Communication and Commitment with Constraints: An Application to Alliances*

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Abstract

An informed and an uninformed agent both contribute to a joint coordination game such that their actions are substitutable and constrained. When agents are allowed to share information prior to the coordination stage, in the absence of commitment, there is full information revelation as long as constraints are not binding. The presence of binding constraints results in only partial revelation of information in equilibrium. The most informative equilibrium is strictly pareto dominant. Allowing for limited commitment strictly increases (ex ante) welfare of both agents. I completely characterize the optimal commitment mechanism for the uninformed agent. Finally, I apply the theoretical results to the problem of information sharing and binding agreements in international alliances.

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1 Introduction

In international alliances, member countries jointly make decisions in the presence of information asymmetry. They share private information and pool resources in order to achieve common objectives (e.g., collective defense, intelligence sharing, peacekeeping). The principles of effective sharing of information and mutual commitment to work together have been central tenets of the NATO (see, e.g., Petersson, 2015; Wittmann, 2009). In a similar vein, more recently, the EU nations have initiated the Permanent Structured Cooperation (PESCO) agreement that aims at creating a *collective defense* policy. Achieving these objectives however usually involves coordination of actions between the member countries. The defining features of such coordination is that the actions of members are (imperfectly) substitutable and there are constraints (e.g. fiscal, military/personnel) on the available action choices.

To achieve coordination, alliances usually organize protocols based on communication (e.g., via diplomatic channels) to exchange private information between the members. When action constraints are not binding, it is possible for members to share information and coordinate efficiently. The incentive problem arises when an informed party is constrained by the set of actions available. In this case, it exacerbates the incentives of an informed member to misrepresent their private information in order to induce a higher action from the uninformed one. The constraints therefore affect countries' capacity to contribute and coordinate efficiently. An important question that arises is how do constraints on actions affect the nature of communication?

Alternatively, alliance members could resort to ex ante commitment contracts (e.g., via binding agreements) that specify decision rules that they can agree upon and commit to. However, when there are multiple decision-makers the precise structure of the optimal rules of commitment — or ex ante contracts — are unclear. Further, whether there is any value at all of commitment — to binding agreements — in alliances is salient from a policy perspective.

To gain insights into these questions, I analyze a coordination game between two agents in which both agents take actions but only one agent is informed about an underlying state of the world. Importantly, agents' actions are imperfectly substitutable and their action sets are constrained. Given information asymmetry and action constraints, I study equilibria and efficiency properties of two decision-making protocols: *i) simultaneous protocol* and *ii) commitment protocol*. The former relies purely on *communication* (cheap talk à la Crawford and Sobel, 1982) between agents with simultaneous decision-making and no commitment. The latter resembles the classical *delegation problem* (Holmström, 1984), and involves one-sided ex-ante *commitment* by the uninformed agent with sequential decision-making.[1]

Central Results. The three main set of results can be stated as follows:

- There is a *negative* relationship between constraints and communication, but, a *positive* relationship between communication and welfare. That is, lesser constraints on the action set imply greater credibility in information transmission. Further more information transmission translates to higher ex-ante welfare for both agents.
- The optimal commitment contract has a simple threshold feature. The uninformed agent *minimizes* miscoordination losses up to a threshold state conditional on providing the necessary informational rents to the informed agent. Beyond this, the actions of both agents are *capped* and *unresponsive* to any information.
- The *value of commitment* is positive for both agents. That is, increasing the degree of commitment by the uninformed agent increases both agents' welfare. Further, the welfare gains from commitment rise when constraints are more binding and when both agents have higher conflicts of interests.

Theoretical Framework. The setup considers a coordination game between two agents

[1] The paper analyzes ex-ante commitment of the uninformed agent, in contrast to the information design literature (see, e.g., Bergemann and Morris (2016) and Kamenica and Gentzkow (2011)) that concerns commitment on the informed agent's side.

with (one-sided) asymmetric information, imperfect substitutability in actions, constraints on the action set, and preference heterogeneity (conflict of interests). Each agent has a joint coordination function that generates an output based on the actions of both agents. Further, the coordination function increases in the actions of both players. The main theoretical innovation is that the coordination function is aggregative in nature (see, e.g., Acemoglu and Jensen, 2013 and Jensen, 2010) and the underlying game is *Bayesian aggregative*.[2]

Agents' preferences depend on an unknown state variable and the joint coordination function. A key assumption on the preferences of agents is that they exhibit a *shared costs* feature. Thus, there is both a private marginal cost and an externality cost due to the other agent's action. The overall cost is therefore proportional to the value of the joint coordination function of each agent. The shared costs feature captures the joint decision-making nature in international alliances that entails burden sharing in the form of, for example, operational costs of coordinating, or reputational costs of participating in military interventions. Finally, I assume that the agents' joint coordination and utility functions are twice continuously differentiable, and concave in the actions. The concavity property implies that there is an unique first best level of the joint coordination function corresponding to each state of the world.

Results. The first theorem establishes the existence and uniqueness of pure strategy equilibria in actions for the Bayesian aggregative game. The result uses tools developed in the aggregative games literature (Acemoglu and Jensen, 2013) and reformulates them to the Bayesian game setup. The underlying aggregative property of the coordination function and imperfect substitutability feature of the preferences drives the existence result, while the concavity of preferences (in particular, both the coordination and utility

[2]An aggregative game is one in which each players' payoff is a function of their own action and a common aggregator function. The aggregator function itself can be represented as additively separable functions of each agents' actions. See also Corchon (1994) and Dubey, Haimanko, and Zapechelnyuk (2006).

functions) is sufficient to ensure uniqueness of equilibrium actions.[3]

Adding communication to the Bayesian aggregative game results in communication equilibria that are *threshold* in nature. When the informed agent does not suffer from binding constraints on actions, all private information is communicated in equilibrium and agents take actions as if there were perfectly informed. This leads to full efficiency for both agents. In the presence of binding constraints, the informed agent communicates truthfully only up to a certain threshold state and pools all information beyond.[4] The intuition is that in the absence of constraints, both agents can take actions such that the joint coordination function for each agent corresponds with their first best levels, precluding the need to misrepresent information. However, full information revelation breaks down when action constraints bind and there is loss of information in equilibrium. With binding constraints the informed agent finds it optimal to pool information at the top to induce a higher action from the uninformed agent. Since the informed agent also takes an action post communication, the agent can use her private information on the pooling interval to her benefit.[5]

Given the multiplicity of communication equilibria, it is important to compare their welfare properties. I show that the communication equilibria exhibit an intuitive pareto ordering - the more informative threshold equilibrium is ex ante pareto dominant. This implies that welfare of both agents is monotonically increasing in the amount of information revealed. The intuition is as follows. When more information is revealed, both agents achieve first best levels of coordination for a greater measure of types. Further, under the most informative equilibrium, the pooling message induces a higher expected

[3]Given the uniqueness of actions for any set of beliefs between the agents, it is possible to fully characterize the actions of agents under any pooling equilibria of the two protocols since only the posterior beliefs change without affecting the effective structure of payoffs in the action stage.

[4]The threshold equilibria are related in spirit to those in Kartik, Ottaviani, and Squintani (2007), and Kartik (2009). In both those papers, there is exaggerated communication in equilibrium, in contrast to the truthful messaging equilibria characterized in this paper.

[5]There are *other* hybrid equilibria from communication that are in some sense subsumed by the threshold equilibria. They bear some semblance in structure to the *central pooling equilibria* in the work of Bernheim and Severinov (2003). See Proposition 1.

action from the uninformed agent. Since the informed agent has discretion in choosing her actions on the pooling interval accordingly, this means that the informed agent's utility is strictly better off on the pooling interval. This novel feature provides greater flexibility to the informed agent and allows for better coordination of the agents' actions. It minimizes the inefficiency from miscoordination, thereby improving the welfare of both agents.

The threshold equilibria introduces miscoordination in agents' actions due to informational asymmetries on the pooling interval. This results in inefficiencies on the pooling interval for both agents. Agents could instead rely on ex ante commitments to mitigate this inefficiency. In international alliances, for example, countries commit to binding agreements that specify mutually agreed upon rules of engagement. To capture this feature, I analyze a *commitment protocol* in which both agents agree to an ex-ante contract in which the uninformed agent commits to a communication dependent incentive compatible *decision rule*. The informed agent decides on the information to communicate and a subsequent (non-contractible) action after observing the decision rule of the uninformed agent. This gives rise to a *limited commitment* (see, e.g., Bester and Strausz, 2001; Krishna and Morgan, 2008) setup in which only the uninformed agent can commit to actions while the informed agent does not have commitment power with respect to both communication and decision-making.[6]

With limited commitment, the informed agent, depending on the contracted commitment rule, can decide what information to reveal and then, what action to take. This adds a layer of complexity to the uninformed agent's commitment problem. Using the revelation principle I restrict attention to direct mechanisms in which the uninformed agent's problem is twofold: *i)* to choose an action rule that satisfies the informed agent's non-contractibility constraint; and *ii)* to minimize the inefficiencies from miscoordination, conditional on satisfying the incentive compatibility conditions.

[6] Notice that this is similar to the optimal delegation problem (Alonso and Matouschek, 2008), except that there is an additional informed decision-maker whose action is not contractible.

Trivially, the informed agent mimics the actions of the simultaneous protocol up to the most informative threshold. Beyond this, the optimal commitment rule exhibits two key features. The uninformed agent minimizes miscoordination losses by ensuring that the informed agent always takes the *maximal action*. This implies the uninformed agent contributes only the residual action required to satisfy the informed agent's IC constraint. Further, the uninformed agent *caps actions* beyond a (higher) threshold of information, meaning that the informed agent's informational rent is capped beyond this threshold.[7] By committing to an ex-ante decision rule, the uninformed agent incentivizes the informed agent to reveal more information in a way that benefits both agents.

Finally, I consider a simple parameterized uniform-quadratic setting that delivers closed-form characterizations of the communication and commitment thresholds, the welfare of agents under the two protocols, and the welfare gains from commitment for both the agents. Several interesting insights emerge. First, the informational threshold for truthful communication and the commitment threshold both increase in the upper bound of the action set of the informed agent. Second, the welfare of agents under both the protocols can be represented as a simple increasing function of the communication threshold. That is, any increase in the upper bound of the action set translates into more truthful communication and therefore higher welfare for the agents. Interestingly, the gains from commitment decrease in this parameter, but increase when both agents have more conflicting interests.

Contribution. From a theoretical standpoint, the paper offers two broad contributions. First, it introduces a Bayesian version of an aggregative game and merges it with the vast literature on strategic communication. Second, the paper models *limited* commitment with two decision makers, in which only the uninformed agent commits to a binding (contractible) decision rule while the informed agent's action is *non-contractible*. The

[7]Without this capping, the uninformed agent would end up providing first best levels to the informed for all possible states, which would be equivalent to full delegation as in Dessein (2002). This form of full delegation is never optimal for the uninformed agent in this paper, similar to the result of Krishna and Morgan (2008).

paper therefore also bridges the literature on communication with coordination motives and contracting with imperfect commitment.

From an applied standpoint, the central results deliver relevant policy prescriptions with respect to informational incentives and decision-making in international alliances. One central finding establishes a negative relationship between action constraints and communication. In NATO, for example, the US leadership has urged the European bloc of countries to contribute their '*fair share*' towards joint defense initiatives of the alliance.[8] The theoretical analysis argues that when countries contribute more, it decreases their decision-making constraints. As a result, communication carries greater credibility that reduces the losses from miscoordination of actions. This in turn improves efficiency and increases welfare of partnering countries. The result therefore sheds light on the importance of contributing to alliances, implying both an *informational* effect and a *welfare* effect due to the slacken constraints on decision-making in the alliance.[9]

Another important implication is that binding agreements between countries are pareto improving from an ex-ante perspective. By committing to different possible contingencies, countries that rely on information from others can minimize miscoordination by targeting their actions better. The gains from such commitments are higher when the action constraints are greater and when countries are more conflicted in their preferences over final outcomes. The results therefore provide an ex ante utilitarian rationale for *binding commitments* within an alliance. Intuitively, my findings suggest that countries with more divergent interests have the most to gain from committing to binding agreements.

[8] "What seems to be clear, however, is that the Obama Administration does not want to lead NATO in the charismatic or traditional way that the US used to do. That has also been manifested in the discussion about the burden sharing within the alliance; that the European members must step up both concerning the leadership of the alliance in European affairs, and concerning actual defense spending." - p.165, Petersson, 2015

[9] In Section 8 I discuss the main insights of the model in light of the recent developments that have focused attention on the importance of contributing to alliances, and the role of binding agreements.

Related Literature

This paper extends and contributes to the vast theoretical and applied literature of that studies communication in interdependent environments. The role of communication with strategic complementarities in actions have been widely studied and applied to many settings (e.g., Alonso, Dessein, and Matouschek, 2008; Baliga and Morris, 2002; Dessein and Santos, 2006; Hagenbach and Koessler, 2010; Rantakari, 2008). Save for Alonso (2007), who considers a principal-agent setting in which an uninformed principal controls the decision rights and actions of the two players are either strategic complements or substitutes, no other paper has looked at incentive problems when players' actions are substitutable.

The literature on delegation (Holmstrom, 1978) has delved into the question of optimal commitment by an uninformed Principal. Alonso and Matouschek (2008) characterize the necessary and sufficient conditions for interval delegation to be optimal under quadratic loss utility functions. Amador and Bagwell (2013) generalize this result for a broader class of welfare functions and also allow for money burning.[10] Melumad and Shibano (1991) characterize a deterministic commitment rule for the uninformed receiver in a standard cheap talk game. The optimal commitment rule in my work, though deterministic, studies two decision makers and limited one-sided commitment.

The optimal mechanism resembles the interval delegation result in that the uninformed agent provides a cap on actions, but the model allows for non-contractible actions (imperfect commitment) by the informed agent. In this regard, it is similar to Bester and Strausz (2001), who study an imperfect commitment problem without transfers, and Krishna and Morgan (2008), who look at contracting with imperfect commitment and monetary transfers. In contrast to these papers, I model a problem in which both agents are decision-makers and there is limited commitment, as the informed agent's action is

[10] In this vein, Ambrus and Egorov (2017) study a perfect commitment contracting problem with money burning and monetary transfers.

non-contractible.[11]

This paper is also related to work on communication and commitment by Forges, Horst, and Salomon (2016) and Forges and Horst (2018). They look at ex-ante commitment contracts that are followed by communication of one-sided private information. Those papers focus on two-player games with no interdependencies, while this paper focuses on coordination games with action substitutability and constraints. This apart, in their work, commitment imposes strong interim and ex-post individual rationality constraints that are absent in my setup.

2 An Example

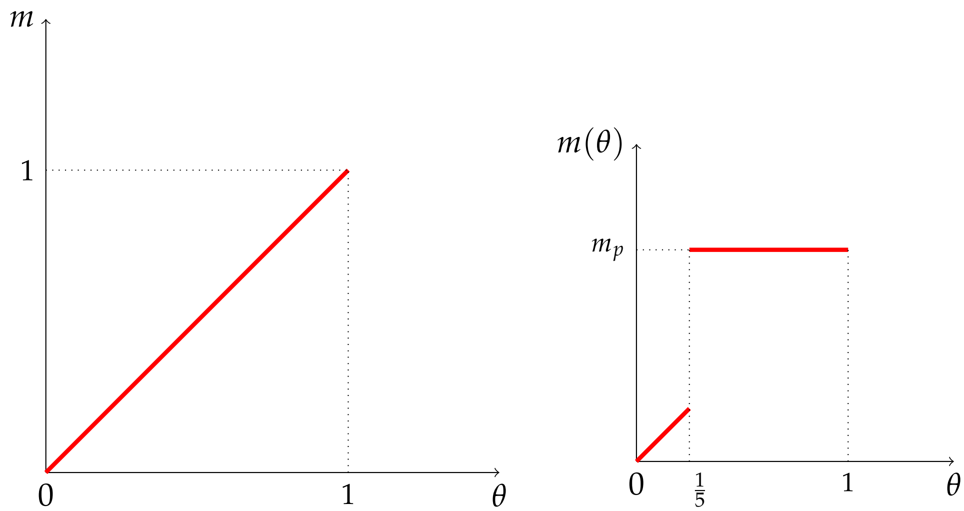
Consider a joint task to be executed by an uninformed agent A_1 and an informed agent A_2. A_2 perfectly observes the state of the world θ, drawn from an uniform distribution $[0, 1]$. The information is soft and A_2 communicates its private information by sending a cheap talk message $m(\theta)$ to A_1. Upon communication, both agents take actions (x_1, x_2) that affect both their payoffs. The utility functions are given by:

$$U^1 = -\left[\left(\frac{x_1 + \eta x_2}{1+\eta}\right) - \theta\right]^2$$

$$U^2 = -\left[\left(\frac{x_2 + \eta x_1}{1+\eta}\right) - \theta - b\right]^2$$

The conflict of interests between the two agents in given by the bias parameter $b > 0$. Observe that both players take actions that contribute to the project and these actions are substitutable in that $\frac{d^2U^i}{dx_1 dx_2} < 0$, where $\eta \in (0, 1)$ captures the degree of substitutability. The two players have a limited action space, i.e. $x_i \in [-a, a]$, that may constrain them. If A_2 truthfully reveals the true state of the world, i.e. $m(\theta) = \theta$, the two players solve the

[11] In contrast, both Bester and Strausz (2001) and Krishna and Morgan (2008) study a problem where the uninformed Principal takes two decisions, one of which is contractible and the other, not.



(a) $a \geq \frac{9}{5}$ (b) $a = 1$

Figure 1: When $a \geq \frac{9}{5}$, the action constraints are not binding for A_2, resulting in full information revelation. On the other hand, when $a \in (\frac{4}{5}, \frac{9}{5})$ there is only partial revelation of information.

following best responses:

$$A_1 : x_1 = (1+\eta)\theta - \eta x_2$$

$$A_2 : x_2 = (1+\eta)(\theta + b) - \eta x_1$$

To simplify exposition, let $b = \frac{2}{5}$ and $\eta = \frac{1}{2}$. The actions after truthful messaging are given by: $x_1^* = \theta - \frac{2}{5}$, $x_2^* = \theta + \frac{4}{5}$. Notice immediately that full information revelation is possible if $a \geqslant \frac{9}{5}$. This is because A_2 can reveal truthfully the highest type $\theta = 1$, and take an action $x_2^*(1) = \frac{9}{5}$. This way, A_2 achieves first best. If instead, $a < \frac{4}{5}$, then no information can be credibly revealed by A_2.[12]

Finally, when $\frac{4}{5} < a < \frac{9}{5}$, A_2 has an incentive to reveal some information. To see this, let $a = 1$. Then, for any $\theta \in [0, \frac{1}{5}]$, A_2 reveals the state truthfully since her optimal

[12]Suppose say $a = \frac{2}{5}$. Then the constraint is binding for all types. A_2 can inflate her signal in order to induce A_1 to allocate more. To see this, instead of $m(0) = 0$, say inflated message is $m(0) = \frac{2}{5}$. Then, A_1 best responds by allocating $x_1^* = \frac{2}{5}$. A_2 then contributes $x_2^* = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$. That is, by inflating her information the informed agent induces a higher action from A_1 whilst achieving first best. However this incentive to misrepresent means that messages do not carry credibility in equilibrium. A_2 can never credibly reveal any information to A_1 and therefore communication is rendered ineffective.

action is within the domain of available actions ($x_2^*(\frac{1}{5}) = 1$). But, for any $\theta > \frac{1}{5}$, A_2 cannot sustain a truthful messaging strategy since the constraints are binding for A_2 (i.e. $x_2 = 1$). Then the optimal action for A_1 is according to its best response function, which is $x_1 = \frac{3}{2}\theta - \frac{1}{2}$. This cannot be an equilibrium since A_2 gets a payoff of $U_2 = -\left(\frac{1+\frac{1}{2}(\frac{3}{2}.\theta-\frac{1}{2})}{\frac{3}{2}} - \theta - \frac{2}{5}\right)^2 \neq 0$ where $\frac{1+\frac{1}{2}(\frac{3}{2}.\theta-\frac{1}{2})}{\frac{3}{2}} < \theta + \frac{2}{5}$ for $m = \theta > \frac{1}{5}$. This implies there is under-allocation from A_2's perspective if it reveals the truth. Therefore, A_2 has an incentive to exaggerate its information to induce agent A_1 to play a higher action. This precludes separation beyond $\theta = \frac{1}{5}$. In fact, all types above this cutoff send a pooling message, m_p. Therefore, there is at most a partially revealing equilibrium in which A_2 is truthful (separates) in the range $\theta \in [0, \frac{1}{5}]$ and pools for $\theta \in (\frac{1}{5}, 1]$ by sending the same message, m_p.[13]

The example suggests a *novel trade-off* for information transmission with substitutability and action constraints. The ability to truthfully reveal information depends on the actions available to the informed player. The informed agent A_2 can communicate more information when the action set available is bigger. For the same reasons, when constraints bind, there is an incentive to inflate private information and extract more actions from the uninformed agent A_1.

