private consumption $\bar{x}$ and $\hat{x}$ at the same equilibrium, which is a contradiction.

■

A.9 Proof of Lemma 6

Proof. The first part of the proof of Lemma 5 that if everybody is contributing to the public good, then $x_i^* = \hat{x} \,\forall\, i$.

Then for arbitrary players i and j:

$$U_x(x_i^*, G^*) = U_x(x_j^*, G^*)$$

Suppose now that, to the contrary, there exist individuals i and j such that $t_{ij} > 0$.

Then from 9 it will also hold that:

$$U_x(x_i^*, G^*) = \alpha U_x(x_j^*, G^*)$$

Clearly, the above equations are contradictory when $\alpha < 1$.

■

A.10 Proof of Proposition 5

Proof.

We tackle the proof in three parts. We first show that when model with transfers has the same equilibrium as the model without transfers we have that $\tilde{x} \leq \epsilon(y_{(1)})$ and this equilibrium is unique

Let us denote by $(x_1^o, ..., x_n^o)$ the unique levels of private good consumption for the n members of the group at the unique equilibrium of the model without transfers, $\mathbf{g}^*(\mathbf{y}, \alpha)$. For this to also be an equilibrium of the model with transfers, a necessary and sufficient condition is that equation 14 must hold for every i with $x_i = x_i^o$. Specifically, for any i and j:

$$x_i^o \leq \epsilon(x_j^o) \tag{17}$$

Since ϵ is a strictly increasing function of x, for it to hold true for all i and j it must hold true for the largest possible x_i^o and the least possible x_j^o. The least possible private good consumption is $x_j^o = y_{(1)}$ i.e. the income of the poorest member if he does not contribute. Since the game $\mathcal{G}(\mathbf{y}, \alpha)$ always has an equilibrium with at least one contributor, the maximum possible consumption is $x_i^o = \tilde{x}$ (since contributions are arranged in increasing order of income, by Lemma 1, and the consumption of a contributor is the highest possible level of private good consumption). This case includes the case of an interior equilibrium where everybody contributes to the public good (including the poorest individual) and there are no transfers (as shown in Lemma 6): $\tilde{x} < y_{(1)} < \epsilon(y_{(1)})$. The uniqueness of the equilibrium without transfers is guaranteed because the game $\mathcal{G}(\mathbf{y}, \alpha)$ has a unique equilibrium. We can further deduce from equation (17) being a necessary and sufficient condition that an equilibrium with transfers and an equilibrium without transfers are mutually exclusive.

We next show that the model with transfers has an equilibrium with transfers iff $\sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}) \geq 0$, and in this case $\underline{x}$, $\bar{x}$ and G^* are determined uniquely. Given the concavity of U, $\bar{x}$ is a strictly increasing function of $\underline{x}$. Note also that due to the concavity of the functions U and V, G is a strictly increasing function of $\bar{x}$. This is not specific to the model with transfers; this property would hold even for the model without transfers. Since G is a strictly increasing function of $\bar{x}$ and $\bar{x}$ is a strictly increasing function of $\underline{x}$, $G = \gamma(\underline{x})$ is strictly increasing in $\underline{x}$. Using the function x^+, we rewrite the constraint that the sum of all transfers given, net of the sum of transfers received and contributions to the public good, must be zero:

$$\sum_i (y_i - \epsilon(\underline{x}))^+ - \sum_i (\underline{x} - y_i)^+ - \gamma(\underline{x}) = 0 \tag{18}$$

Equation (18) is entirely defined in terms of $\underline{x}$. Define the function

$$F(\underline{x}) = \sum_i (y_i - \epsilon(\underline{x}))^+ - \sum_i (\underline{x} - y_i)^+ - \gamma(\underline{x})$$

Finding an equilibrium for the model with transfers thus corresponds to finding a root for the equation $F(\underline{x}) = 0$. Given our assumptions, $F(\underline{x})$ is a strictly decreasing function of $\underline{x}$ if $\underline{x} > y_i$ for at least one i. For a unique solution to exist to $F(\underline{x}) = 0$, we should have that $F(x_1) > 0$ and $F(x_2) < 0$ for some x_1 and x_2 in the domain of $F(\underline{x})$ with $x_1 < x_2$. Consider any $x_2 > \max_i y_i$. It is easy to show that $F(x_2)$ is strictly negative given our assumptions. Thus, $F(x_2) < 0$. The lower bound for $\underline{x}$ (or the minimum possible value) is $y_{(1)}$, the minimum income for the group. At $\underline{x} = y_{(1)}$, $\sum_i (y_{(1)} - y_i)^+ = 0$. However, $\sum_i (y_i - \epsilon(y_{(1)}))^+ \geq 0$, and $-\gamma(y_{(1)}) < 0$. This difference is positive iff

$$\sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}) \geq 0$$

Otherwise, this difference is strictly negative and the function $F(\underline{x})$ lies everywhere below zero under the domain of F and there is no solution to the equation $F(\underline{x}) = 0$.