3 The Model

Consider a joint project between two agents $\mathcal{I} = \{1, 2\}$. The payoff from the project is dependent on state $\theta \in \Theta$ and the actions of both agents. The state $\theta \in \Theta$ is distributed according to a cdf F and a corresponding density f with full support. Agent A_2 receives a perfectly observable private signal about the state θ while agent A_1 is uninformed. The set of possible actions available to the agents is constrained and given by $x_i \in V \subseteq \mathbb{R}^+$,

[13] Partitional equilibria of the kind developed by Crawford-Sobel are also ruled out on the interval $(\frac{1}{5}, 1]$. The incentive to exaggerate ensures that if there are two partitions, say, boundary types in the lower partition would find it profitable to deviate to the higher partition, precluding the existence of an indifferent type. See Lemma 1.

where the set V is closed and compact with $\inf(V) = \underline{k}$, $\sup(V) = \bar{k}$, and $\underline{k}, \bar{k} \in \mathbb{R}^+$. Each agent's utility is given by $U(\phi^i(x_i, x_{-i}), \theta, b_i)$, where $\phi^i(.)$ is the agent-specific *joint action function* (henceforth *coordination function*). The coordination function $\phi^i(.)$ depends on agent i's action x_i, as well as the action of the other agent, x_{-i}. The function is represented by a mapping $\phi^i : V \times V \to Z \subset \mathbb{R}$. The bias parameter b_i measures the conflict of interest between the two agents. This captures the extent to which the goals of the agents differ relative to the underlying state of the project.

The utility function $U : V^2 \times \Theta \times \mathbb{R} \to \mathbb{R}$ is twice continuously differentiable, $U_{11}(.) < 0$, $U_{12}(.) > 0$, and $U_{13}(.) > 0$ such that U has a unique maxima for any given pair (θ, b_i). For sake of exposition, let the bias of the uninformed agent be normalized to $b_1 = 0$ and that of the informed to $b_2 = b > 0$.[14] Let $\bar{\phi}^1_\theta \equiv \arg\max_{\phi^1} U\left(\phi^1, \theta\right)$ and $\bar{\phi}^2_\theta \equiv \arg\max_{\phi^2} U\left(\phi^2, \theta, b\right)$ be the first best levels of joint actions for the two agents respectively, for a given θ. The above setup induces a Bayesian game with one-sided incomplete information given by $\Gamma = \left(\mathcal{I}, V, \Theta, F, \{\phi^i\}_{i \in \mathcal{I}}\right)$.

The coordination function $\phi^i(.)$ is increasing, twice continuously differentiable and concave in the actions of both agents, x_1 and x_2. I assume the following structure on the coordination function $\phi^i(.)$:

Assumption 1. *Increasing marginal contribution:* $0 < \frac{\partial \phi^i(.)}{\partial x_i} = \frac{\partial \phi^j(.)}{\partial x_j} < \infty$

Assumption 2. *Positive spillover:* $0 < \frac{\partial \phi^i(.)}{\partial x_j} = \frac{\partial \phi^j(.)}{\partial x_i} < \infty$

Assumption 3. *Imperfect substitutability:* $\frac{\left(\frac{\partial \phi^i}{\partial x_i}\right)}{\left(\frac{\partial \phi^i}{\partial x_j}\right)} > 1$

Assumption 1 ensures that the function is strictly increasing and bounded in each agent's own action, while the second assumption ensures the same with respect to the other agent's action. Assumption 3 implies that the *marginal contribution effect* dominates

[14] Notice that including a conflict of interest is not necessary for the analysis and the main results. Instead one could vary the coordination functions to generate the same trade-offs.

the *spillover effect*. The assumptions guarantee that the utility function satisfies the condition $\frac{d^2U}{dx_i dx_j} < 0$, implying that actions of the two agents are substitutable.[15] The payoffs exhibit a *shared costs* feature, instead of the free riding (marginal costs) property that is commonly observed in games with action substitutability (e.g., Dubey et al., 2006). That is, there is a cost associated with the positive externality generated by each agent's action on the other agent's coordination function. As a result, each agent's cost is determined by the total value of their respective coordination function, and not only by individual actions. In alliances, for example, this could be a reputational cost incurred for partnering in a military operation with another country, or nation-building costs that are incurred jointly when countries work together for post-conflict rehabilitation efforts.

Finally, I make the following assumption that considers a class of games with an *generalized aggregative property* (Acemoglu and Jensen, 2013; Jensen, 2010).

Assumption 4. *Aggregator function: There exists a continuous and additively separable aggregator* $\psi : V \times V \rightarrow S \subset \mathbb{R}$ *and a transformation* $\tilde{\phi}^i : V \times S \rightarrow Z$ *such that,*

$$\phi^i(x_i, x_{-i}) = \tilde{\phi}^i(x_i, \psi(x))$$

The additive separability of $\psi(x)$ implies there exists strictly increasing functions $H, h_1, h_2 : \mathbb{R} \rightarrow \mathbb{R}$ such that $\psi(x) = H(h_1(x_1) + h_2(x_2))$ (see, e.g., Gorman, 1968).[16] Given a generalized aggregator function, the underlying game can be represented as a *Bayesian Aggregative Game* .

[15]To see this consider the derivative $\frac{dU}{dx_i} = \frac{\partial U}{\partial \phi^i}\frac{\partial \phi^i}{\partial x_i}$. Differentiating this expression with respect to x_{-i} gives $\frac{d^2U}{dx_i dx_j} = \frac{\partial^2 U}{\partial {\phi^i}^2}\frac{\partial \phi^i}{\partial x_j}\frac{\partial \phi^i}{\partial x_i} < 0$ since the coordination function is additively separable in (x_1, x_2).

[16]In the example of Section 2 the aggregator is a linear function. That is $h_i(x_i) = x_i$, $\psi(x) = \frac{\eta}{1+\eta}(x_1 + x_2)$, and $\tilde{\phi}^i(x_i, \psi(x)) = \frac{1-\eta}{1+\eta}x_i + \frac{\eta}{1+\eta}(x_1 + x_2)$. Alternatively, $\phi^i(x_i, x_j) = \frac{\alpha x_i + \beta x_j}{\alpha+\beta}$ where $\alpha > \beta$ such that $h_i(x_i) = x_i$, $\psi(x) = \frac{\beta}{\alpha+\beta}(x_1 + x_2)$, and $\tilde{\phi}^i(x_i, \psi(x)) = \frac{\alpha-\beta}{\alpha+\beta}x_i + \frac{\beta}{\alpha+\beta}(x_1 + x_2)$. In this case, the parameters are sufficient to induce conflicting interests.

Definition 1. *Let* $\tilde{\Gamma} = \left(\mathcal{I}, V, \Theta, F, \{\tilde{\phi}^i\}_{i\in\mathcal{I}}\right)$ *represent a Bayesian Aggregative Game such that,*

$$U\left(\phi^i(x_1, x_2), \theta\right) = U\left(\tilde{\phi}^1(x_1, \psi(x)), \theta\right)$$

$$U\left(\phi^2(x_2, x_1), \theta, b\right) = U\left(\tilde{\phi}^2(x_2, \psi(x)), \theta, b\right)$$

A pure strategy Bayesian Nash Equilibrium (henceforth BNE) of the game without communication is defined as follows:

Definition 2. *(BNE) A profile of actions* $\left\{x^B(\theta)\right\}_{\theta\in\Theta}$*, where* $x^B(\theta) \equiv \left(x_1^B, x_2^B(\theta)\right)$ *constitutes a Nash equilibrium of the Bayesian Aggregative Game if,*

$$x_1^B \equiv \arg\max_{x_1\in V} \mathbb{E}_\theta\left[U\left(\tilde{\phi}^1(x_1, \psi(x^B(\theta))), \theta\right)\right]$$

$$\forall\theta\in\Theta : x_2^B(\theta) \equiv \arg\max_{x_2\in V} U\left(\tilde{\phi}^2(x_2, \psi(x^B(\theta))), \theta, b\right)$$

If the Bayesian Aggregative Game admits an equilibrium, and moreover, an unique one for any given prior f, then it follows that adding communication would similarly result in uniqueness of equilibrium actions. Specifically, the posterior beliefs of A_1 is affected by the messaging equilibrium, but existence and uniqueness of the best responses in the Bayesian game post-communication would continue to hold. With this in mind, the following theorem establishes the existence and uniqueness of BNE for a Bayesian Aggregrative Game.

Theorem 1. *There exists an unique BNE of the Bayesian Aggregative Game.*

Proof. See Appendix A.1. □

The main tool required to obtain the result is the notion of *aggregate backward response correspondence*.[17] Equilibrium of the Bayesian Aggregative game is a fixed point of the aggregate backward response correspondence. To guarantee existence of a fixed point,

[17] The detailed notations are confined to the Appendix.

the underlying game has to be a *nice aggregative game*.[18] Assumptions 1-4 ensures that the *backward response correspondences* are upper hemi-continuous and there exists a fixed point (Kakutani's theorem) of the game. In the presence of information asymmetry, only the informed agent is able to adjust actions according to her private information while the uninformed A_1 takes an expected action (possibly the boundary action $x_1^B = inf V$) such that the local maxima conditions are satisfied. The equilibrium actions of the agents in the game is defined as a set that consists of a pair of actions $x(\theta) = (x_1, x_2(\theta))$ and an aggregate function $Q(\theta) = \psi((x(\theta)))$ for every $\theta \in \Theta$. Since there is an unique value of the coordination function for every type θ, and the coordination function itself is concave in the actions of agents, it follows from standard arguments that the equilibrium actions are unique. Specifically, there is an unique pair of actions for every $\theta \in \Theta$ such that $\tilde{\phi}^i(x_i(\theta), \psi(x(\theta))) = \bar{\phi}_\theta$ for $i = \{1, 2\}$.

4 Communication Equilibria

I now revert back to the original formulation of the model to characterize the set of communication equilibria prior to the action stage. Following Kartik (2009), let $M = \bigcup_\theta M_\theta$ be a Borel space of messages available to A_2 such that $\forall \theta, \theta^{'} \in \Theta : M_\theta \bigcap M_{\theta'} = \emptyset$. The strategic communication game, or the "*Simultaneous Protocol*", proceeds in two stages.

- In the first stage, A_2 observes the true state $\theta \in \Theta$ and sends a message $m \in M$ to A_1. The messaging strategy is defined by a mapping $\mu : \Theta \to M$ where the message $m = \mu(\theta)$.
- In the second stage, both agents simultaneously take actions $\alpha_1 : M \to V$ and $\alpha_2 : \Theta \times M \to V$.

[18] See Definition 6 in Acemoglu and Jensen (2013).

An equilibrium of the *simultaneous protocol* game is a Perfect Bayesian Equilibrium in monotone messaging (pure) strategies[19] that satisfies the following properties:

- A_1 and A_2 simultaneously choose actions $(x_1^*(m), x_2^*(\theta, m))$ that maximize their expected utility,

$$x_1^*(m) \equiv \arg\max_{x_1 \in V} \mathbb{E}_{\theta|m} \left[U\left(\phi^1(x_1, x_2^*(\theta, m)), \theta\right)\right] \text{ subject to } x_1 \in V \tag{1}$$

$$x_2^*(\theta, m) \equiv \arg\max_{x_2 \in V} \left[U\left(\phi^2(x_2, x_1^*(m)), \theta, b\right)\right] \text{ subject to } x_2 \in V \tag{2}$$

- The coordination function maximizes each player's expected utility conditional on their information, ie, $\phi^{1^*}(x_1^*(m), x_2^*(\theta, m)) \equiv \arg\max_{\phi^1} U\left(\phi^1(x_1, x_2), \theta\right)$ and $\phi^{2^*}(x_2^*(\theta, m), x_1^*(m)) \equiv \arg\max_{\phi^2} U\left(\phi^2(x_2, x_1), \theta, b\right)$

- The posterior beliefs, given by a cdf $P(\theta \mid m)$, are updated using Bayes' rule where possible, given the messaging rule $\mu^*(\theta)$

- Given beliefs and second stage actions $(x_1(m), x_2(\theta, m))$, A_2's messaging strategy maximizes expected payoff in the first stage,

$$\mu^*(\theta) \in \arg\max_{m \in M} \int_{\theta \in \Theta} U\left(\phi^2(x_2(\theta, m), x_1(m)), \theta, b\right) dP(\theta|m)$$

A PBE always exists in games with cheap talk. A babbling equilibrium in which agent A_2's message is ignored and A_1 takes an action based on the prior distribution of the state, is equivalent to a BNE of the aggregative game described in Theorem 1. Going forward, I try to identify conditions under which more informative communication equilibria emerge.

[19]In the analysis, I restrict attention to message monotonicity in that if $\theta' > \theta''$ then $\mu(\theta') \geq \mu(\theta'')$. Refer to Kartik (2009) for more on this.

Full Information Revelation

When can the two agents share information efficiently? In other words, can all A_2 truthfully reveal all private information to A_1, i.e. $\mu(\theta) = \theta$ for all $\theta \in \Theta$? To see if a fully revealing equilibrium exists, it is important to understand the incentives of the informed agent A_2. For truthful messaging to be an equilibrium, A_2 must achieve first best for every possible state θ. Since A_2 is constrained, the bounds on her action set given by $\inf V = \underline{k}$ and $\sup V = \bar{k}$ directly affects A_2's ability to achieve first best. Therefore, the domain of action set V acts as an *incentive compatibility* constraint for truth-telling. To better understand the incentives for truthful communication, I reformulate the second stage problem when the informed agent A_2's action domain is *unrestricted*.

Definition 3. ***Unconstrained actions**: Let $\bar{x}_2(\theta, m)$ be the optimal action of A_2 when i) $x_2 \in \mathbb{R}$; and ii) message m is believed by A_1 to be the true state.*

$$\bar{x}_2(\theta, m) \text{ solves } \max_{x_2 \in \mathbb{R}} U\left(\phi^2(x_2, \bar{x}_1(m)), \theta, b\right) \text{ subject to}$$
$$\bar{x}_1(m) \equiv \arg\max_{x_1 \in V} U\left(\phi^1(x_1, \bar{x}_2(\theta, m)), m\right)$$

Further, when communication is truthful ($m = \theta$), let the optimal action of players under the unconstrained problem be $\bar{x}_1(\theta)$ and $\bar{x}_2(\theta) = \bar{x}_2(\theta, \theta)$.

Assumption 5. $\underline{k} \leq \bar{x}_2(0) \leq \bar{k}$

Definition 4. *Highest type incentive compatibility (**HTIC**)*[20] *: $\bar{x}_2(1) \leq \bar{k}$*

Definition 3 does not prescribe the action of A_2 in equilibrium. Instead, $\bar{x}_2(\theta, m)$ characterizes the response of an informed agent when the message misrepresents the true state but is believed to be true by a '*naive*' A_1. Assumption 5 ensures informative equilibria to exist. *HTIC* implies that the best response of A_2 after revealing the highest state $\theta = \sup \Theta$ is within the domain of V. This enables the informed agent to achieve

[20] HTIC is not related to the *No incentive to separate* (NITS) condition proposed by Chen, Kartik, and Sobel (2008).

first best levels of the coordination function $\bar{\phi}_\theta$ under a perfectly revealing messaging strategy.[21]

Theorem 2. *A fully separating equilibrium exists if and only if HTIC condition is satisfied.*

Proof. See Appendix A.2 □

Definition 4 provides a necessary and sufficient condition for monotone separating (full revelation) equilibrium. When *HTIC* is satisfied, it implies that $\bar{x}_2(\theta) \leq \bar{k}$ for every $\theta \in \Theta$. The *HTIC* condition ensures that the action constraints are never binding for A_2 under truthful revelation. This implies the informed agent can reveal her information and achieve full efficiency. If *HTIC* is violated but full separation exists, then there are types for whom the actions are constrained by $\bar{k}$. Therefore the coordination function is strictly lower than the first best (*under-provision*). The types for whom the actions are constrained prefer to exaggerate their message and pretend to be a higher type. This incentive to exaggerate implies the fully revealing equilibrium breaks down .

Partial Information Revelation

Next, I focus on characterizing threshold equilibria when action constraints bind. The following assumption ensures an intuitive characterization of informative threshold equilibria.

Assumption 6. $\underline{k} \leq \bar{x}_2(0,1) \leq \bar{k}$

Assumption 6 is a stronger version of Assumption 5 and ensures that any exaggeration by A_2 is *feasible* in that it would result in an action by A_2 that is within the domain of V. The informed agent has an incentive to exaggerate her private information when there exists states for which truthful communication can never be credible. When *HTIC* condition fails, there exists an unique cutoff $\bar{\theta}$ such that $\bar{x}_2(\bar{\theta}) = \bar{k}$.

[21] Throughout the paper, I will refer to miscoordination of the form $\phi^i(.) > \bar{\phi}^i_\theta$ as *over-provision* and $\phi^i(.) < \bar{\phi}^i_\theta$ as *under-provision*.

Let $G = \{\theta : \bar{x}_2(\theta) > \bar{k}\}$ be the set of states for which truthful revelation causes the action constraint of A_2 to bind. Therefore, this cutoff type $\bar{\theta} = \sup\{\Theta \backslash G\}$ provides an upper bound on the extent of truthful communication. I show next that none of the messages beyond $\bar{\theta}$ are credible in any equilibria of the communication game.[22]

Lemma 1. *When HTIC is violated, all types in set G pool on the same message in every equilibrium of the communication game.*

Proof. See Appendix A.3 □

The intuition behind Lemma 1 is the following. Suppose it was possible for A_2 to partition the set G into two - $G_1 = (\bar{\theta}, \bar{\theta}_g]$ and $G_2 = (\bar{\theta}_g, \tilde{\theta}]$. Then, there are types that are pooled in the first partition for whom A_2's optimal action is constrained by the bound $\bar{k}$. For these types, A_2 would have an incentive to exaggerate and pool with the higher partition G_2, precluding informative partitions on G in equilibrium. Therefore, in the presence of constraints, two things hold: *i)* at most, there is only partial revelation of information; and *ii)* no credible information is conveyed beyond $\bar{\theta}$. The next theorem characterizes the set of all partially revealing threshold equilibria.

Theorem 3. *When HTIC is violated, there are Partially Revealing Threshold Equilibria (PRTE) such that, $\forall \theta^* \in [0, \bar{\theta}]$: $m(\theta) \in M_\theta$ if $\theta \in [0, \theta^*]$ and $m(\theta) = \bar{m} \in \bigcup\limits_{t \in (\theta^*, \tilde{\theta}]} M_t$ if $\theta \in (\theta^*, \tilde{\theta}]$. The actions of agents are given by:*

- $\forall m(\theta) \in \bigcup_{\theta \in [0,\theta^*]} M_\theta : x_1^*(m(\theta)) = \bar{x}_1(\theta), x_2^*(\theta, m(\theta)) = \bar{x}_2(\theta)$
- *If* $m(\theta) = \bar{m}$,
  - $x_1^*(\bar{m}) \equiv \arg\max\limits_{x_1 \in V} \int\limits_{\theta \in (\theta^*, \tilde{\theta}]} U\left(\phi^1(x_1, x_2^*(\theta, \bar{m})), \theta\right) dP(\theta | \bar{m})$

[22] This resembles the credibility notion of *self-signaling*, identified by Aumann (1990), and Farrell and Rabin (1996). When the unconstrained action is above the bound, it implies that the action constraints are binding, and the equilibrium actions is $x_2^*(\theta) = \bar{k}$. Given imperfect substitutability, the informed agent's action has a *positive spillover* implying that $U_1(\phi^{2^*}(\bar{k}, x_1^*(\theta)), \theta, b) > 0$. This *'positive spillover effect'* implies that communication ceases to be credible, since A_2 (strictly) prefers to induce a higher action from A_1, by inflating her private information. See Baliga and Morris (2002) for more on this point.

– $x_2^*(\theta, \bar{m}) \equiv \arg\max_{x_2 \in V} U\left(\phi^2(x_2, x_1^*(\bar{m})), \theta, b\right)$

Proof. See Appendix A.4 □

Two things stand out from Theorem 3. First, there is complete pooling above a certain cutoff state, while every message within the cutoff is truthful.[23] Second, there is multiplicity of threshold equilibria. Further, A_2 could partition the information within $[0, \bar{\theta})$ instead of revealing them truthfully. This is so because, under any PRTE, the constraints are satisfied with slack for any type in this interval. As a result, there is always a possibility to pool any type $\theta \in [0, \bar{\theta})$ with lower types within the interval such that the incentive compatibility conditions are satisfied. This gives rise to possibly multiple partitions in $[0, \bar{\theta})$.[24] The following proposition characterizes all such *monotone hybrid equilibria*.

Proposition 1. *Monotone Hybrid Equilibria (MHE): Fix a PRTE with threshold $\theta^* < \bar{\theta}$. For every such θ^* equilibrium, there exists $i \in \{1, 2, ..., N\}$ and,*

- *Types $\langle t_0 = \underset{\sim}{\theta}, t_1 = \theta_1, ...t_N = \theta^* \rangle$, actions $\langle k_1, k_2, ..., k_N \rangle$, and messages $m_i = (t_{i-1}, t_i)$, $m_{N+1} = (\theta^*, \tilde{\theta}]$ such that $\forall i \in \{1, 2, ..., N\}$,*
    - $x_1^*(m_i) = \underset{x_1 in V}{\text{argmax}} \int_{\theta \in m_i} U\left(\phi^1(x_1, x_2^*(\theta, m_i)), \theta\right) dP(\theta \mid m_i)$
    - $x_2^*(\theta, m_i) = \underset{x_2 \in V}{\text{argmax}}\, U\left(\phi^2(x_2, x_1^*(m_i)), \theta, b\right)$ *and* $x_2^*(t_i, m_i) = k_i$
    - $\forall \theta \in m_i : \phi^2(x_2^*(\theta, m_i), x_1^*(m_i)) = \bar{\phi}^2_\theta$

- *For the pooling message $m_{N+1} = (\theta^*, \tilde{\theta}]$,*
    - $x_1^*(m_{N+1}) \equiv \arg\max_{x_1 \in V} \int_{\theta \in m_{N+1}} U\left(\phi^1(x_1, x_2^*(\theta, m_{N+1})), \theta\right) dP(\theta | m_{N+1})$
    - $x_2^*(\theta, m_{N+1}) \equiv \arg\max_{x_2 \in V} U\left(\phi^2(x_2, x_1^*(m_{N+1})), \theta, b\right)$

[23] On a similar vein, Ottaviani and Squintani (2006) construct a cutoff equilibrium in which messages are revealing (albeit inflated) below the threshold, and for states above the cutoff, information transmission is partitional in nature. See also Kartik (2009) and Kartik et al. (2007).

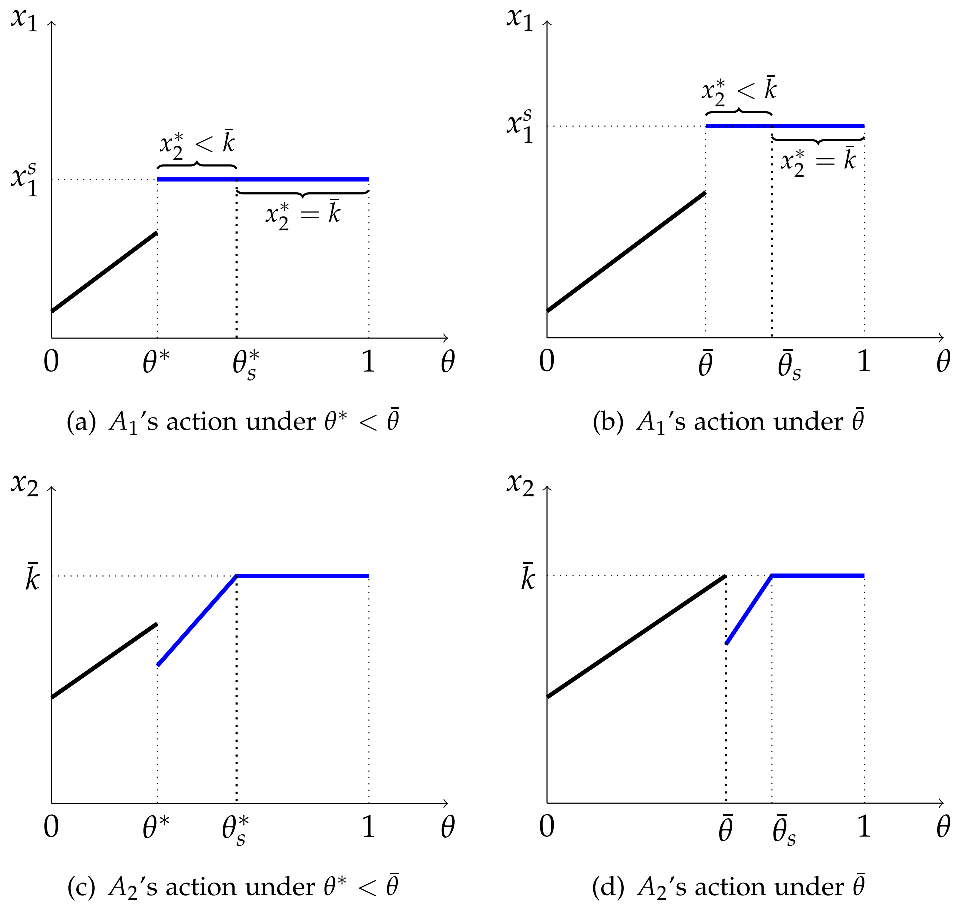
[24] Notice however that in all such equilibria, the types belonging to $G = (\bar{\theta}, \tilde{\theta}]$ are always pooled together.

Proof. Appendix A.5 □

In the hybrid equilibria, the informed agent chooses multiple message pools up to a certain threshold and pools all the information above this threshold. The novel feature of these equilibria is that they are different in structure to the classical partitional equilibria in that they are not monotonically increasing in size. The size of the message pools instead depends on the equilibrium action of the marginal type in the interval, i.e., the indifference condition is pinned down by the action of the boundary type for any two adjacent messages. Incentive compatibility dictates that the boundary type's action does not bind, resulting in first best levels of coordination function under the two adjacent messages.

Efficiency

As is the case with cheap talk models, there is multiplicity of equilibria in this setup. An important question that arises is the relationship between information thresholds and welfare of agents. To analyze this, it is important to characterize the action of agents when information is pooled. The informed agent A_2 is able to take an action after the communication stage which allows her to undo some of the inefficiencies from pooling information. The best-response of A_1 takes this into account. Specifically, A_1 chooses an action such that A_2 is unable to achieve first best levels of coordination for some types within the interval, i.e. there always exists a measure of types such that $x_2(\theta, \bar{m}) = \bar{k}$. Figure 2 illustrates this point ($\Theta \equiv [0, 1]$). Notice that there is non-monotonicity in A_2's action at θ^* because of the discontinuous jump in A_1's response upon receiving the pooling message. Since A_1's action has a discontinuity at θ^*, the informed agent A_2 readjusts her action resulting in a discontinuous downward jump at θ^*. A_1's action in equilibrium is such that there is an interval of types —$(\theta^*_s, \tilde{\theta}]$ —for which the action constraint is binding for A_2 (see parts (c) and (d) of Figure 2).



(a) A_1's action under $\theta^* < \bar{\theta}$

(b) A_1's action under $\bar{\theta}$

(c) A_2's action under $\theta^* < \bar{\theta}$

(d) A_2's action under $\bar{\theta}$

Figure 2: i) ━ *interval of separation*: $m(\theta) \in M_\theta$; ii) ━ *interval of pooling*: $\bar{m}$

Proposition 2. *The most informative equilibrium, $\theta^* = \bar{\theta}$, is ex ante efficient for both agents.*

Proof. See Appendix A.6 □

Both agents benefit from more information sharing. For A_1, higher threshold implies first best on the separating interval and lesser variance on the pooling interval. These twin effects reinforce each other in the more informative threshold equilibrium. For A_2, higher threshold implies that constraints on the action set are binding for a smaller measure of types. It also increases the action of A_1 on the pooling interval, reducing miscoordination. Both these effects provide A_2 with greater ex-ante welfare under the more informative equilibria. Figure 3 shows these trade offs. On the left, under a less informative threshold, A_1's action is lower on the pooling interval and this affects the

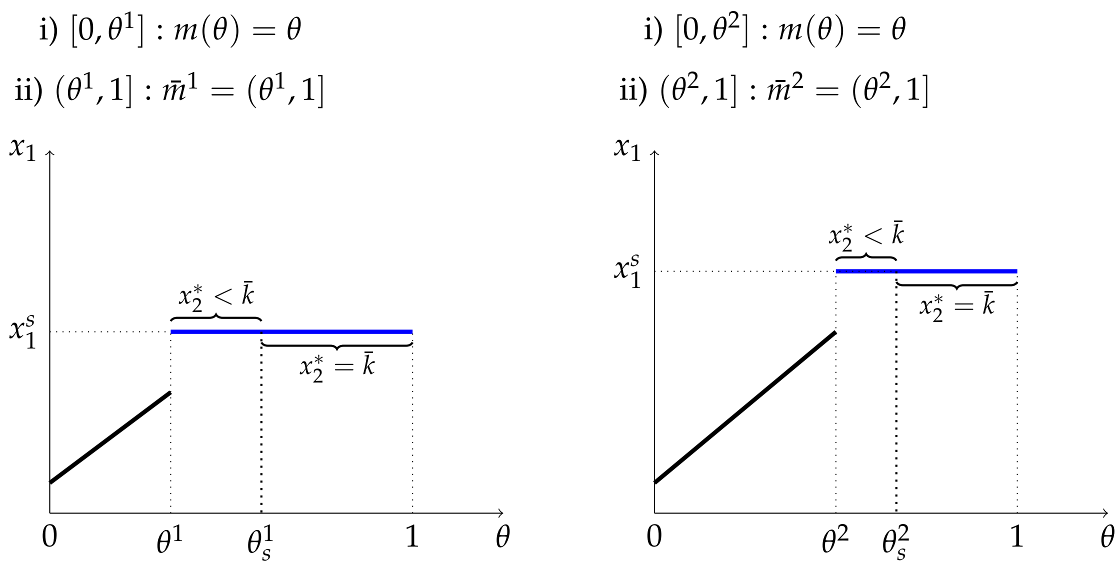


Figure 3: a) $\theta^1 < \theta^2$; b) $\theta_s^1 < \theta_s^2$; c) $x_1^s(\bar{m}^1) < x_1^s(\bar{m}^2)$

informed agent's ability to achieve first best ($\bar{\theta}_s^1 < \bar{\theta}_s^2$).

Notice that the equilibrium welfare under any *MHE* with $\theta_N = \theta^*$ is strictly dominated by a *PRTE* in which information is revealed fully up to θ^*. This is straightforward to observe. For the informed agent there is no difference in welfare between *MHE* and *PRTE*. For the uninformed agent, however, the *PRTE* is strictly better on the separating interval $[\underset{\sim}{\theta}, \theta^*]$ since there is no additional variance on this interval. In *MHE*, by contrast, A_2 pools information which increases the variance for A_1.

Corollary 1. *Given any MHE with $\theta_N = \theta^* < \bar{\theta}$, the corresponding PRTE with an information threshold θ^* is ex ante pareto dominant for both agents.*

5 Optimal Commitment

In this section, I focus on how commitment mitigates inefficiencies that arise from purely communication based decision-making. Countries in an alliance typically commit to international agreements (e.g., NATO Strategic Concept or EU PESCO agreement) that specify levels of commitment and contributions contingent on underlying information.

To characterize this, I study a one-sided *commitment mechanism* (or *commitment rule*) $(\mathcal{M}, T_1)$ under which the uninformed agent commits to a deterministic decision rule contingent on the message space, i.e. $T_1 : \mathcal{M} \to V$. The informed agent chooses a subsequent action strategically (no commitment) by best responding to the commitment rule of A_1, i.e. $T_2 : \Theta \times V \to V$. In other words, the action of A_1 is *contractible* while that of A_2 is *non-contractible*.[25]

Without loss of generality, I restrict attention to deterministic direct mechanisms $\mathbb{D} = (\Theta, T_1)$ where $\mathcal{M} = \Theta$ is the message space and the decision rule for A_1 is the mapping $T_1 : \Theta \to V$. Further, there are no contingent transfers between the agents (Alonso and Matouschek, 2008; Melumad and Shibano, 1991). This is in line with observed practices in international alliances that disallow conditional transfers between countries, but instead allow for direct contributions to the alliance. The optimal commitment rule problem for A_1 is given by the following:

$$\underset{x_1^c(\theta)\in V}{\operatorname{argmax}} \int_{\Theta} U\left(\phi^1\left(x_1^c(\theta), x_2^c(\theta, x_1^c(\theta))\right), \theta\right) dF$$

subject to:

$$\text{(NC)} \qquad x_2^c(\theta, x_1^c(\theta)) \equiv \underset{x_2\in V}{\operatorname{argmax}}\, U\left(\phi^2(x_2, x_1^c(\theta)), \theta, b\right)$$

$$\text{(IC)} \qquad \forall \theta, \theta^{'} \in \Theta :$$

$$U\left(\phi^2\left(x_2(\theta, x_1^c(\theta)), x_1^c(\theta)\right), \theta, b\right) \geq U\left(\phi^2\left(x_2(\theta, x_1^c(\theta^{'})), x_1^c(\theta^{'})\right), \theta, b\right)$$

The optimal commitment rule for the uninformed agent resembles the classical *constrained delegation* problem studied by Holmstrom (1978). The crucial point of departure is that A_1 is subject to a *non-contractibility* (NC) constraint in addition to the standard

[25] This is in stark contrast to problems of *imperfect commitment* studied by Bester and Strausz (2001) and Krishna and Morgan (2008). In Bester and Strausz (2001) for example, the uninformed principal makes two decisions, out of which only one is contractible. In a similar vein Krishna and Morgan (2008) study imperfect contracting in which the principal commits to contingent transfers but is unable to commit to an action.

incentive compatibility constraints, and commits to the rule before A_2 communicates.[26] The commitment protocol allows the uninformed agent to contribute a sequence of actions on Θ conditional on satisfying the IC and NC constraints.

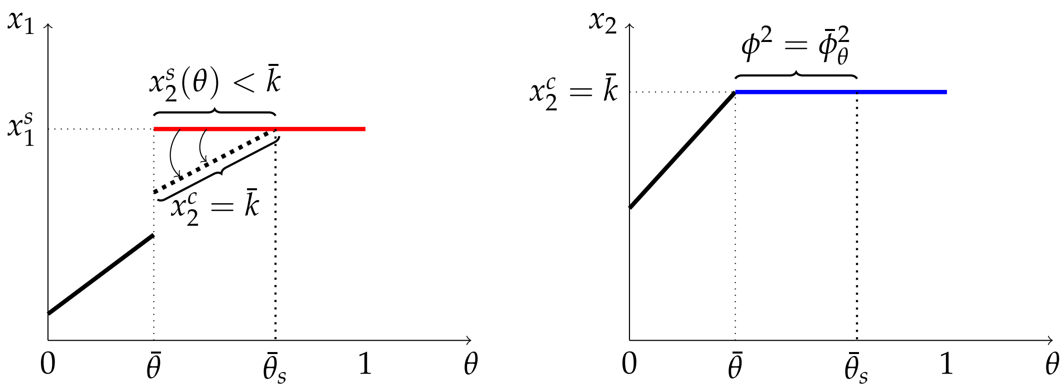
The solution to the commitment problem is akin to minimizing the miscoordination losses for the agent A_1. On the interval $[\underset{\sim}{\theta}, \bar{\theta}]$ this amounts to choosing exactly the same actions as in the simultaneous protocol, i.e. $\bar{x}_1(\theta)$. A_1 incentivizes A_2 to report truthfully and subsequently take an action $\bar{x}_2(\theta)$ such that both agents achieve first best levels of coordination, i.e. $\bar{\phi}^i_\theta$. On the pooling interval, henceforth $\bar{m}_p \equiv (\bar{\theta}, \tilde{\theta}]$, A_1's problem becomes equivalent to finding a monotone increasing sequence of actions that minimizes expected miscoordination. The following lemma provides the fundamental intuition for A_1's choice of actions on $\bar{m}_p$.

Lemma 2. *If $x^c_1(\theta)$ is strictly increasing on any interval (θ_1, θ_2) within $\bar{m}_p$, then $x^c_2(\theta, x^c_1(\theta)) = \bar{k}$ for all types in this interval.*

Proof. See Appendix A.7. □

The intuition behind Lemma 2 follows from two observations. First, in order to satisfy the NC and IC constraint of A_2, any strictly increasing sequence of actions by A_1 on $\bar{m}_p$ must provide informational rents to A_2. The informational rent ensures first best levels of the coordination function to the informed agent (i.e., $\phi^2 = \bar{\phi}^2_\theta$). Second, the choice of a decision rule must simultaneously reduce the extent of losses from miscoordination for A_1. Lemma 2 argues that among several decision rules that satisfy A_2's IC constraint, the one that minimizes A_1's miscoordination loss is the one where A_2 contributes $x^c_2 = \bar{k}$. If not, A_1 could decrease her actions until A_2's action is equal to $\bar{k}$, and improve her expected payoff.

[26] There are no participation constraints in the commitment problem. However, including a participation constraint such that $\int_\Theta U\left(\phi^2\left(x_2(\theta, x^c_1(\theta)), x^c_1(\theta)\right), \theta, b\right) dF \geq \tilde{u}$ would not affect the solution to the commitment problem as long as $\tilde{u}$ is within reasonable thresholds. For example, if $\tilde{u}$ is less than the expected payoff from the simultaneous protocol, this would satisfy agent A_2's participation constraint (see Proposition 6).



(a) A_1's action (b) A_2's action

Figure 4: A_1 can commit to an action that is strictly lower than x_1^s on the interval $(\bar{\theta}, \bar{\theta}_s)$. Notice this is possible since A_2 can always increase her action to $\bar{k}$ and still achieve first best $\bar{\phi}_\theta^2$.

Figure 4 illustrates this argument. Consider for example the action of A_1 on the pooling interval under the simultaneous protocol. Instead of taking an action $x_1^s(\bar{m}_p)$ on $(\bar{\theta}, \bar{\theta}_s)$, A_1 can pivot down and commits to a sequence of actions such that $x_1^c(\theta) < x_1^s(\bar{m}_p)$. Further, the sequence of actions can be chosen such that A_2 best responds with an action equal to $\bar{k}$ (see Figure 4(b)). A_2 achieves first best levels $\bar{\phi}_\theta^2$ while for A_1 the miscoordination is lower compared to the simultaneous protocol.

The presence of a NC constraint overturns some of the established results in the delegation literature.[27] Specifically, given NC, any rule that takes the same action over the entire pooing interval cannot be optimal for A_1. The intuition is that A_2 can readjust her actions and achieve first best which results in over-provision and miscoordination for A_1. The extent of miscoordination is exacerbated by the fact that $x_2^c(.) < \bar{k}$ for some of the types. From Lemma 2, this cannot be an optimal rule. By similar reasoning, multiple pools, multiple separating regions, or a separating interval following a pooling interval can also be ruled out. The following claims establish these key results (the broad intuition is detailed below and the formal proofs are confined to the appendix).

[27] See e.g. Ambrus and Egorov (2017), Alonso and Matouschek (2008) and Melumad and Shibano (1991). In these papers typically the decision rule involves multiple pooling intervals followed by or interspersed with separating regions.

Claim 1. *On the interval $\bar{m}_p$ there is no single flat segment such that $\forall \theta \in \bar{m}_p : x_1^c(\theta) = z \geq \bar{x}_1(\bar{\theta})$.*

Claim 2. *On the interval $\bar{m}_p$ there cannot exist more than one pooling (flat) segments. That is, $\mathbb{I} = \{z_i : \exists \theta_i, \theta_i' \in \bar{m}_p$ and $\forall \theta \in (\theta_i, \theta_i')$ s.t $x_1^c(\theta) = z_i\}$ and $z_j \neq z_l$ such that $\#\mathbb{I} > 1$ cannot be optimal for A_1.*

Suppose $x_1^c(\theta) = \bar{x}_1(\bar{\theta})$. Then $\forall \theta \in \bar{m}_p : x_2(\theta) = \bar{k}$. This cannot be optimal since A_1 can always do better by committing a bit more and satisfying A_2's NC/IC constraints. Instead, suppose $x_1^c(\theta) = z > \bar{x}_1(\bar{\theta})$. Say, for the sake of argument that $z = x_1^s$, i.e. A_1 mimics the action under simultaneous protocol. This again cannot be optimal since agent A_2's action is less than $\bar{k}$ on the interval $(\bar{\theta}, \bar{\theta}_s)$. A_1 can instead always commit to lesser and induce A_2 to contribute $\bar{k}$. Given the findings from Lemma 2 this increases the expected payoff to agent A_1 by minimizing miscoordination on $\bar{m}_p$. For similar reasons, multiple decision pools are infeasible due to the NC constraint. If suppose, for sake of argument there were two different pools with actions z_1 and z_2 respectively (wlog $z_1 < z_2$). Then either the agent on the z_1 pooling region achieves first best or she deviates to the higher pooling region. In the former case it implies $x_2^c < \bar{k}$ which is inefficient for the same reasons as argued earlier. In the latter case, jumping to the higher pool violates IC and the decision rule cannot be efficient for A_1 as a result.