Hence, if $\sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}) \geq 0$, the equation $F(\underline{x}) = 0$ has a unique solution. The thresholds $\underline{x}$, $\bar{x}$ and the amount of public good G are uniquely determined, even though individual levels of contributions and transfers may not be unique. If $\sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}) < 0$, $F(\underline{x}) = 0$ has no solution, and $\underline{x}$ is undetermined.

In the final step of the proof we show that $\tilde{x} \leq \epsilon(y_{(1)}) \iff \sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}) \leq 0$.

$$\tilde{x} \leq \epsilon(y_{(1)})$$
$$\Leftrightarrow \sum_i (y_i - \tilde{x})^+ \geq \sum_i (y_i - \epsilon(y_{(1)}))^+$$

Since λ and ϵ are both increasing functions:

$$\Leftrightarrow \sum_i (y_i - \tilde{x})^+ - \lambda(\tilde{x}) \geq \sum_i (y_i - \epsilon(y_{(1)}))^+ - \lambda(\epsilon(y_{(1)}))$$

We know from Proposition 17 that if $\tilde{x} \leq \epsilon(y_{(1)})$ then there are no transfers. Thus, $\sum_i (y_i - \tilde{x})^+ = G = \lambda(\tilde{x})$ and $\sum_i (y_i - \tilde{x})^+ - \lambda(\tilde{x}) = 0$. Then,

$$0 \geq \sum_i (y_i - \epsilon(y_{(1)}))^+ - \lambda(\epsilon(y_{(1)})) = \sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)})$$

And that completes the proof.

■

References

ANDREONI, J. (1988a): "Privately Provided Public Goods in a Large Economy: The Limits of Altruism," *Journal of Public Economics*, 35, 57–73.

——— (1988b): "Why Free Ride? Strategies and Learning in Public Goods Experiments," *Journal of Public Economics*, 37(3), 291–304.

——— (1989): "Giving with Impure Altruism: Applications to Charity and Ricardian Equivalence," *Journal of Political Economy*, 97, 1447–1458.

——— (1990): "Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving," *Economic Journal*, 100, 464–477.

——— (1995): "Cooperation in Public-Goods Experiments: Kindness or Confusion?," *The American Economic Review*, 85(4), 891–904.

ARORA, S., AND B. SANDITOV (2016): "Social network and private provision of public goods," *Journal of Evolutionary Economics*, 26, 195–218.

ARROW, K. (1981): "Optimal and Voluntary Income Distribution," in *Economic Welfare and the Economics of Soviet Socialism: Essays in Honor of Abram Bergson*, ed. by S. Rosefielde, pp. 679–681. Cambridge University Press.

BANERJEE, A., AND N. GRAVEL (2018): "Contribution To A Public Good Under Subjective Uncertainty," CSH-IFP Working Papers - 10.

BECKER, G. S. (1974): "A Theory of Social Interactions," *Economic Journal*, 82(6), 1063–1093.

——— (1981): "Altruism in the family and selfishness in the market," *Economica*, 48, 1–15.

BERGSTROM, T., L. BLUME, AND H. VARIAN (1986): "On the private provision of public goods," *Journal of public economics*, 29, 25–49.

BOMMIER, A., A. GOERGER, A. GOUSSEBAILE, AND J. P. NICOLAI (2019): "Altruistic Foreign Aid and Climate Change Mitigation," Available at SSRN: http://dx.doi.org/10.2139/ssrn.3485458.

Bourlès, R., Y. Bramoullé, and E. Perez-Richet (2017): "Altruism in Networks," *Econometrica*, 85(2), 675–689.

Cherchye, L., S. Cosaert, T. Demuynck, and B. De Rock (2017): "Group consumption with caring individuals," *Discussion Paper Series DPS 17.15*, pp. 147–162.

Chiappori, P. A. (1988): "Rational household labor supply," *Econometrica*, 56, 63–89.

——— (1992): "Collective labor supply and welfare," *Journal of Political Economy*, 100, 437–467.

Cornes, R. C., and T. Sandler (1986): *The Theory of Externalities, Public Goods and Club Goods*. Cambridge University Press, Cambridge.