Claim 3. *On $\bar{m}_p$, there cannot be a flat segment followed by a strictly increasing interval.*

Claim 4. *There cannot be discontinuous strictly increasing separating intervals on $\bar{m}_p$.*

Claim 5. *There cannot be a fully separating decision rule on $\bar{m}_p$, i.e., $\forall \theta', \theta'' \in \bar{m}_p : x_1^c(\theta') \neq x_1^c(\theta'')$.*

Claim 3 follows from noting that on a flat segment either A_2's IC is satisfied for all types in that interval or there is inefficiency for some types. If it is the former, then A_1 can improve its payoff (see Claim 1-2) and extracting $\bar{k}$ from A_2. If it is the latter, on the other hand, there exists types that do not achieve first best on the pooling region which means they can always deviate to the (strictly increasing) separating interval and benefit from higher actions of A_1, thereby violating IC constraint for truth-telling. Multiple separating segments are also inefficient because they create a discontinuous jump in the actions of A_1. This implies at the point where there is a jump, the informed agent's action falls below $\bar{k}$. This cannot be an optimal decision rule (see Lemma 2). Finally, Claim 5 rules out fully aligned contracts that provide first best to A_2 on $\bar{m}_p$. Clearly this would entail over-provision for A_1 which implies $U_1 < 0$ on the whole interval.[28]

The consequence of Claim 1-Claim 5 is that the optimal commitment rule for agent A_1 has an intuitively simple structure on the interval $\bar{m}_p$. It involves only a single separating interval up to a threshold $\bar{\theta}_c$ followed by a pooling action $x_1^c(\bar{\theta}_c)$ on the rest of the interval, as in Figure 5. Therefore the commitment rule problem reduces to choosing this optimal cutoff $\bar{\theta}_c$ given the NC and IC constraints. The following proposition provides the necessary characterization.

Proposition 3. *The optimal commitment mechanism for A_1 is given by the following:*

1. $\forall \theta \in [\underset{\sim}{\theta}, \bar{\theta}] : x_1^c(\theta) = \bar{x}_1(\theta)$

2. $\exists \bar{\theta}_c \in \bar{m}_p$ *that solves,*

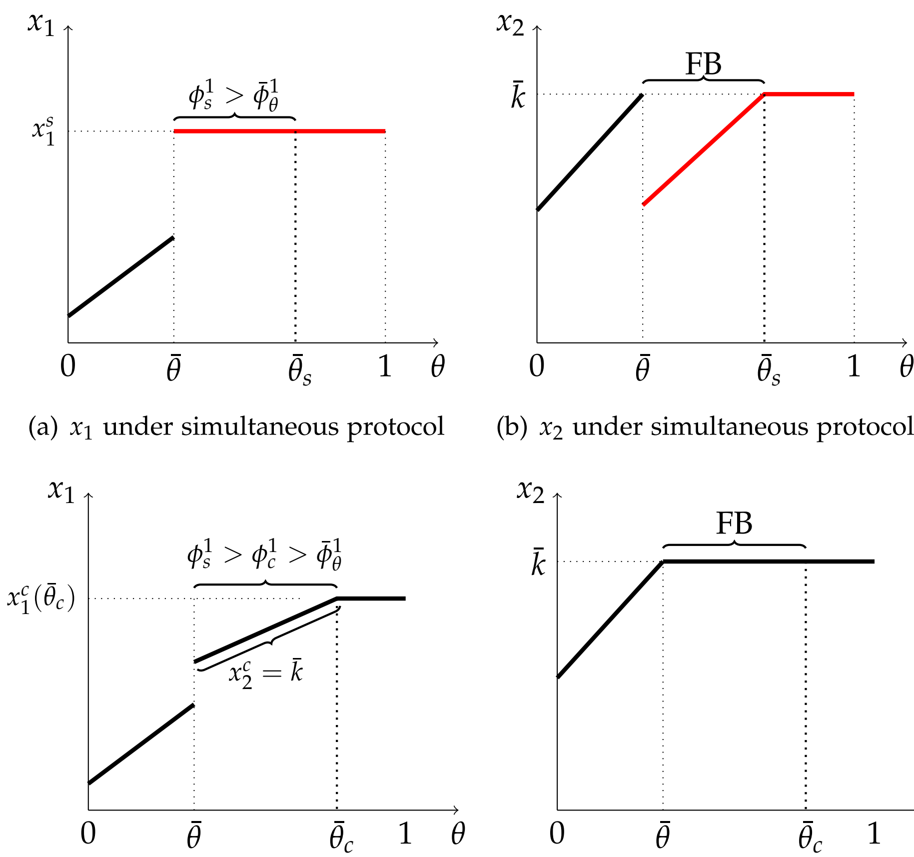
$$\bar{\theta}_c \equiv \underset{t \in \bar{m}_p}{\operatorname{argmax}} \int_{\bar{\theta}}^{t} U\left(\phi^1\left(x_1^c(\theta), \bar{k}\right), \theta\right) dF + \int_{t}^{\tilde{\theta}} U\left(\phi^1\left(x_1^c(t), \bar{k}\right), \theta\right) dF$$

$$\textit{such that } \forall \theta \in (\bar{\theta}, \bar{\theta}_c] : x_1^c(\theta) \equiv \underset{x_1 \in V}{\operatorname{argmax}}\, U\left(\phi^2(\bar{k}, x_1), \theta, b\right), \phi^2\left(\bar{k}, x_1^c(\bar{\theta}_c)\right) = \bar{\phi}^2_{\bar{\theta}_c}$$

3. $\forall \theta \in (\bar{\theta}_c, \tilde{\theta}] : x_1^c(\theta) = x_1^c(\bar{\theta}_c)$

Proof. See Appendix A.8 □

[28]This is similar to arguments in Krishna and Morgan (2008) that rule out fully aligned contracts that are optimal to the agent for all types of her private information. See Corollary 1 for more.



(a) x_1 under simultaneous protocol (b) x_2 under simultaneous protocol

(c) x_1 under commitment rule (d) x_2 under commitment rule

Figure 5: Under the ex ante commitment, A_1 can pivot down on $\bar{m}_p$ (see 5(a) and 5(c)). This induces $\bar{k}$ from A_2 (see 5(b) and 5(d)) under commitment.

The optimal decision rule *i)* mimics the simultaneous protocol on $[\underset{\sim}{\theta}, \bar{\theta}]$; *ii)* strictly increases in the region $(\bar{\theta}, \bar{\theta}_c]$ keeping A_2's action pegged at $\bar{k}$; and *iii)* is invariant on the rest of the interval. Two important properties of the optimal commitment rule becomes clear from Proposition 3. First, the commitment rule *neutralizes* the NC constraint by extracting the *maximal action* from A_2 on the interval $\bar{m}_p$. Since actions are imperfect substitutes, this decision rule also minimizes miscoordination for A_1. Second, the rule entails *capping of actions* beyond the threshold $\bar{\theta}_c$. As a result the optimal rule is *discontinuous* at exactly $\bar{\theta}$ and nowhere else. By pegging A_2's action at $\bar{k}$ on the interval $\bar{m}_p$, A_1 minimizes the informational inefficiencies that were present in simultaneous decision-making.

Proposition 4. *The optimal commitment rule improves ex ante welfare of both agents compared to the simultaneous protocol.*

Proof. See Appendix A.9 □

The interesting conclusion of Proposition 4 is that both agents benefit from commitment. While it is well understood that commitment improves the uninformed player's welfare (Glazer and Rubinstein, 2008), a number of papers on delegation find that the welfare effects of delegation is *ambiguous* and depends crucially on the extent to which the players' interests are *aligned* (see, e.g., Alonso and Matouschek, 2008; Dessein, 2002). Proposition 4 suggests that irrespective of the alignment of interests and the extent of action constraints, commitment is always pareto improving for both agents.

The rationale for this result follows from noting that thresholds are ordered such that $\bar{\theta}_s < \bar{\theta}_c$. The intuition for this is the following. Since A_1 induces A_2 to take the *maximal* action $\bar{k}$ on the interval $\bar{m}_p$ (Figure 5(d)), the marginal utility for A_1 is strictly increasing at $\bar{\theta}_s$ under commitment. This directly translates into higher welfare for A_1 under commitment. For A_2 this means the cap on actions with commitment is also higher compared to the simultaneous protocol. That is A_2 achieves first best up to $\bar{\theta}_c$ and on the remaining interval of types, the miscoordination from under-provision is lower under commitment compared to the simultaneous protocol.

6 Uniform-Quadratic Example

Reconsider the example examined in Section 2. As in the example let the utility functions of the two agents be a quadratic loss function of the following form,

$$U^1 = -\left[\left(\frac{x_1 + \eta x_2}{1+\eta}\right) - \theta\right]^2$$

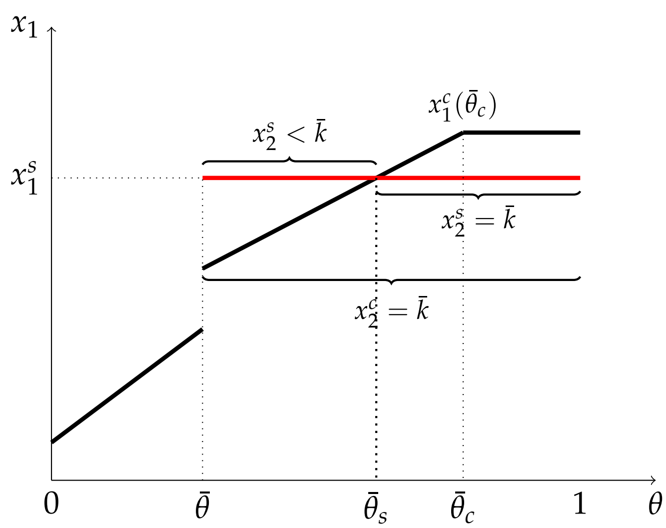


Figure 6: The optimal mechanism exhibits two key features on the interval $\bar{m}_p$: *i)* there is *maximal* action from A_2, i.e. $x_2^c = \bar{k}$; *ii)* A_1 places a *cap* on actions given by $x_1^c(\bar{\theta}_c)$.

$$U^2 = -\left[\left(\frac{x_2 + \eta x_1}{1+\eta}\right) - \theta - b\right]^2$$

In accordance with the analysis of the previous sections let $\phi^1(x_1, x_2) = \left(\frac{x_1+\eta x_2}{1+\eta}\right)$ and $\phi^2(x_2, x_1) = \left(\frac{x_2+\eta x_1}{1+\eta}\right)$ be the respective joint coordination functions of the two agents. The parameter $\eta \in (0, 1)$ measures the extent of interdependence between the two agents' actions. Let the action set of the two players be $V = [0, \bar{k}]$, where the lower bound is normalized, $\underline{k} = 0$.[29] As in Section 2, the state is uniformly distributed ($\theta \in \mathcal{U}[0, 1]$) and perfectly observed only by A_2. All the subsequent results are represented as functions of the exogenous variables $(\eta, b, \bar{k})$. This way, the analysis of the uniform quadratic setting leads to interesting comparative statics with respect to these parameters. All the analysis and proofs are relegated to Appendix B.

[29]This is without loss of generality since the lower bound does not affect the incentives of the informed player in the model.

7 An Application to International Alliances: Discussion

The theoretical results capture the trade-offs involved when there are incentives for coordination between multiple decision-makers, and decisions are substitutable. This is true when, for example, countries within an alliance work together to achieve common objectives. The theory presented in the paper offers important insights for decision-making in alliances.

Why contribute to alliances? Implications for information sharing and efficiency

One of the main insights of the analysis is that the action set affects information revelation and efficiency of ex-ante outcomes. The intuition is straightforward. A greater upper bound increases the truthful communication threshold of the informed agent (*information effect*) and this in turn increases efficiency (Proposition 2). For the uninformed agent too, a greater upper bound is critical. For example if the optimal expected action is above the available set of actions, then the informed agent's actions are constrained when information is pooled. The inability to take an optimal action leads to inefficiency. The informed agent also suffers miscoordination for a greater measure of types as a result. This decreases welfare of both agents.

In the context of alliances, the need for contributing resources cannot be understated. For example, US Presidents have long advocated for greater partnership and contributions from EU countries. Bush in 2006, Obama in 2014, and more recently Trump in 2018 have all pushed for a greater share of resource contributions from NATO's European allies.[30] The NATO, since 2006, has agreed to and adopted a resolution for implementing the "*two per cent guideline*" for member nations in which each ally would contribute 2 percent of GDP (proportional contributions) for NATO's collective defense initiatives.

[30] See *CNBC* press article on July 11, 2018, for more on this topic.

The fact that European members of NATO have not contributed sufficiently to the joint defense budget has been well documented (Petersson, 2015).[31] This paper provides an informational and coordination rationale for more contributions. Having *skin in the game* is pareto improving for two reasons. First, it improves information revelation between members. Second, it reduces miscoordination that arises due to lack of sufficient resources.

Why commitment in alliances? Implications for binding agreements

The optimal commitment mechanism characterized in Proposition 3 has both normative and positive implications. Specifically, the mechanism provides a framework for how binding agreements could be negotiated between parties seeking to work together in an alliance. The two main features of the optimal mechanism are *i)* maximal action by informed agent beyond a threshold of information, and *ii)* capping of actions by the uninformed agent. The latter is indirectly observed through commitment clauses proscribed in alliance agreements that specify *rules of thumb* for members' defense spending (e.g. 2 percent of GDP in NATO or 2.5-3 percent of GDP under PESCO). Such *rules of thumb* clauses place an implicit commitment on countries to contribute without free-riding on others, and at the same time also provide an upper bound (cap) on the levels of spending they are obligated to undertake. All countries barring the US contribute lesser or equal to the proscribed amount in the case of NATO, for example.[32]

The first feature (*maximal actions*) makes a normative point on the structure of ex-ante agreements. One possible way to interpret the maximal action clause is the requirement for hawkish (greater bias) countries to contribute the highest possible resources when the information goes beyond threshold. The optimal mechanism, according to Proposition 3,

[31] Petersson (2015) writes, quoting from Obama's "West Point speech" on May 28, 2014, "We cant have a situation in which the United States is consistently spending over 3 percent of our GDP on defense," and Europe is spending 1 percent. "The gap becomes too large," he continued, and the alliance needed to make sure that everybody was doing their fair share."

[32] See Wittmann (2009) for an extensive analysis of NATO's *Strategic Concept* agreement.

must be such that more biased members contribute their full capacity to an operation in order to minimize miscoordination. In the war on terror over the last two decades, for example, the US regularly collected intelligence information about potential threats and possible opportunities to attack terrorist targets. Since US is more biased, an agreement where the US contributes all its resources while the other members contribute only the residual required is the optimal mechanism from an ex-ante perspective.

The role of commitment in minimizing miscoordination is another central result of the paper. Proposition 4 argues that there are welfare gains for the informed agent from commitment. This is important since it provides a rationale for participating in such commitment contracts. Specifically, if the ex-ante reservation utility (outside options) for the informed agent without commitment is the expected payoff from the simultaneous protocol, then the informed agent would always prefer the commitment protocol.[33] The commitment protocol therefore provides an utilitarian rationale for binding agreements between countries.

Commitment minimizes miscoordination losses – from under-provision for the informed agent and over-provision for the uninformed – and leads to a welfare improvement. An important feature is that, unlike delegation, the commitment protocol allows for the informed party to make decisions at the interim stage. This form of non-contractibility is particularly appealing when studying international alliances. The Permanent Structured Cooperation (PESCO) agreement, for example, enables this form of decision-making authority to participating countries. The PESCO agreement from 2018 reads, and I quote, "The aim is to jointly develop defence capabilities and make them available for EU military operations....The difference between PESCO and other forms of cooperation is the legally binding nature of the commitments undertaken by the participating Member States. The decision to participate was made voluntarily by each partici-

[33]Notice, however, that at the interim stage the participation constraints are stronger and may not be satisfied. See Forges et al. (2016) and Forges and Horst (2018) for more on communication contracts with interim participation constraints.

pating Member State, and decision-making will remain in the hands of the participating Member States in the Council." The commitment protocol presented in Section 5 captures this form of *autonomy* in decision-making via the inclusion of a non-contractibility constraint on the informed agent's action.

Another important finding is that the value of commitment increases as the agents' interests diverge. Managing conflict of interest is an important component of alliances. In the Iraq war of 2003, the US-Britain alliance faced diverging interests that affected the effectiveness of their joint military exercise. Sir John Chilcott's *Iraq Inquiry* commissioned in 2009 and published in 2016 notes that divergence in interests between US and Britain post-invasion resulted in major hurdles for efficient cooperation between the troops. They found that decision-makers in UK were myopic from an early stage of the war in Iraq and focused attention of troop withdrawal, while the US military establishment had a more longer-horizon view, and oriented their policies towards long-term interests.[34] While such contingencies are difficult to anticipate and commit to ex ante, what the results of Proposition 4 imply is that the gains from committing to binding agreements is higher when the potential for divergence is greater. This is counter-intuitive since countries that diverge in their goals tend not to commit in the first place. Commitment to work together is therefore harder to achieve. The analysis of my paper shows that the value of commitment is higher precisely when parties diverge more.

[34]In Section 9.8, p 23 of the report, Sir Chilcott records the observation by US's Gen Jackson: "The perception, right or wrong, in some – if not all – US military circles is that the UK is motivated more by the short-term political gain of early withdrawal than by the long-term importance of mission accomplishment." Also see https://webarchive.nationalarchives.gov.uk/20171123122743/http://www.iraqinquiry.org.uk/the-report/ for more on the Iraq Inquiry report.

8 Extensions

Lying costs

The equilibrium in both protocols exhibits some level of lying by the informed agent. Experimental evidence suggests that there is an intrinsic propensity to say the truth even when the information conveyed is *soft* (Gneezy, 2005; Hurkens and Kartik, 2009), suggesting an aversion to lying. In international alliances, misrepresentation of information could lead to distrust in diplomatic relations and reputational losses, especially when it is possible to learn about the true state of the world ex-post.

Introducing lying costs (Kartik, 2009) changes the incentives of the informed agent drastically. Suppose, for sake of exposition, lying costs are minimized when the messages are truthful (i.e. $\mu(\theta) = \theta$). Then, the presence of lying costs eliminates all but the most informative equilibrium under both simultaneous and sequential protocols. The intuition is that there is now a lying cost associated with wrongful reporting for no marginal benefit in utility. A_2 incurs a *wasteful* lying cost by exaggerating, or pooling with the other types and sending $\bar{m}_p$. This implies that there is an unique separating equilibrium on $[0, \bar{\theta}]$ such that $\mu(\theta) = \theta$ and the multiplicity problem associated with the baseline model disappears.

What is left to consider is the equilibrium messaging on the pooling interval, $\bar{m}_p = (\bar{\theta}, \tilde{\theta}]$. One way to interpret my results is by considering them as the limit case of a game with lying costs. As the intensity of lying costs goes to zero, the equilibrium messaging is truthful on $[0, \bar{\theta}]$ and all other types send the message $\bar{m}_p$. Specifically, when the intensity of lying is very small, there is an incentive to (almost) costlessly exaggerate beyond $\bar{\theta}$, resulting in no further information transmission. On the other hand, when the lying costs are sufficiently high, there is full separation as the incentives to exaggerate are counteracted by the lying costs.

The interesting case is when the lying costs are sufficiently high but not prohibitively so. It is then possible for alternate equilibrium messaging strategies to emerge. For example, agent A_2 could *bunch* state space and send the same (possibly inflated) message for every type in this partition, resulting in clustering of A_2's private information on the interval $\bar{m}_p$.[35]

Verifiable Information Disclosure

So far, the analysis has focused mainly on transmission of *soft information*. In many projects the nature of information is verifiable (Grossman (1981); Milgrom (1981)). The informed agent can disclose verifiable information about project quality, for example. Alternatively, the project contract might specify evidence provision as a requirement. When information can be verified, the incentives for communication change completely. There is *unraveling* in the sense that A_2 would always find it optimal to reveal every state truthfully, leading to full information disclosure even in the presence of action constraints. This is straightforward to observe. On the pooling interval, the highest state $\tilde{\theta}$ is better off disclosing the true type due to the *positive spillover* effect. By an induction argument, the same holds for types to the left of the highest type. This way, in the limit it follows that every type in $\bar{m}_p$ would find it optimal to reveal her type truthfully resulting in full disclosure.

9 Conclusion

The paper investigates the role of communication and commitment when there are (one-sided) information asymmetries between agents. When agents' decisions are substitutable and they face action constraints, under simultaneous decision-making, there is

[35] Chen (2011) finds clustering and inflated messaging in a completely different setup. In Chen's work, there is a small prior probability that an informed sender is honest (always reports truthfully) and the uninformed receiver is naive (always believes the message). This leads to message inflation and clustering at the top end of the message spectrum.

only partial information revelation in equilibrium. There is a positive relationship between amount of information revealed and efficiency, in that welfare of both agents are strictly increasing in the extent of information shared.

The paper considers the case where an uninformed agent has commitment power and the informed agent's action is non-contractible. With one-sided commitment, the uninformed agent commits to an optimal mechanism that minimizes the miscoordination losses up to a threshold and caps her actions beyond this threshold. The optimal commitment mechanism increases the ex-ante expected payoff of both agents compared to the simultaneous protocol. The value of commitment is increasing in the divergence of interests between agents and decreasing when the action constraints are lesser.

There are other potential incentive problems associated with the presence of constraints that are worth exploring. For example, with two sided incomplete information constraints might exacerbate the communication barriers between agents. As information is more dispersed the inefficiencies emerging from action constraints might worsen. Alternatively, when players' actions are strategic complements, the presence of constraints might still affect the credibility of information transmission. Another possible avenue for future research is to endogenize the investment in the action set. Though constraints were assumed to be exogenous in this paper, it is plausible for agents to invest in an action set ex-ante at some marginal cost. Since domain of the action set determines the extent of information revealed, this investment decision might differ according to what the underlying decision-making protocol is. All these scenarios require a more detailed analysis, and are left for future work.

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A Appendix

A.1 Proof of Theorem 1

In order to establish the equilibrium result some additional notations are required. I will define them in a manner similar to Acemoglu and Jensen (2013), except that they are augmented to incorporate the additional features of the underlying Bayesian game. Let the aggregator for any $\theta \in \Theta$ be defined as $Q(\theta) = H(h_1(x_1) + h_2(x_2(\theta)))$ where $Q : \Theta \to S \subset \mathbb{R}$ and $Q(\Theta) = \{Q(\theta) : \theta \in \Theta\} \in 2^S$. Notice that the action of uninformed A_1 is unaffected by θ and is taken in expectation, in any BNE. Since the aggregator is additively separable in the actions of agents the best-responses of any agent can be written purely as a function of the other agents action. Let $X_1 = h_1(x_1)$, $X_2(\theta) = h_2(x_2(\theta))$, and $X_2(\Theta) = \{X_2(\theta) : \theta \in \Theta\}$ for expositional purposes. The reduced best-reply correspondence for the two agents can be written as,

$$R_1(X_2(\Theta)) = \arg\max_{x_1 \in V} \int_{\theta \in \Theta} U\left(\tilde{\phi}^1(x_1, H(h_1(x_1) + X_2(\theta))), \theta\right) dF$$

$$R_2(X_1, \theta) = \arg\max_{x_2 \in V} U\left(\tilde{\phi}^2(x_2, H(X_1 + h_2(x_2))), \theta, b\right)$$

As before, I define $R_2(X_1, \Theta) = \{R_2(X_1, \theta) : \theta \in \Theta\}$ to be the collection of best responses for every type of private information of A_2. Given the R_i's, it is possible to define precisely the *backward response correspondence* $B_1 : 2^S \to 2^V \cup \emptyset$ and $B_2 : S \times \Theta \to 2^V \cup \emptyset$ for the two agents respectively.

$$B_1(Q(\Theta)) = \left\{x_1 \in V : x_1 \in R_1\left(\{H^{-1}(Q(\theta)) - h_1(x_1)\}_{\theta \in \Theta}\right)\right\}$$

$$B_2(Q(\theta), \theta) = \left\{x_2 \in V : x_2 \in R_2\left(H^{-1}(Q(\theta)) - h_2(x_2(\theta)), \theta\right)\right\}$$

In a similar vein, the *aggregate backwards response correspondence* $Z : 2^S \times \Theta \to 2^S \cup \emptyset$ is,

$$Z(Q(\Theta), \theta) \equiv \{\psi(x_1, x_2(\theta)) \in S : x_1 \in B_1(Q(\Theta)) \text{ and } x_2(\theta) \in B_2(Q(\theta), \theta)\}$$

$$\mathcal{Z}(Q(\Theta)) = \left\{ Z(Q(\Theta), \theta) \in 2^S : \theta \in \Theta \right\}$$

The aggregate correspondence $\mathcal{Z}(Q(\Theta))$ captures the relationship between the set of action pairs $X(\theta) = (x_1, x_2(\theta))$ and the equilibrium aggregates $Q(\theta)$. The equilibrium of the Bayesian Aggregative game is simply the fixed point of the correspondence $\mathcal{Z}$, i.e. $Q(\Theta) \in \mathcal{Z}(Q(\Theta))$. Rewriting the (expected) payoffs of the agents in terms of the aggregator function,

$$\Pi_1(x_1, Q(\Theta)) = \int_{\theta \in \Theta} U\left(\tilde{\phi}^1(x_1, Q(\theta)), \theta\right) dF$$

$$\Pi_2(x_2, Q(\theta)|\theta) = U\left(\tilde{\phi}^2(x_2, Q(\theta)), \theta, b\right)$$

$$\Pi_2(x_2, Q(\Theta)|\Theta) = \{\Pi(x_2, Q(\theta)|\theta) \in \mathbb{R} : \theta \in \Theta\}$$

The FOC can be then expressed as follows:

$$\frac{d\Pi_1(x_1, Q(\Theta))}{dx_1} = \int_{\theta \in \Theta} U_1\left(\tilde{\phi}^1(x_1, Q(\theta)), \theta\right) \left[\tilde{\phi}^1_1(x_1, Q(\theta)) + \tilde{\phi}^1_2(x_1, Q(\theta)) H'(H^{-1}(Q)) \frac{dh_1}{dx_1}\right] dF$$

$$\frac{d\Pi_2(x_2, Q(\theta)|\theta)}{dx_2} = U_1\left(\tilde{\phi}^2(x_2, Q(\theta)), \theta, b\right) \left[\tilde{\phi}^2_1(x_2, Q(\theta)) + \tilde{\phi}^2_2(x_2, Q(\theta)) H'(H^{-1}(Q)) \frac{dh_2}{dx_2}\right]$$

From both the expressions, it is clear that,

$$\left[\tilde{\phi}_1^i(x_i, Q(\theta)) + \tilde{\phi}_2^i(x_i, Q(\theta))H'(H^{-1}(Q))\frac{dh_i}{dx_i}\right] > 0$$

The functions $U(.)$ and $\psi(.)$ are twice continuously differentiable and the action set V is compact and closed. The last expression above therefore follows directly from the assumptions on the functional forms of U and ψ. Further by construction the best-reply correspondences $R_1(X_2(\Theta))$ and $R_2(X_1, \theta)$ exhibit *decreasing selection* in $X_2(\Theta)$ and X_1 respectively since the game is generalized aggregative and actions are strategic substitutes. The best-reply correspondences are therefore upper hemi-continuous and convex valued. The aggregate $Q(\theta)$ is upper semi-continuous implying that the backward response correspondences (B_i) are upper hemi-continuous. Since the backward response correspondence is upper hemi-continuous and the state space is closed and compact, it follows that for any convergent sequences $Q^t(\Theta) \to Q(\Theta)$ and $(Q^t(\Theta), \theta^t) \to (Q(\Theta), \theta)$ it must hold that if $x_1^t \in B_1(Q^t(\Theta))$ and $x_2^t(\theta^t) \in B_2(Q^t(\Theta), \theta^t)$, then given that B_1 and B_2 have a closed graph (since R_1 and R_2 have closed graphs), $x_1 \in B_1(Q(\Theta))$ and $x_2(\theta) \in B_2(Q(\Theta), \theta)$. The set valued aggregate correspondence $Z : 2^S \times \Theta \to 2^S \cup \emptyset$ is therefore upper hemi-continuous. Applying Kakutani's fixed point theorem it is straightforward that the set valued mapping Z has a fixed point (see e.g. Corollary 1 in Jensen (2010)).

Existence implies that there is a collection of $\{(x_1, x_2(\theta))\}_{\theta \in \Theta}$ such that the FOC of the two players are satisfied. This implies that boundary actions $x_1, x_2(\theta) \in \{\underline{k}, \bar{k}\}$ may be optimal for the agents. To see this, take the case of A_1. For a local maxima, it must be that $\frac{d\Pi_1(x_1, Q(\Theta))}{dx_1} = 0$. Since $\left[\tilde{\phi}_1^1(x_1, Q(\theta)) + \tilde{\phi}_2^1(x_1, Q(\theta))H'(H^{-1}(Q))\frac{dh_1}{dx_1}\right] > 0$, if U_1 is everywhere negative for $x_1 = \inf V = \underline{k}$, then it must be that $\frac{d\Pi_1(x_1, Q(\Theta))}{dx_1} < 0$. However, since the agent is constrained, the boundary strategy is the optimal one. Specifically, A_1

may find it optimal to play $\underline{k}$ and it may be that $\left.\frac{d\Pi_1(x_1,Q(\Theta))}{dx_1}\right|_{x_1=\underline{k}} \leq 0$. For any interior strategy $x_1 \in (\underline{k}, \bar{k})$ to be an equilibrium best-response for A_1 the FOC has to hold implying $\Lambda_1(x_1, Q(\Theta)) = \frac{d\Pi_1(x_1,Q(\Theta))}{dx_1} = 0$. Given the concavity assumption on U, it follows that the *uniform local solvability*[36] condition holds satisfying the sufficient condition for a local maxima.

To prove uniqueness, I will first construct the appropriate equilibrium actions and then show that it must be unique. I will consider equilibrium actions for A_1 that are interior, i.e. $x_1 \in (\underline{k}, \bar{k})$. As stated before, informed agent A_2 plays an action for every type of her private information. Given that $b > 0$ and single crossing condition $U_{13} > 0$, the FOC for A_2 implies either $U_1 = 0$ ($\tilde{\phi}^2_1(x_2(\theta), Q(\theta)) = \bar{\phi}^2_\theta$) at $x_2(\theta)$, or $U_1 > 0$ and $x_2(\theta) = \bar{k}$. Let the complete information action for the two players, for any $\theta \in \Theta$, be $(\tilde{x}_1(\theta), \tilde{x}_2(\theta))$ such that $\tilde{\phi}^i(\tilde{x}_i(\theta), \psi(\tilde{x})) \equiv \text{argmax}_{\tilde{\phi}^i} U(\tilde{\phi}^i, \theta, b_i)$ for both agents. Further, let $\inf \Theta = \underset{\sim}{\theta}$ and $\sup \Theta = \tilde{\theta}$. Given this, the following lemma provides a structure to the profile of actions in equilibrium.

Lemma 3. *The equilibrium (interior) action x^e_1 of A_1 must be such that $\exists \bar{\theta}_e \in (\underset{\sim}{\theta}, \tilde{\theta})$ such that $x^e_1 = \tilde{x}_1(\bar{\theta}_e)$.*

Proof. The proof requires looking at the two extreme cases, i.e. $\theta \in \{\underset{\sim}{\theta}, \tilde{\theta}\}$.

Case 1. $x^e_1 > \tilde{x}_1(\tilde{\theta})$

Since the functions H and ψ are both increasing in x_i, it must follow that $x_2(\tilde{\theta}) < \tilde{x}_2(\tilde{\theta})$. Let $x^b_1 = \tilde{x}_1(\tilde{\theta}) + \Delta_1(\tilde{\theta})$ and $x_2(\tilde{\theta}) = \tilde{x}_2(\tilde{\theta}) - \Delta_2(\tilde{\theta})$ where both $\Delta_1 > 0$ and $\Delta_2 > 0$. A_2 is maximizing her utility which implies that $\tilde{\phi}^2\left(x_2(\tilde{\theta}), \psi(x^b_1, x_2(\tilde{\theta}))\right) = \bar{\phi}^2_{\tilde{\theta}}$.
Consider the following equality, $\tilde{\phi}^1\left(\tilde{x}_1(\tilde{\theta}), \psi(\tilde{x}_1(\tilde{\theta}), \tilde{x}_2(\tilde{\theta}))\right) = \bar{\phi}^1_{\tilde{\theta}}$. Applying total differ-

[36] Uniform local solvability implies that when if $\Lambda_1(x_1, Q(\Theta)) = 0 \implies \frac{d\Lambda_1(x_1,Q(\Theta))}{dx_1} < 0$ for all $x_1 \in V$ and $Q(\Theta) \in 2^S$. See Definition 8 in Acemoglu and Jensen (2013) for more.

entiation to the LHS of this expression,

$$d\tilde{\phi}^1 = \frac{\partial \tilde{\phi}^1}{\partial x_1} dx_1 + \frac{\partial \tilde{\phi}^1}{\partial \psi} d\psi = \frac{\partial \tilde{\phi}^1}{\partial x_1} \Delta_1(\tilde{\theta}) + \frac{\partial \tilde{\phi}^1}{\partial \psi} d\psi$$

Similarly,

$$d\tilde{\phi}^2 = \frac{\partial \tilde{\phi}^2}{\partial x_2} dx_2 + \frac{\partial \tilde{\phi}^2}{\partial \psi} d\psi = -\frac{\partial \tilde{\phi}^2}{\partial x_2} \Delta_2(\tilde{\theta}) + \frac{\partial \tilde{\phi}^2}{\partial \psi} d\psi = 0$$

$$\implies d\psi = \frac{\frac{\partial \tilde{\phi}^2}{\partial x_2}}{\frac{\partial \tilde{\phi}^2}{\partial \psi}} . \Delta_2(\tilde{\theta})$$

Substituting it into the expression for $d\tilde{\phi}^1$,

$$d\tilde{\phi}^1 = \frac{\partial \tilde{\phi}^1}{\partial x_1} \Delta_1(\tilde{\theta}) + \frac{\partial \tilde{\phi}^2}{\partial x_2} \Delta_2(\tilde{\theta}) > 0$$

This implies that $\tilde{\phi}^1(.) > \bar{\phi}^1_{\tilde{\theta}}$ and $U_1 < 0$ for A_1 when action is above $\tilde{x}_1(\tilde{\theta})$. From continuity, this is true for any generic $\Delta_1(\theta) > 0$ and $\Delta_2(\theta) > 0$ such that $\theta < \tilde{\theta}$. A similar argument holds for the case when $x_1^e < \tilde{x}_1(\underset{\sim}{\theta})$.

Case 2. $x_1^e < \tilde{x}_1(\underset{\sim}{\theta})$

It follows that in this case $x_2(\underset{\sim}{\theta}) > \tilde{x}_2(\underset{\sim}{\theta})$. Let $x_1^e = \tilde{x}_1(\underset{\sim}{\theta}) - \Delta_1(\underset{\sim}{\theta})$ and $x_2(\underset{\sim}{\theta}) = \tilde{x}_2(\underset{\sim}{\theta}) + \Delta_2(\underset{\sim}{\theta})$ where both $\Delta_1 > 0$ and $\Delta_2 > 0$. As previously, $\tilde{\phi}^2\left(x_2(\underset{\sim}{\theta}), \psi(x_1^e, x_2(\underset{\sim}{\theta}))\right) = \bar{\phi}^2_{\underset{\sim}{\theta}}$.
Clearly,

$$d\tilde{\phi}^1 = -\frac{\partial \tilde{\phi}^1}{\partial x_1} \Delta_1(\tilde{\theta}) - \frac{\partial \tilde{\phi}^2}{\partial x_2} \Delta_2(\tilde{\theta}) < 0$$

This implies that $\tilde{\phi}^1(.) < \bar{\phi}^1_{\underset{\sim}{\theta}}$ and $U_1 > 0$ for A_1 when action is below $\tilde{x}_1(\underset{\sim}{\theta})$. Since the best responses of the agents are upper hemi-continuous, applying intermediate value theorem (Bolzano's theorem) implies that there must be a $\bar{\theta}_e \in [\underset{\sim}{\theta}, \tilde{\theta}]$ and $x_1^e \in [\tilde{x}_1(\underset{\sim}{\theta}), \tilde{x}_1(\tilde{\theta})]$ such that $x_1^e = \tilde{x}_1(\bar{\theta}_e)$. □

Consider two such equilibrium profile of actions $\mathcal{E}_1 = \left\{x_1^{e_1}, (x_2^1(\theta))_{\theta \in \Theta})\right\}$ and $\mathcal{E}_2 =$

$\left\{x_1^{e_2}, (x_2^2(\theta))_{\theta\in\Theta}\right\}$ such that (wlog) $x_1^{e_1} < x_1^{e_2}$. I claim the following.

CLAIM. $\bar{\theta}_e^1 < \bar{\theta}_e^2$

Proof. Since the functions ψ and h_i are increasing in the actions of the agents and $\tilde{\phi}^i$ satisfies decreasing selection, it follows that $x_1^{e_1} < x_1^{e_2} \iff x_2^1(\theta) \geq x_2^2(\theta)$ for all $\theta \in \Theta$ and there exists an interval $[\underset{\sim}{\theta}, \theta')$ such that $x_2^1(\theta) > x_2^2(\theta)$ for all $\theta \in [\underset{\sim}{\theta}, \theta')$ (follows from continuity). Further, if for any type θ it is true that $x_2^1(\theta) = x_2^2(\theta)$, then it must be such that $x_2^1(\theta) = x_2^2(\theta) = \bar{k}$ and $\tilde{\phi}^2(x_2^i(\theta), \psi(x^i(\theta))) < \bar{\phi}_\theta^2$.
There are two cases possible. First, $x_2^1(\bar{\theta}_e^1) < \bar{k}$ and $x_2^2(\bar{\theta}_e^2) = \bar{k}$ in which case it is straightforward to observe that from the increasing property of the functions $\bar{\theta}_e^1 < \bar{\theta}_e^2$. In the second case, $x_2^1(\bar{\theta}_e^1) < x_2^2(\bar{\theta}_e^2) < \bar{k}$. In this case it still holds that $\tilde{\phi}^1\left(x_1^{e_1}, \psi(x_1^{e_1}, x_2^1(\bar{\theta}_e^1))\right) = \bar{\phi}^1_{(\bar{\theta}_e^1)}$ and $\tilde{\phi}^1\left(x_1^{e_2}, \psi(x_1^{e_2}, x_2^2(\bar{\theta}_e^2))\right) = \bar{\phi}^1_{(\bar{\theta}_e^2)}$. However $\bar{\phi}^1_{(\bar{\theta}_e^1)} < \bar{\phi}^1_{(\bar{\theta}_e^2)}$ and single crossing condition $U_{12} > 0$ further implies that $\bar{\theta}_e^1 < \bar{\theta}_e^2$. □

Split the interval $[\underset{\sim}{\theta}, \tilde{\theta}]$ into three intervals, $\left\{[\underset{\sim}{\theta}, \bar{\theta}_e^1), [\bar{\theta}_e^1, \bar{\theta}_e^2), [\bar{\theta}_e^2, \tilde{\theta}]\right\}$. Define $\tilde{\phi}^1\left(x_1^{e_1}, \theta\right) \equiv \tilde{\phi}^1\left(x_1^{e_1}, \psi(x_1^{e_1}, x_2^1(\theta))\right)$, $\tilde{\phi}^1\left(x_1^{e_2}, \theta\right) \equiv \tilde{\phi}^1\left(x_1^{e_2}, \psi(x_1^{e_2}, x_2^2(\theta))\right)$, and d_θ as the following:

$$d_\theta = \tilde{\phi}^1\left(x_1^{e_2}, \theta\right) - \tilde{\phi}^1\left(x_1^{e_1}, \theta\right)$$

Consider the three sets separately.

Case. $\theta \in [\underset{\sim}{\theta}, \bar{\theta}_e^1)$

From the above arguments, $x_1^{e_1} < x_1^{e_2} < \tilde{x}_1(\underset{\sim}{\theta})$ and $x_2^2(\underset{\sim}{\theta}) < x_2^1(\underset{\sim}{\theta}) < \tilde{x}_2(\underset{\sim}{\theta})$. Rewriting the above actions as $x_1^{e_i} = \tilde{x}_1(\underset{\sim}{\theta}) + \Delta_1^i(\underset{\sim}{\theta})$ and $x_2^i(\underset{\sim}{\theta}) = \tilde{x}_2(\underset{\sim}{\theta}) - \Delta_2^i(\underset{\sim}{\theta})$ where $i = \{1,2\}$. It is immediately clear that $\Delta_1^1(\underset{\sim}{\theta}) < \Delta_1^2(\underset{\sim}{\theta})$ and $\Delta_2^1(\underset{\sim}{\theta}) < \Delta_2^2(\underset{\sim}{\theta})$. From Lemma 3, $\tilde{\phi}^1\left(x_1^{e_2}, \underset{\sim}{\theta}\right) > \tilde{\phi}^1\left(x_1^{e_1}, \underset{\sim}{\theta}\right) > \bar{\phi}^1_{\underset{\sim}{\theta}} \implies d_{\underset{\sim}{\theta}} > 0$. Since $\tilde{\phi}^1\left(x_1^{e_1}, \theta\right)$ and $\tilde{\phi}^1\left(x_1^{e_2}, \theta\right)$ are both continuous functions in x_2, it is straightforward to see that $\Delta_1^i(\theta) = x_1^{e_i} - \tilde{x}_1(\theta)$ and $\Delta_2^i(\theta) = \tilde{x}_2(\theta) - x_2^i(\theta)$ are such that the sequences $\left(\Delta_1^1(\theta)\right)^n \to \Delta_1^1(\bar{\theta}_e^1) = 0$, $\left(\Delta_1^2(\theta)\right)^n \to \Delta_1^2(\bar{\theta}_e^1) > 0$,

$\left(\Delta_2^1(\theta)\right)^n \to \Delta_2^1(\bar{\theta}_e^1) = 0$, and $\left(\Delta_2^2(\theta)\right)^n \to \Delta_2^2(\bar{\theta}_e^1) > 0$. On the interval $[\underset{\sim}{\theta}, \bar{\theta}_e^1)$ take the sequence $(d_\theta)^n$. Given that the sequences $\left(\Delta_1^i(\theta)\right)^n$ and $\left(\Delta_2^i(\theta)\right)^n$ are convergent, it follows that the sequences $\left(\left(\tilde{\phi}^1\left(x_1^{e_1}, \theta\right)\right)^n, \left(\tilde{\phi}^1\left(x_1^{e_2}, \theta\right)\right)^n\right)$ are such that $\left(\tilde{\phi}^1\left(x_1^{e_1}, \theta\right)\right)^n \to \tilde{\phi}^1\left(x_1^{e_1}, \bar{\theta}_e^1\right)$ and $\left(\tilde{\phi}^1\left(x_1^{e_2}, \theta\right)\right)^n \to \tilde{\phi}^1\left(x_1^{e_2}, \bar{\theta}_e^1\right)$. Since $\tilde{\phi}^1\left(x_1^{e_1}, \bar{\theta}_e^1\right) = \bar{\phi}^1_{\bar{\theta}_e^1}$, it follows that $d_{\bar{\theta}_e^1} = \tilde{\phi}^1\left(x_1^{e_2}, \bar{\theta}_e^1\right) - \bar{\phi}^1_{\bar{\theta}_e^1} > 0$. Therefore, the sequence $(d_\theta)^n$ on the compact set $S \subset \mathbb{R}$ converges *pointwise* such that $(d_\theta)^n \to d_{\bar{\theta}_e^1}$. Finally, $\tilde{\phi}^1\left(x_1^{e_2}, \theta\right) > \tilde{\phi}^1\left(x_1^{e_1}, \theta\right) > \bar{\phi}^1_\theta$ on this interval, $U_1 < 0$ for A_1. Since $U_{11} < 0$, this further implies that,

$$U_1\left(\tilde{\phi}^1(x_1^{e_2}, \theta), \theta\right) - U_1\left(\tilde{\phi}^1(x_1^{e_1}, \theta), \theta\right) < 0 \quad \text{for all} \quad \theta \in [\underset{\sim}{\theta}, \bar{\theta}_e^1) \tag{3}$$

Case. $\theta \in [\bar{\theta}_e^1, \bar{\theta}_e^2)$

As before, $\tilde{x}_1(\bar{\theta}_e^1) = x_1^{e_1} < x_1^{e_2}$ and $x_2^2(\bar{\theta}_e^1) < x_2^1(\bar{\theta}_e^1) = \tilde{x}_2(\bar{\theta}_e^1)$. Rewriting the actions as $x_1^{e_1} = \tilde{x}_1(\theta) - \Delta_1^1(\theta)$, $x_1^{e_2} = \tilde{x}_1(\theta) + \Delta_1^2(\theta)$, $x_2^1(\theta) = \tilde{x}_2(\theta) + \Delta_2^1(\theta)$, and $x_2^2(\theta) = \tilde{x}_2(\theta) - \Delta_2^2(\theta)$, it immediately follows that $\Delta_1^1(\bar{\theta}_e^1) = \Delta_2^1(\bar{\theta}_e^1) = 0$ and $\Delta_1^2(\bar{\theta}_e^2) = \Delta_2^2(\bar{\theta}_e^2) = 0$. Again from the previous case, we know that $\tilde{\phi}^1\left(x_1^{e_2}, \bar{\theta}_e^1\right) > \tilde{\phi}^1\left(x_1^{e_1}, \bar{\theta}_e^1\right) = \bar{\phi}^1_{\bar{\theta}_e^1} \implies d_{\bar{\theta}_e^1} > 0$.

We can then rewrite $(\Delta_1^i(\theta), \Delta_2^i(\theta))$ as the following: $\Delta_1^1(\theta) = \tilde{x}_1(\theta) - x_1^{e_1}$, $\Delta_1^2(\theta) = x_1^{e_2} - \tilde{x}_1(\theta)$, $\Delta_2^1(\theta) = x_2^1(\theta) - \tilde{x}_2(\theta)$, and $\Delta_2^2(\theta) = \tilde{x}_2(\theta) - x_2^2(\theta)$ such that the sequences $\left(\Delta_1^1(\theta)\right)^n \to \Delta_1^1(\bar{\theta}_e^2) > 0$, $\left(\Delta_1^2(\theta)\right)^n \to \Delta_1^2(\bar{\theta}_e^2) = 0$, $\left(\Delta_2^1(\theta)\right)^n \to \Delta_2^1(\bar{\theta}_e^2) > 0$, and $\left(\Delta_2^2(\theta)\right)^n \to \Delta_2^2(\bar{\theta}_e^2) = 0$. Given that the sequences $\left(\Delta_1^i(\theta)\right)^n$ and $\left(\Delta_2^i(\theta)\right)^n$ are convergent, it follows that the sequences $\left(\left(\tilde{\phi}^1\left(x_1^{e_1}, \theta\right)\right)^n, \left(\tilde{\phi}^1\left(x_1^{e_2}, \theta\right)\right)^n\right)$ are such that $\left(\tilde{\phi}^1\left(x_1^{e_1}, \theta\right)\right)^n \to \tilde{\phi}^1\left(x_1^{e_1}, \bar{\theta}_e^2\right)$ and $\left(\tilde{\phi}^1\left(x_1^{e_2}, \theta\right)\right)^n \to \tilde{\phi}^1\left(x_1^{e_2}, \bar{\theta}_e^2\right)$. From Lemma 3, $\bar{\phi}^1_{\bar{\theta}_e^2} = \tilde{\phi}^1\left(x_1^{e_2}, \bar{\theta}_e^2\right) > \tilde{\phi}^1\left(x_1^{e_1}, \bar{\theta}_e^2\right) \implies d_{\bar{\theta}_e^2} > 0$. On the interval $[\bar{\theta}_e^1, \bar{\theta}_e^2)$, consider the sequence $(d_\theta)^n$. It follows that since $d_{\bar{\theta}_e^1} = \tilde{\phi}^1\left(x_1^{e_2}, \bar{\theta}_e^1\right) - \bar{\phi}^1_{\bar{\theta}_e^1} > 0$ and $d_{\bar{\theta}_e^2} = \bar{\phi}^1_{\bar{\theta}_e^2} - \tilde{\phi}^1\left(x_1^{e_1}, \bar{\theta}_e^2\right) > 0$, the sequence $(d_\theta)^n$ converges *pointwise* such that $(d_\theta)^n \to d_{\bar{\theta}_e^2}$. Finally, $\tilde{\phi}^1\left(x_1^{e_2}, \theta\right) > \bar{\phi}^1_\theta > \tilde{\phi}^1\left(x_1^{e_1}, \theta\right)$ on this interval meaning $U_1 > 0$ under the action $x_1^{e_1}$ and $U_1 < 0$ under $x_1^{e_2}$ for agent A_1. This trivially implies that,

$$U_1\left(\tilde{\phi}^1(x_1^{e_2},\theta),\theta\right) - U_1\left(\tilde{\phi}^1(x_1^{e_1},\theta),\theta\right) < 0 \quad \text{for all} \quad \theta \in [\bar{\theta}_e^1, \bar{\theta}_e^2) \tag{4}$$

Case. $\theta \in [\bar{\theta}_e^2, \tilde{\theta}]$

In this case, we can rewrite the equilibrium actions as $x_1^{e_i} = \tilde{x}_1(\theta) - \Delta_1^i(\theta)$, and $x_2^i(\theta) = \tilde{x}_2(\theta) + \Delta_2^i(\theta)$. At $\theta = \bar{\theta}_e^2$, the initial conditions on the Δ_i's are $\Delta_1^1(\bar{\theta}_e^2) > 0$, $\Delta_1^2(\bar{\theta}_e^2) = 0$, $\Delta_2^1(\bar{\theta}_e^2) > 0$ and $\Delta_2^2(\bar{\theta}_e^2) = 0$. We can then rewrite $(\Delta_1^i(\theta), \Delta_2^i(\theta))$ as the following: $\Delta_1^i(\theta) = \tilde{x}_1(\theta) - x_1^{e_i}$ and $\Delta_2^i(\theta) = x_2^i(\theta) - \tilde{x}_2(\theta)$ such that the sequences $\left(\Delta_1^i(\theta)\right)^n \to \Delta_1^i(\tilde{\theta}) > 0$ and $\left(\Delta_2^i(\theta)\right)^n \to \Delta_2^i(\tilde{\theta}) > 0$. However, we also know that $\Delta_1^1(\theta) \geq \Delta_1^2(\theta)$ and $\Delta_2^1(\theta) \geq \Delta_2^2(\theta)$ on this interval. Starting from $d_{\bar{\theta}_e^2} = \bar{\phi}^1_{\bar{\theta}_e^2} - \tilde{\phi}^1\left(x_1^{e_1}, \bar{\theta}_e^2\right) > 0$, the sequences $\left(\Delta_1^i(\theta)\right)^n$ and $\left(\Delta_2^i(\theta)\right)^n$ are convergent implying $\left(\Delta_1^i(\theta)\right)^n \to \Delta_1^i(\tilde{\theta})$ and $\left(\Delta_2^i(\theta)\right)^n \to \Delta_2^i(\tilde{\theta})$. Therefore, it follows directly from the preceding observation that the sequences $\left(\left(\tilde{\phi}^1\left(x_1^{e_1},\theta\right)\right)^n, \left(\tilde{\phi}^1\left(x_1^{e_2},\theta\right)\right)^n\right)$ are such that,

$$\left(\tilde{\phi}^1\left(x_1^{e_1},\theta\right)\right)^n \to \tilde{\phi}^1\left(x_1^{e_1},\tilde{\theta}\right) \qquad \left(\tilde{\phi}^1\left(x_1^{e_2},\theta\right)\right)^n \to \tilde{\phi}^1\left(x_1^{e_2},\tilde{\theta}\right)$$

Given that there is miscoordination on this interval, from Lemma 3 this miscoordination is higher when the action x_1 is smaller, i.e. $\bar{\phi}^1_{\tilde{\theta}} > \tilde{\phi}^1\left(x_1^{e_2},\tilde{\theta}\right) > \tilde{\phi}^1\left(x_1^{e_1},\tilde{\theta}\right)$.
Finally, as before, on the interval $[\bar{\theta}_e^2, \tilde{\theta}]$ consider the sequence $(d_\theta)^n$. It follows that since $d_{\bar{\theta}_e^2} = \bar{\phi}^1_{\bar{\theta}_e^2} - \tilde{\phi}^1\left(x_1^{e_1},\bar{\theta}_e^2\right) > 0$ and $d_{\tilde{\theta}} = \tilde{\phi}^1\left(x_1^{e_2},\tilde{\theta}\right) - \tilde{\phi}^1\left(x_1^{e_1},\tilde{\theta}\right) > 0$, the sequence $(d_\theta)^n$ converges *pointwise* such that $(d_\theta)^n \to d_{\bar{\theta}_e^2}$. Since $\bar{\phi}^1_\theta > \tilde{\phi}^1\left(x_1^{e_2},\theta\right) > \tilde{\phi}^1\left(x_1^{e_1},\theta\right)$ on this interval meaning $U_1 > 0$ under both actions $x_1^{e_1}$ and $x_1^{e_2}$, and $U_{11} < 0$, it follows that,

$$U_1\left(\tilde{\phi}^1(x_1^{e_2},\theta),\theta\right) - U_1\left(\tilde{\phi}^1(x_1^{e_1},\theta),\theta\right) < 0 \quad \text{for all} \quad \theta \in [\bar{\theta}_e^2, \tilde{\theta}] \tag{5}$$

To conclude the proof for uniqueness, I re-examine the FOC for a maxima for agent A_1

given by the equation,

$$\Lambda_1(x_1, Q(\Theta)) = \int\limits_{\theta\in\Theta} U_1\left(\tilde{\phi}^1(x_1, Q(\theta)), \theta\right)\left[\tilde{\phi}^1_1(x_1, Q(\theta)) + \tilde{\phi}^1_2(x_1, Q(\theta))H'(H^{-1}(Q))\frac{dh_1}{dx_1}\right] dF \qquad (6)$$

Rewriting the terms in the integral as,

$$\tilde{U}_1(x_1^{e_i}, \theta) = U_1\left(\tilde{\phi}^1(x_1^{e_i}, Q(\theta)), \theta\right) =$$

$$\Upsilon(x_1^{e_i}, \theta) = \tilde{\phi}^1_1(x_1^{e_i}, Q^{e_i}(\theta)) + \tilde{\phi}^1_2(x_1^{e_i}, Q^{e_i}(\theta))H'(H^{-1}(Q))\left.\frac{dh_1}{dx_1}\right|_{x_1=x_1^{e_i}}$$

Given equations 3-5 and the concavity properties of the coordination functions $\tilde{\phi}^i$, $Q(.)$ and (H, h),

$$\tilde{U}_1(x_1^{e_2}, \theta) = \tilde{U}_1(x_1^{e_1}, \theta) - \delta(\theta) \qquad \text{such that } \delta(\theta) > 0$$

$$\Upsilon(x_1^{e_2}, \theta) = \Upsilon(x_1^{e_1}, \theta) - v(\theta) \qquad \text{such that } v(\theta) > 0$$

Therefore if $x_1^{e_1}$ is indeed an equilibrium,

$$\int\limits_{\theta\in\Theta} \tilde{U}_1(x_1^{e_1}, \theta).\Upsilon(x_1^{e_1}, \theta)dF = 0$$

Similarly, if $x_1^{e_2}$ is also an equilibrium,

$$\int\limits_{\theta\in\Theta} \left(\tilde{U}_1(x_1^{e_1}, \theta) - \delta(\theta)\right).\left(\Upsilon(x_1^{e_1}, \theta) - v(\theta)\right) dF < 0$$

This is valid for any two equilibria that satisfy the conditions for a local maxima. Therefore the equilibrium actions are unique. **QED**

A.2 Proof of Theorem 2

Sufficiency

Suppose HTIC condition was satisfied. Then the following statements hold true under truthful messaging $m(\theta) \in M_\theta$ by A_2:

$$\bar{x}_1(\theta) = x_1^*(\theta) \equiv \underset{x_1 \in V}{\operatorname{argmax}}\, U\left(\phi^1(x_1, x_2^*(\theta)), \theta\right) \quad \text{for all } \theta \in \Theta \setminus \tilde{\theta}$$

$$\bar{x}_2(\theta) = x_2^*(\theta) \equiv \underset{x_2 \in V}{\operatorname{argmax}}\, U\left(\phi^2(x_2, x_1^*(\theta)), \theta, b\right) \quad \text{for all } \theta \in \Theta \setminus \tilde{\theta}$$

$$\phi^2(\bar{x}_2(\theta), \bar{x}_1(\theta)) = \phi^2(x_2^*(\theta), x_1^*(\theta)) = \bar{\phi}_\theta^2 \quad \text{for all } \theta \in \Theta \setminus \tilde{\theta}$$

The first equivalence is due to the uniqueness of best responses and the fact that $x_1^*(\theta) < \bar{k}$ for $\theta \in \Theta \setminus \tilde{\theta}$ by single crossing property of the utility function ($U_{12} > 0$). The second follows immediately from HTIC and the first condition being satisfied. The last equality trivially holds true. This ensures there is no inefficiency and A_2 always achieves first best levels of coordination for all θ. Hence, there exists an equilibrium in which there is *full separation* and information is completely revealed.

Necessity

Suppose there is full separation but HTIC is violated, i.e. $\bar{x}_2(\tilde{\theta}) > \bar{k}$.

CLAIM. $\exists \bar{\theta} < \tilde{\theta} : \bar{x}_2(\bar{\theta}) = x_2^*(\bar{\theta}) = \bar{k}$

Proof. *We know that for any two $\theta_1, \theta_2 \in \Theta$ such that $\theta_1 \neq \theta_2$, $\bar{x}_2(\theta_1) \neq \bar{x}_2(\theta_2)$. This follows from continuity and single crossing property of the utility functions. Since $\bar{x}_2(\underset{\sim}{\theta}) > \underline{k}$ and $\bar{x}_2(\tilde{\theta}) > \bar{k}$, and the fact that the sequence $(\bar{x}_2(\theta))^n \to \bar{x}_2(\underset{\sim}{\theta})$ (pointwise convergent), implies there must exist a subsequence $(\bar{x}_2(\theta))^m$, $m < n$ converging to $\bar{k}$. This immediately implies $(\bar{x}_2(\theta))^m \to \bar{k} = \bar{x}_2(\bar{\theta})$.*

Consider the set of types $\theta \in (\bar{\theta}, \tilde{\theta}]$. Since $\bar{x}_2(\theta) > \bar{k}$, it directly follows that the equilibrium actions are such that $x_2^*(\theta) = \bar{k}$. This further means that,

$$x_1^*(\theta) \equiv \underset{x_1 \in V}{\text{argmax}}\, U\left(\phi^1(x_1, \bar{k}), \theta\right) > \bar{x}_1(\theta)$$

To construct the appropriate sequences, let the equilibrium actions $x_i^*(\theta)$ be written as,

$$x_1^*(\theta) = \bar{x}_1(\theta) + \kappa_1(\theta)$$

$$x_2^*(\theta) = \bar{x}_2(\theta) - \kappa_2(\theta)$$

Notice that A_1's equilibrium actions are readjusted according to $\bar{x}_1(\theta)$ while A_2's actions are indexed to $x_2(\theta)$. This immediately implies that $\kappa_1(\theta) > 0$ and $\kappa_2(\theta) > 0$ are both increasing in θ. The readjusted action for A_1 solves the following,

$$x_1^*(\theta) \equiv \underset{x_1 \in V}{\text{argmax}}\, U\left(\phi^1(x_1, \bar{k}), \theta\right) \quad \text{for all } \theta \in (\bar{\theta}, \tilde{\theta}]$$

This is equivalent to choosing a x_1 such that $\phi^1(x_1, \bar{k}) = \bar{\phi}_\theta^1$. That is, the uninformed agent anticipates that A_2's action is constrained by $\bar{k}$ and best responds to that. Since A_1 achieves $\bar{\phi}_\theta^1$ irrespective of whether A_2's action is $\bar{x}_2(\theta)$ or $x_2^*(\theta) = \bar{k}$, total differentiation of the ϕ^i's at any $\theta \in (\bar{\theta}, \tilde{\theta}]$ yields the following:

$$\begin{aligned} d\phi^1 &= \frac{\partial \phi^1}{\partial x_1} dx_1 + \frac{\partial \phi^1}{\partial x_2} dx_2 \\ d\phi^1 &= \frac{\partial \phi^1}{\partial x_1} \kappa_1(\theta) - \frac{\partial \phi^1}{\partial x_2} \kappa_2(\theta) = 0 \end{aligned}$$

$$\implies \kappa_1(\theta) = \frac{\frac{\partial \phi^1}{\partial x_2}}{\frac{\partial \phi^1}{\partial x_1}} . \kappa_2(\theta)$$

Since $\frac{\partial \phi^1}{\partial x_2} < \frac{\partial \phi^1}{\partial x_1}$, it directly implies that $\kappa_1(\theta) < \kappa_2(\theta)$. Totally differentiating ϕ^2 gives,

$$\begin{aligned} d\phi^2 &= \frac{\partial \phi^2}{\partial x_2} dx_2 + \frac{\partial \phi^2}{\partial x_1} dx_1 \\ d\phi^2 &= -\frac{\partial \phi^2}{\partial x_2} \kappa_2(\theta) + \frac{\partial \phi^2}{\partial x_1} \kappa_1(\theta) \end{aligned}$$

Again the fact that $\kappa_1(\theta) < \kappa_2(\theta)$ and $\frac{\partial \phi^2}{\partial x_1} < \frac{\partial \phi^2}{\partial x_2}$ implies that $d\phi^2 < 0$ when the actions are constrained, i.e. $\phi^2(\bar{k}, x_1^*(\theta)) < \bar{\phi}_\theta^2$ on this interval.

$$\phi^2(\bar{k}, \bar{x}_1(\theta)) < \phi^2(\bar{k}, x_1^*(\theta)) < \phi^2(\bar{x}_2(\theta), \bar{x}_1(\theta)) = \bar{\phi}_\theta^2$$

The first inequality is due to Assumption 3 and the second follows from the above arguments. Given these profile of actions under full separation, all we need to show is a profitable deviation for any $\theta in (\bar{\theta}, \tilde{\theta}]$. Suppose, wlog, θ pretends to be a higher type $\theta + \epsilon$ such that,

$$x_1^*(\theta + \epsilon) \equiv \underset{x_1 \in V}{\text{argmax}}\, U\left(\phi^1(x_1, \bar{k}), \theta\right)$$

From earlier arguments, we know that $\phi^1(x_1^*(\theta + \epsilon), \bar{k}) = \bar{\phi}^1_{\theta+\epsilon}$. Further, $x_1^*(\theta + \epsilon) = \bar{x}_1(\theta + \epsilon) + \kappa_1(\theta + \epsilon) > x_1^*(\theta)$. The deviation action for A_2 with private information θ but pretends to be $\theta + \epsilon$, given by $x_2^d(\theta, \theta + \epsilon)$ is,

$$x_2^d(\theta, \theta + \epsilon) \equiv \underset{x_2 \in \mathbb{R}}{\text{argmax}}\, U\left(\phi^2(x_2, x_1^*(\theta + \epsilon)), \theta, b\right)$$

Notice that the optimal deviation action is unconstrained ($x_2 \in \mathbb{R}$) similar to $\bar{x}_1(\theta)$. There are two possible cases for the optimal deviation action. I consider both of them below.

Case. $x_2^d(\theta, \theta + \epsilon) \leq \bar{k} \implies x_2^*(\theta, \theta + \epsilon) = x_2^d(\theta, \theta + \epsilon)$

Trivially this implies $\bar{\phi}_\theta^2 = \phi^2\left(x_2^*(\theta, \theta + \epsilon), x_1^*(\theta + \epsilon)\right) > \phi^2\left(\bar{k}, x_1^*(\theta)\right)$. Therefore the deviation is profitable.

Case. $x_2^d(\theta,\theta+\epsilon) > \bar{k} \implies x_2^*(\theta,\theta+\epsilon) = \bar{k}$

In this case, clearly the following inequalities hold:

$$\bar{\phi}_\theta^2 = \phi^2\left(x_2^d(\theta,\theta+\epsilon), x_1^*(\theta+\epsilon)\right) > \phi^2\left(\bar{k}, x_1^*(\theta+\epsilon)\right) > \phi^2\left(\bar{k}, x_1^*(\theta)\right) = \phi^2\left(\bar{k}, \bar{x}_1(\theta)+\kappa_1(\theta)\right)$$

The first inequality is obviously true. The second follows from the positive spillover assumption and the fact that $x_1^*(\theta+\epsilon) > x_1^*(\theta)$. Therefore, by deviating from full separation and sending a message $m(\theta) \in M_{\theta+\epsilon}$, the informed agent is able to reduce the miscoordination as $\phi^2\left(\bar{k}, x_1^*(\theta+\epsilon)\right)$ is closer to $\bar{\phi}_\theta^2$. Since U_1 is increasing, this implies that,

$$U\left(\phi^2\left(\bar{k}, x_1^*(\theta+\epsilon)\right), \theta, b\right) > U\left(\phi^2\left(\bar{k}, x_1^*(\theta)\right), \theta, b\right)$$

Therefore under full separation and when *HTIC* condition does not hold, there are always types for whom there is a profitable deviation, precluding separation. This completes the proof. **QED**

A.3 Proof of Lemma 1

Suppose not and there exists an equilibrium messaging strategy in which there are two partitions (wlog), (θ', θ_g) and (θ_g, θ'') such that $\theta_g \in G$ and $\theta'' > \theta_g$. (I do not impose any further restrictions on θ' and θ''.) Let the messages associated with the two partitions be $\bar{m}_1$ and $\bar{m}_2$. For purposes of this proof, I will refer to the messages as characterizing the set of types sending them, i.e. $\bar{m}_1 = (\theta', \theta_g)$ and $\bar{m}_2 = (\theta_g, \theta'')$. I will show that the type θ_g will prefer to send the higher message $\bar{m}_2$.

The equilibrium action of the agents under the two pooling messages are,

$$\begin{aligned}
x_1^*(\bar{m}_i) &\equiv \underset{x_1 \in V}{\operatorname{argmax}} \int_{\theta \in \bar{m}_i} U\left(\phi^1(x_1, x_2^*(\theta, \bar{m}_i)), \theta\right) dP(\theta|\bar{m}_i) \\
x_2^*(\theta, \bar{m}_i) &\equiv \underset{x_2 \in V}{\operatorname{argmax}}\, U\left(\phi^2(x_2, x_1^*(\bar{m}_i)), \theta, b\right)
\end{aligned}$$

CLAIM. *The action of A_1 is such that $x_1^*(\bar{m}_1) < x_1^*(\theta_g)$ and $x_1^*(\bar{m}_2) < x_1^*(\theta'')$.*

Proof. Consider the action of A_1 under $\bar{m}_1$. Suppose $x_1^*(\bar{m}_1) = x_1^*(\theta_g)$, where

$$x_1^*(\theta_g) \equiv \underset{x_1 \in V}{\operatorname{argmax}} \, U\left(\phi^1(x_1, \bar{k}), \theta_g\right)$$

We know from previous arguments in Theorem 2 that $\bar{x}_2(\theta_g) > \bar{k}$ which implies that $x_2^*(\theta_g) = \bar{k}$ and $x_1^*(\theta_g) \equiv \operatorname{argmax}_{x_1} U\left(\phi^1(x_1, \bar{k}), \theta_g\right) > \bar{x}_1(\theta_g)$. Take the type θ'. If the best response of A_2 given the action $x_1^*(\theta_g)$ is such that,

$$x_2^*(\theta', \bar{m}_1) \equiv \underset{x_2 \in V}{\operatorname{argmax}} \, U\left(\phi^2(x_2, x_1^*(\theta_g)), \theta', b\right)$$

Then there are two cases to consider.

Case. $x_2^*(\theta', \bar{m}_1) = \bar{k}$

From single crossing, for all $\theta \in \bar{m}_1$ it holds that $x_2^*(\theta) = \bar{k}$ and trivially for all $\theta \in \bar{m}_1 \backslash \{\theta_g\}$, $\bar{\phi}^1_{\theta_g} = \phi_1(x_1^*(\theta_g), \bar{k}) > \bar{\phi}_\theta \implies U_1 < 0$ on this interval for A_1. Therefore $x_1^*(\bar{m}_1) < x_1^*(\theta_g)$.

Case. $x_2^*(\theta', \bar{m}_1) < \bar{k}$

In this case, take the sequence of $(x_2^*(\theta, \bar{m}_1))^n \to \bar{k}$. From continuity of the payoff function, clearly the set of types for which $x_2^* = \bar{k}$ is compact and closed. Let $\bar{\theta}_1 = \inf G_1 \equiv \{\theta : x_2^*(\theta, \bar{m}_1) = \bar{k}\}$. For all the types in G_1, $\phi^1(x_1^*(\theta_g), \bar{k}) > \bar{\phi}^1_\theta \implies U_1 < 0$ for A_1 and $\phi^2(\bar{k}, x_1^*(\theta_g)) \leq \bar{\phi}^2_\theta \implies U_1 \geq 0$ for A_2. At $\bar{\theta}_1$, $\phi^1(x_1^*(\theta_g), \bar{k}) > \bar{\phi}^1_{\bar{\theta}_1}$ and $\phi^2(\bar{k}, x_1^*(\theta_g)) = \bar{\phi}^2_{\bar{\theta}_1}$.

$$\phi^1_d(\theta) = \phi^1\left(x_1^*(\theta_g), x_2^*(\theta, \bar{m}_1)\right) - \bar{\phi}^1_\theta$$

$\phi^1_d(\theta)$ captures the extent of miscoordination on the interval $(\theta', \bar{\theta}_1)$. The sequence of equilibrium actions of A_2 starting from $\left(x_2^*(\bar{\theta}_1, \bar{m}_1)\right)$ is convergent, i.e. $(x_2^*(\theta, \bar{m}_1))^n \to (x_2^*(\theta', \bar{m}_1)) < \bar{k}$. The decreasing sequence $(x_2^*(\theta, \bar{m}_1))^n$ is convergent implies that $\left(\bar{\phi}^2_\theta\right)^n \to$

$\left(\bar{\phi}^2_{\theta'}\right)$. Since both ϕ^i's are continuous, $d\bar{\phi}^1_\theta = d\bar{\phi}^2_\theta$. However,

$$d\phi^1 = \frac{\partial \phi^1}{\partial x_1}.dx_1 + \frac{\partial \phi^1}{\partial x_2}.dx_2 = \frac{\partial \phi^1}{\partial x_2}.dx_2 < \frac{\partial \phi^2}{\partial x_2}.dx_2 = d\phi^2$$

That is, the rate of convergence of ϕ^1 is slower than that of ϕ^2 due to imperfect substitutability of actions. Therefore the mis-coordination term for A_1, given by $\left(\phi^1_d(\theta)\right)_{\theta \in (\theta', \bar{\theta}_1)}$, where $\phi^1_d(\bar{\theta}_1) > 0$ is bounded away from zero. That is $\left(\phi^1_d(\theta)\right)^n \to \phi^1_d(\theta') > 0$. Therefore $U_1 < 0$ for A_1 on this interval as well. This completes the proof. □

CLAIM. $x_1^*(\bar{m}_1) < x_1^*(\bar{m}_2)$.

Proof. Suppose $x_1^*(\bar{m}_2) = x_1^*(\bar{m}_1)$. Then, since $x_1^*(\bar{m}_1) < x_1^*(\theta_g)$ it implies that for all $\theta \in \bar{m}_2$, $x_2^*(\theta) = \bar{k}$ and $\phi^1(x_1^*(\bar{m}_1), \bar{k}) < \bar{\phi}^1_{\theta_g} < \bar{\phi}^1_\theta$ (single crossing). This further implies $U_1 > 0$ over the whole interval and there is a profitable deviation for A_1 at $x_2 = x_1^*(\bar{m}_1)$. □

Now, all we need to show is that the indifference condition at θ_g breaks down. That is,

$$\phi^2\left(\bar{k}, x_1^*(\bar{m}_1)\right) < \phi^2\left(x_2^*(\theta_g, \bar{m}_2), x_1^*(\bar{m}_2)\right) \leq \bar{\theta}^2_{\theta_g}$$

Where the first term is the coordination function for θ_g type A_2 under the pooling message $\bar{m}_1$ ($x_1^*(\bar{m}_1) = \bar{k}$). To show that the above inequality holds, we define $x_2^*(\theta_g, \bar{m}_2)$ as the following,

$$x_2^*(\theta_g, \bar{m}_2) \equiv \underset{x_2 \in V}{\operatorname{argmax}}\, U\left(\phi^2(x_2, x_1^*(\bar{m}_2)), \theta, b\right)$$

If $x_2^*(\theta_g, \bar{m}_2) < \bar{k}$ then $\phi^2\left(x_2^*(\theta_g, \bar{m}_2), x_1^*(\bar{m}_2)\right) = \bar{\phi}^2_{\theta_g}$ in which case sending the message is clearly better for θ_g type in that,

$$U\left(\bar{\phi}^2_{\theta_g}, \theta_g\right) > U\left(\phi^2(\bar{k}, x_1^*(\bar{m}_1)), \theta_g\right)$$

Alternately, if $x_2^*(\theta_g, \bar{m}_2) = \bar{k}$ then $\phi^2\left(\bar{k}, x_1^*(\bar{m}_1)\right) < \phi^2\left(\bar{k}, x_1^*(\bar{m}_2)\right) < \bar{\phi}^2_{\theta_g}$ since there is positive spillover at θ_g. This is valid for any pair of (θ', θ'') as long as $\theta'' > \theta_g$. This completes the proof. **QED**

A.4 Proof of Theorem 3

Consider the following set of strategies for the agents for a given threshold θ^*:

- (*Equilibrium path*) If $m \in \bigcup_{[\underset{\sim}{\theta},\theta^*]} M_\theta$: $x_1^*(\theta) = \bar{x}_1(\theta)$ and $x_2^*(\theta) = \bar{x}_2(\theta)$
- If $m = \bar{m}$:
  - $x_1^*(\bar{m}) \equiv \arg\max_{x_1 \in V} \int_{\theta \in \Theta \setminus [\underset{\sim}{\theta},\theta^*]} U\left(\phi^1(x_1, x_2^*(\theta, \bar{m})), \theta\right) dP(\theta|\bar{m})$
  - $x_2^*(\theta, \bar{m}) \equiv \arg\max_{x_2 \in V} U\left(\phi^2(x_2, x_1^*(\bar{m})), \theta, b\right)$
- (*Off equilibrium path*) If $\bar{m} \in \bigcup_{\theta \in \Theta \setminus \{[\underset{\sim}{\theta},\theta^*] \cup m^{-1}(\bar{m})\}} M_\theta$: $\quad p(\theta^*|\bar{m}) = 1$
  - $x_1^*(\bar{m}) = x_1^*(\theta^*)$
  - $x_2(\theta, \bar{m}) \equiv \underset{x_2 \in V}{\text{argmax}}\, U\left(\phi^2(x_2, x_1^*(\bar{m})), \theta, b\right)$

In the *PRTE* defined above there is a region of *separation* $[\underset{\sim}{\theta}, \theta^*]$ and a region of *pooling* where all types above the cutoff $\theta*$ send the same message $\bar{m}$. Clearly, on the *separating interval* both agents achieve their (unique) first best $\bar{\phi}^i_\theta$ and cannot do better. The off-equilibrium beliefs are constructed such that any message that is not a separating one or $\bar{m}$ is believed to be from θ^*, the highest separating type. To show that this constitutes an equilibrium, I characterize the equilibrium response of the agents under $\bar{m}$ in the following lemma.

Lemma. *For any information threshold θ^*, agent A_1's equilibrium action on receiving the message $\bar{m} = (\theta^*, \tilde{\theta}]$ is given by $x_1^*(\bar{m})$ that solves,*

$$\arg\max_{x_1 \in V} \int_{\theta^*}^{\theta_s^*} U\left(\phi^1(x_1, x_2^*(\theta, \bar{m})), \theta\right) dP(\theta \mid \bar{m}) + \int_{\theta_s^*}^{\tilde{\theta}} U\left(\phi^1(x_1, \bar{k}), \theta\right) dP(\theta \mid \bar{m}) \tag{7}$$

Proof. The lemma states that there is a cutoff state θ_s^* such that for all $\theta \in (\theta^*, \theta_s^*]$ the best response of A_2 is within the constraint $\bar{k}$ (i.e. $\phi^2(x_2^*(\theta, \bar{m}), x_1^*(\bar{m})) = \bar{\phi}_\theta^2$). For the remaining types in the pooling interval $\theta \in \bar{m} \setminus (\theta^*, \theta_s^*]$, $x_2^*(\theta, \bar{m}) = \bar{k}$. Define $x_1^{fb}(\tilde{\theta}) \equiv \underset{x_1 \in V}{\operatorname{argmax}}\, U\left(\phi^2(\bar{k}, x_1), \tilde{\theta}, b\right)$ as the action of A_1 that provides first best levels of coordination function to A_2 when the type is $\tilde{\theta}$, i.e. $\phi^2(\bar{k}, x_1^{fb}(\tilde{\theta})) = \bar{\phi}_{\tilde{\theta}}^2$.
To prove the lemma, it suffices to prove that $x_1^*(\bar{m}) < x_1^{fb}(\tilde{\theta})$. The inequality follows directly from noting that if $x_1^*(\bar{m}) = x_1^{fb}(\tilde{\theta})$, then for all $\theta \in \bar{m}$, $x_2^*(\theta, \bar{m}) < \bar{k}$ and $\phi(x_1^{fb}(\tilde{\theta}), \bar{k}) > \bar{\phi}_{\tilde{\theta}}^1$, from single crossing and continuity. However, as argued earlier (see Lemma 1), this also implies that $\phi^1(x_1^{fb}(\tilde{\theta}), x_2^*(\theta, \bar{m})) > \bar{\phi}_\theta^1$ for all $\theta \in \bar{m}$ due to imperfect substitutability of agents' actions. Therefore there must exist a $\theta_s^* \in (\theta^*, \tilde{\theta})$. This completes the proof. □

Given the nature of best responses described above, it immediately implies that for A_2, all types in $(\theta^*, \theta_s^*]$ achieve first best levels of coordination under the pooling message and therefore are at least weakly better off. For all the types in $(\theta_s^*, \tilde{\theta}]$, $x_2^*(\theta, \bar{m}) = \bar{k}$ and $\bar{\phi}_\theta^2 > \bar{\phi}^2\left(\bar{k}, x_1^*(\bar{m})\right) > \phi^2(\bar{k}, x_1^*(\theta^*))$. Since $U_1 > 0$ on this interval it immediately follows that when the action of A_1 is higher it is strictly better for A_2 (positive spillover effect). This completes the proof. **QED**

A.5 Proof of Proposition 1

Take any PRTE with threshold θ^*. I make the following claim.

Claim: $\forall \theta' \in (0, \theta^*), \exists\, \epsilon > 0 :\ \forall \theta \in (\theta' - \epsilon, \theta'],$

$$U\left(\phi^2\left(x_2^*(\theta), x_1^*(\theta)\right), \theta, b\right) = U\left(\phi^2\left(x_2^*(\theta, m_{(\theta'-\epsilon, \theta']}), x_1^*(m_{(\theta'-\epsilon, \theta']})\right), \theta, b\right)$$

Where the message $m_{(\theta'-\epsilon, \theta']}$ simply implies that the type is in the interval $(\theta' - \epsilon, \theta']$. The claim just states that for any separating type θ', it is possible to find a pooling interval of types $m_{pool} = m_{(\theta'-\epsilon,\theta']}$ such that the indifference condition holds for all types within this interval, i.e. each of the types in the pooling interval is indifferent between the separating message and the pooling one. The indifference (IC) condition merely requires that A_2 is able to achieve $\bar{\phi}^S_\theta$ which is possible as long as best responses are within the constraints.

To show this, all we need to check for are the indifference conditions of the boundary types $\theta' - \epsilon$ and θ',

$$U\left(\phi^2\left(x_2^*(\theta'), x_1^*(\theta')\right), \theta', b\right) = U\left(\phi^2\left(x_2^*(\theta', m_{(\theta'-\epsilon,\theta']}), x_1^*(m_{(\theta'-\epsilon,\theta']})\right), \theta', b\right)$$

$$U\left(\phi^2\left(x_2^*(\theta' - \epsilon), x_1^*(\theta' - \epsilon)\right), \theta' - \epsilon, b\right) =$$
$$U\left(\phi^2\left(x_2^*(\theta' - \epsilon, m_{(\theta'-\epsilon,\theta']}), x_1^*(m_{(\theta'-\epsilon,\theta']})\right), \theta' - \epsilon, b\right)$$

The latter condition follows from noting that any upward deviation is always within the domain of available actions (from Assumption 5). That is, $x_1^*(\theta' - \epsilon) > x_1^*(m_{(\theta'-\epsilon,\theta']})$ from single crossing ($U_{12} > 0$) and $x_2^*(\theta' - \epsilon) < x_2^*(\theta' - \epsilon, m_{(\theta'-\epsilon,\theta']})$ due to imperfect substitutability. However,

$$\phi^2\left(x_2^*(\theta' - \epsilon), x_1^*(\theta' - \epsilon)\right) = \phi^2\left(x_2^*(\theta' - \epsilon, m_{(\theta'-\epsilon,\theta']}), x_1^*(m_{(\theta'-\epsilon,\theta']})\right) = \bar{\phi}^2_{\theta'-\epsilon}$$

This implies that A_2 achieves first best levels of coordination function for the type $\theta' - \epsilon$ irrespective of whether the message is a separating or pooling one. The former condition states that the type θ' would pool with lower types and be indifferent from separating. To see this, notice that $x_2^*(\theta') = k' < \bar{k}$ under a separating (truthful) message. By continuity, there must exist a ϵ-deviation such that the $x_2^*(\theta', m_{(\theta'-\epsilon,\theta']}) \in (k', \bar{k}]$. If this were not true, then $\lim_{\epsilon \to 0} x_2^*(\theta', m_{(\theta'-\epsilon,\theta']}) = k' < \bar{k}$, a contradiction. As before, since $x_2^*(\theta') < x_2^*(\theta', m_{(\theta'-\epsilon,\theta']})$ it follows (from Assumption 3 and SC) that $x_1^*(\theta') > x_1^*(m_{(\theta'-\epsilon,\theta']})$ but

$\phi^2\left(x_2^*(\theta^{'}), x_1^*(\theta^{'})\right) = \phi^2\left(x_2^*(\theta^{'}, m_{(\theta^{'}-\epsilon,\theta^{'}]}), x_1^*(m_{(\theta^{'}-\epsilon,\theta^{'}]})\right) = \bar{\phi}^2_{\theta^{'}}$. If not, A_2 can always increase actions up to the point where it achieves first best. This completes the proof. **QED**

A.6 Proof of Proposition 2

Let $W_1(\theta^*)$ and $W_2(\theta^*)$ be the ex ante welfare of the two agents respectively. I will write them down in terms of the cutoff threshold θ^*.

A_1 **Welfare:**

$$W_1(\theta^*) = \int_{\underset{\sim}{\theta}}^{\theta^*} U\left(\phi^1\left(x_1^*(\theta), x_2^*(\theta)\right), \theta\right) dF(\theta) + \int_{\theta^*}^{\tilde{\theta}} U\left(\phi^1\left(x_1^*(\bar{m}), x_2^*(\theta, \bar{m})\right), \theta\right) dF(\theta)$$

Taking the derivative of A_1's welfare with respect to θ^*,

$$\frac{dW_1(\theta^*)}{d\theta^*} = \left[U\left(\phi^1\left(x_1^*(\theta^*), x_2^*(\theta^*)\right), \theta^*\right) - U\left(\phi^1\left(x_1^*(\bar{m}), x_2^*(\theta^*, \bar{m})\right), \theta^*\right)\right] f(\theta^*) > 0$$

for any $\theta^* \leq \bar{\theta}$ since $\phi^1\left(x_1^*(\theta^*), x_2^*(\theta^*)\right) = \bar{\phi}^1_{\theta^*}$, the first best levels of coordination. Further, there is a discontinuous jump at θ^* following a pooling message, implying that $\left|\phi^1\left(x_1^*(\theta^*), x_2^*(\theta^*)\right) - \phi^1\left(x_1^*(\bar{m}), x_2^*(\theta^*, \bar{m})\right)\right| > 0$ at θ^*.

A_2 **Welfare:**

Take any two cutoff equilibria $\theta^1, \theta^2 \leq \bar{\theta}$, call them $PRTE_1$ and $PRTE_2$, such that $\theta^1 < \theta^2$ (wlog). Let the corresponding pooling messages associated with the PRTE be $\bar{m}^1 = (\theta^1, 1]$ and $\bar{m}^2 = (\theta^2, 1]$ respectively. I will establish that A_2 is better off with the more informative equilibrium θ^2. Similar to arguments made in A.4, for cutoff equilibria θ^1, θ^2

there exists a corresponding θ_s^1 and θ_s^2 such that $x_2^*(\theta_s^1, \bar{m}^1) = x_2^*(\theta_s^2, \bar{m}^2) = \bar{k}$. From single crossing property, A_1s action must be higher for the pooling message $\bar{m}^2$ corresponding to the threshold θ^2, i.e. $x_1^*(\bar{m}^2) > x_1^*(\bar{m}^1)$. If this is true, then $\theta_s^1 < \theta_s^2$. Suppose not, and $\theta_s^1 > \theta_s^2$. Then, $x_2^*(\theta_s^2, \bar{m}^1) < x_2^*(\theta_s^1, \bar{m}^1) = \bar{k}$. But $x_2^*(\theta_s^2, \bar{m}^1) \geq x_2^*(\theta_s^2, \bar{m}^2) = \bar{k}$. This is a contradiction. Therefore the claim holds. In order to prove the result for A_2, I consider two possible scenarios.

Scenario (a): *When* $\theta_s^1 < \theta^2$. That is, $\theta^1 < \theta_s^1 < \theta^2 < \theta_s^2$. The welfare to A_2 under the two PRTE's is given by,

$$W_2(\theta^1) = \int_{\underset{\sim}{\theta}}^{\theta^1} U\left(\phi^2\left(x_2^*(\theta), x_1^*(\theta)\right), \theta, b\right) dF + \int_{\theta^1}^{\tilde{\theta}} U\left(\phi^2\left(x_2^*(\theta, \bar{m}^1), x_1^*(\bar{m}^1)\right), \theta, b\right) dF$$

$$W_2(\theta^2) = \int_{\underset{\sim}{\theta}}^{\theta^2} U\left(\phi^2\left(x_2^*(\theta), x_1^*(\theta)\right), \theta, b\right) dF + \int_{\theta^2}^{\tilde{\theta}} U\left(\phi^2\left(x_2^*(\theta, \bar{m}^2), x_1^*(\bar{m}^2)\right), \theta, b\right) dF$$

Under $PRTE_1$, A_2's equilibrium action is within the bound for the interval $(0, \theta_s^1]$. Since $\theta_s^1 < \theta_s^2$, A_2's action is also within the bound over the interval $(0, \theta_s^1]$ under $PRTE_2$. Therefore, what is left to be checked are those states in which the constraints are binding for A_2. In $PRTE_1$, this corresponds to the interval $(\theta_s^1, 1]$. On the same interval, I compare the expected (ex ante) utility under $PRTE_2$. I will refer to this utility as the residual welfare that results from inefficiency, $W_2^{RES}(\theta^1)$ and $W_2^{RES}(\theta^1)$ respectively.

$$W_2^{RES}(\theta^1) = \int_{\theta_s^1}^{\theta_s^2} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF + \int_{\theta_s^2}^{\tilde{\theta}} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF$$

$$W_2^{RES}(\theta^2) = \int_{\theta_s^1}^{\theta^2} U\left(\phi^2\left(x_2^*(\theta), x_1^*(\theta)\right), \theta, b\right) dF + \int_{\theta^2}^{\theta_s^2} U\left(\phi^2\left(x_2^*(\theta, \bar{m}^2), x_1^*(\bar{m}^2)\right), \theta, b\right) dF + \int_{\theta_s^2}^{\tilde{\theta}} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^2)\right), \theta, b\right) dF$$

Taking the expression $W_2^{RES}(\theta^1)$ and expanding the first term, we get,

$$\int_{\theta_s^1}^{\theta^2} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF + \int_{\theta^2}^{\theta_s^2} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF$$

Comparing the above expression with the first two terms of $W_2^{RES}(\theta^2)$,

$$\int_{\theta_s^1}^{\theta^2} U\left(\phi^2\left(x_2^*(\theta), x_1^*(\theta)\right), \theta, b\right) dF + \int_{\theta^2}^{\theta_s^2} U\left(\phi^2\left(x_2^*(\theta, \bar{m}^2), x_1^*(\bar{m}^2)\right), \theta, b\right) dF > \int_{\theta_s^1}^{\theta^2} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF + \int_{\theta^2}^{\theta_s^2} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF$$

This follows from pair-wise comparison of the terms,

$$\int_{\theta_s^1}^{\theta^2} U\left(\phi^2\left(x_2^*(\theta), x_1^*(\theta)\right), \theta, b\right) dF > \int_{\theta_s^1}^{\theta^2} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF \quad (8)$$

$$\int_{\theta^2}^{\theta_s^2} U\left(\phi^2\left(x_2^*(\theta, \bar{m}^2), x_1^*(\bar{m}^2)\right), \theta, b\right) dF > \int_{\theta^2}^{\theta_s^2} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF \quad (9)$$

Similarly comparing the last term of $W_2^{RES}(\theta^1)$ and $W_2^{RES}(\theta^2)$,

$$\int_{\theta_s^2}^{\tilde{\theta}} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^2)\right), \theta, b\right) dF > \int_{\theta_s^2}^{\tilde{\theta}} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF \tag{10}$$

The inequality 8 follows from noting that on the interval $(\theta_s^1, \theta^2]$, A_2 achieves $\bar{\phi}_\theta^2$ under the higher threshold equilibrium.

$$\forall t \in (\theta_s^1, \theta^2] : U\left(\phi^2\left(x_2^*(\theta), x_1^*(\theta)\right), \theta, b\right) > U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right)$$

Similarly, inequality 9 is true since on the interval $(\theta^2, \theta_s^2]$, A_2 induces A_1 to allocate more with message $\bar{m}^2$ and correspondingly changes its action to achieve first best $\bar{\phi}_\theta^2$.

$$\forall \theta \in (\theta^2, \theta_s^2] : U\left(\phi^2\left(x_2^*(\theta, \bar{m}^2), x_1^*(\bar{m}^2)\right), \theta, b\right) > U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right)$$

The last inequality 10 follows from noting that since $x_1^*(\bar{m}^1) < x_1^*(\bar{m}^2)$, it is valid that $\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right) < \phi^2\left(\bar{k}, x_1^*(\bar{m}^2)\right)$ and because there is a positive spillover at the bound for A_2, ie $U_1 \mid_{\theta \in (\theta_s^2, \tilde{\theta}]} > 0$,

$$\forall \theta \in (\theta_s^2, \tilde{\theta}] : U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^2)\right), \theta, b\right) > U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right)$$

Comparing the terms pairwise therefore yields the required result, $W_2^{RES}(\theta^2) > W_2^{RES}(\theta^1)$.

Scenario (b): *When* $\theta_s^1 > \theta^2$. That is, $\theta^1 < \theta^2 < \theta_s^1 < \theta_s^2$.

In this case, as earlier, I will look at states in which there is inefficiency generated by information pooling and compare the residual welfare.

$$W_2^{RES}(\theta^1) = \int_{\theta_s^1}^{\theta_s^2} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF + \int_{\theta_s^2}^{\tilde{\theta}} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF$$

$$W_2^{RES}(\theta^2) = \int_{\theta_s^1}^{\theta_s^2} U\left(\phi^2\left(x_2^*(\theta,\bar{m}^2), x_1^*(\bar{m}^2)\right), \theta, b\right) dF + \int_{\theta_s^2}^{\tilde{\theta}} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^2)\right), \theta, b\right) dF$$

Pairwise comparison yields,

$$\int_{\theta_s^1}^{\theta_s^2} U\left(\phi^2\left(x_2^*(\theta,\bar{m}^2), x_1^*(\bar{m}^2)\right), \theta, b\right) dF > \int_{\theta_s^1}^{\theta_s^2} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF \tag{11}$$

$$\int_{\theta_s^2}^{\tilde{\theta}} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^2)\right), \theta, b\right) dF > \int_{\theta_s^2}^{\tilde{\theta}} U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right) dF \tag{12}$$

The inequalities 11 and 12 follow from arguments made earlier. Specifically, on $(\theta_s^1, \theta_s^2]$ A_2 is able to achieve $\bar{\phi}_\theta^2$ with the cutoff equilibrium θ^2 and is therefore strictly better off compared to the equilibrium threshold θ^1. In the interval $(\theta_s^2, \tilde{\theta}]$, there is inefficiency from miscoordination in that $\phi^2(.) < \bar{\phi}_\theta^2$. However, since A_2 induces a higher action from A_1 under θ^2 equilibrium, $x_1^*(\bar{m}^2) > x_1^*(\bar{m}^1)$, it follows that $\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right) < \phi^2\left(\bar{k}, x_1^*(\bar{m}^2)\right) < \bar{\phi}_\theta^2$ and given $U_1 > 0$ on this interval,

$$\forall \theta \in (\theta_s^2, \tilde{\theta}] : U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^2)\right), \theta, b\right) > U\left(\phi^2\left(\bar{k}, x_1^*(\bar{m}^1)\right), \theta, b\right)$$

Therefore, $W_2^{RES}(\theta^2) > W_2^{RES}(\theta^1)$. This completes the proof. **QED**

A.7 Proof of Lemma 2

The key to proving this is to look at all pairs of actions (x_1, x_2) that achieve the first best for A_2, in order to satisfy its IC and NC constraints. Given Assumption 1 and Assumption 2, for any $\theta \in \bar{m}_p$, there are different action pairs (x_1, x_2) such that $\phi^2(x_2, x_1) = \bar{\phi}_\theta^2$. I proceed by constructing the set of ϕ^1 that corresponds with all admissible pairs (x_1, x_2)

such that for any θ, $\phi^2(x_2, x_1) = \bar{\phi}^2_\theta$. The following defines this admissible set:

$$\forall \theta \in \bar{m}_p, (x_2, x_1) \in V : \mathcal{A}_\theta = \left\{ \phi^1(x_1, x_2) : \phi^2(x_2, x_1) = \bar{\phi}^2_\theta \right\}$$

I will impose further structure on the set $\mathcal{A}_\theta$ for the interval $\bar{m}_p$. From continuity property of $\phi^1(.)$ and $\phi^2(.)$, the set $\mathcal{A}_\theta$ is compact. Further, let $\sup \mathcal{A}_\theta = \phi^1_{sup}(\theta)$ and $\inf \mathcal{A}_\theta = \phi^1_{inf}(\theta)$. Let $x_1^{inf}(\theta)$ be such that $\phi^2\left(\bar{k}, x_1^{inf}(\theta)\right) = \bar{\phi}^2_\theta$.

CLAIM. $\forall \theta \in \bar{m}_p : \phi^1(x_1^{inf}(\theta), \bar{k}) = \phi^1_{inf}(\theta)$

Proof. Note that x_2 varies from $\underline{k}$ to $\bar{k}$ and x_1 is just the residual contribution that ensures $\phi^2(.) = \bar{\phi}^2_\theta$. Applying total differentiation to ϕ^2, we get the following:

$$d\phi^2 = \frac{\partial \phi^2}{\partial x_2}.dx_2 + \frac{\partial \phi^2}{\partial x_1}.dx_1$$

Since $\phi^2(.) = \bar{\phi}^2_\theta$, a constant in $\mathcal{A}_\theta$, $d\phi^2 = 0$. Substituting this in the above equation and rearranging,

$$\left| \frac{dx_1}{dx_2} \right| = \frac{\frac{\partial \phi^2}{\partial x_2}}{\frac{\partial \phi^2}{\partial x_1}} > 1$$

Similarly,

$$d\phi^1 = \frac{\partial \phi^1}{\partial x_2}.dx_2 + \frac{\partial \phi^1}{\partial x_1}.dx_1$$

$$\frac{d\phi^1}{dx_2} = \frac{\partial \phi^1}{\partial x_2} + \frac{\partial \phi^1}{\partial x_1}.\frac{dx_1}{dx_2} = \frac{\partial \phi^1}{\partial x_2} - \left| \frac{dx_1}{dx_2} \right|.\frac{\partial \phi^1}{\partial x_1} \tag{13}$$

$$\implies \frac{d\phi^1}{dx_2} < \left[\frac{\partial \phi^1}{\partial x_1} - \left| \frac{dx_1}{dx_2} \right|.\frac{\partial \phi^1}{\partial x_1} \right] = \frac{\partial \phi^1}{\partial x_2}.\left[1 - \left| \frac{dx_1}{dx_2} \right| \right] < 0 \tag{14}$$

Equation 14 follows from imperfect substitutability in that $\frac{\partial \phi^1}{\partial x_1} > \frac{\partial \phi^1}{\partial x_2}$. This further establishes that ϕ^1 is decreasing in the actions of A_2. Therefore the infimum of the set $\mathcal{A}_\theta$

corresponds with the pair of actions in which A_2 takes the maximal action $\bar{k}$ and A_1, the residual $x_1^{inf}(\theta)$. □

CLAIM. $\forall\theta \in \bar{m}_p : \phi^1_{inf}(\theta) > \bar{\phi}^1_\theta$

Proof. From lemma 4 it is clear there is an ordering over ϕ^1. Specifically, $\phi^1_{sup}(\theta) > ...... > \phi^1_{inf}(\theta)$. Suppose $\phi^1_{inf}(\theta) > \bar{\phi}^1_\theta$ were not true. Then, either $\phi^1_{sup}(\theta) > ..... > \bar{\phi}^1_\theta > ... > \phi^1_{inf}(\theta)$ or $\bar{\phi}^1_\theta > \phi^1_{sup}(\theta) > ..... > \phi^1_{inf}(\theta)$. If the former was true, then A_2 can achieve first best by truthfully revealing the state θ. That is, A_2 could have revealed truthfully up to some higher threshold $\bar{\theta}$, which violates the most informative threshold equilibrium $\bar{\theta}$. The latter cannot be true because of the imperfect substitutability assumption and a positive conflict of interest. Therefore it must hold that $\phi^1_{sup}(\theta) > ...... > \phi^1_{inf}(\theta) > \bar{\phi}^1_\theta$. □

From the above claims, it is clear that on the interval $\bar{m}_p$, there is over-provision for agent A_1 as long as A_2 achieves first best. However, precisely for this reason, it implies that $U_1 < 0$ and therefore the following holds:

$$\forall\theta \in \bar{m}_p : \phi^1_{inf}(\theta) \equiv \underset{\phi^1 \in \mathcal{A}_\theta}{\operatorname{argmax}}\, U(\phi^1, \theta) \tag{15}$$

That is, of all action pairs (x_1, x_2) that satisfy A_2's IC constraint for truth-telling, the one that maximizes A_1's utility is the one that minimizes this miscoordination from over-provision, which coincides with $x_2^c = \bar{k}$. Suppose there was a strictly increasing interval $(\theta_1, \theta_2) \in \bar{m}_p$ such that $\exists\theta' \in (\theta_1, \theta_2) : x_2^c(\theta', x_1^c(\theta')) < \bar{k}$. Then, given that IC and NC must be satisfied, $\phi^1\left(x_1^c(\theta'), x_2^c(\theta', x_1^c(\theta'))\right) \in \mathcal{A}_t\theta'$. But clearly from the earlier arguments, A_1 can always instead choose to take an action $x_1^{inf}(\theta')$ such that $x_2^c(\theta', x_1^{inf}(\theta')) = \bar{k}$. This satisfies IC of A_2 since $\phi^1(x_1^{inf}(\theta'), \bar{k}) \in \mathcal{A}_{\theta'}$ and increases the payoff to A_1 since $U\left(\phi^1(x_1^{inf}(\theta'), \bar{k}), \theta'\right) > U\left(\phi^1\left(x_1^c(\theta'), x_2^c(\theta', x_1^c(\theta'))\right), \theta'\right)$.

A.8 Proof of Proposition 3

To show the optimal commitment rule indeed takes the form described in Proposition 3, I will start by proving Claim 1- Claim 5.

A.8.1 Proof of Claim 1

Suppose the claim weren't true and say A_1, wlog, allocates $x_1^c(\theta) = z, \forall\theta \in \bar{m}_p$. There are three possible cases to consider.

Case i) $z = \bar{x}_1(\bar{\theta}) \equiv \underline{z}$

In this case, A_2's NC constraint dictates that $x_2^c = \bar{k}$ for every possible type in $\bar{m}_p$, since $U_{12} > 0$. If this is so, then $\forall\theta \in \bar{m}_p : \phi^1(\bar{x}_1(\bar{\theta}), \bar{k}) = \bar{\phi}^1_{\bar{\theta}} < \bar{\phi}^1_{\theta}$. This implies that the expected marginal utility of A_1 is greater than zero and given $U_{11} < 0$, there is an incentive for A_1 to increase her actions. Therefore, $z \neq \bar{x}_1(\bar{\theta})$.

Case ii) $z = \bar{z} \equiv \underset{x_1 \in V}{\operatorname{argmax}}\, U\left(\phi^2(\bar{k}, x_1), \tilde{\theta}, b\right)$

This corresponds with the case where A_1 provides first best joint coordination levels to A_2 for all $\theta \in \bar{m}_p$, i.e. $x_2^c(\tilde{\theta}, \bar{z}) = \bar{k}$ and,

$$\forall\theta \in (\bar{\theta}, \tilde{\theta}) : x_2^c(\theta, \bar{z}) < \bar{k}$$

Let $\bar{z} = \bar{x}_1(\bar{\theta}) + \bar{\Delta}_1(\tilde{\theta})$ and $x_2^c(\theta, \bar{z}) = \bar{k} - \bar{\Delta}_2(\theta)$ such that $\bar{\Delta}_2(\tilde{\theta}) = 0$ on the interval $\bar{m}_p$. Then, due to Assumption 3 it must be that the difference $\phi^2\left(\bar{k}, \bar{z}\right) - \bar{\phi}^2_{\tilde{\theta}} > \phi^1\left(\bar{z}, \bar{k}\right) - \bar{\phi}^1_{\tilde{\theta}}$. This follows from noting that $\phi^2\left(\bar{k}, \bar{z}\right) = \bar{\phi}^2_{\tilde{\theta}}$ by definition and since there is imperfect substitutability it follows that keeping x_2 fixed, $d\phi^1 = \frac{\partial\phi^1}{dx_1}.dx_1 > \frac{\partial\phi^2}{dx_1}.dx_1 = d\phi^2$ implying that $\phi^1\left(\bar{z}, \bar{k}\right) > \bar{\phi}^1_{\tilde{\theta}}$. Since $x_2^c(\theta, \bar{z}) < \bar{k}$ and $\phi^2(.) = \bar{\phi}^2_{\theta}$ on the interval $\bar{m}_p$, it must be from continuity of the coordination functions that the sequence $\left(\phi^1\left(\bar{x}_1(\bar{\theta}) + \bar{\Delta}_1(\tilde{\theta}), \bar{k} - \bar{\Delta}_2(\theta)\right) - \bar{\phi}^1_{\theta}\right)^n \to \tilde{\epsilon} > 0$ i.e. the sequence is pointwise bounded away from zero. Therefore there is over-provision over the entire interval $\bar{m}_p$ and the marginal utility for A_1 is decreasing at $\bar{z}$.

Case ii): $z \equiv \tilde{z} \in (\underline{z}, \bar{z})$

If this were true, then the following inequalities hold.

$$x_2^c(\tilde{\theta}, \tilde{z}) = \bar{k}$$

$$x_2^c(\bar{\theta}, \tilde{z}) < \bar{k}$$

The first follows directly from NC constraint and the second holds since $\phi^2(\bar{k}, \tilde{z}) > \phi^2(\bar{k}, \bar{z}) = \bar{\phi}^2_{\bar{\theta}}$. But if this were true, then there exists some types such that A_2 allocates less than $\bar{k}$ and still achieves first best. That is,

$$\exists \theta' \in (\bar{\theta}, \tilde{\theta}) \quad \text{such that} \quad x_2^c(\theta, \tilde{z}) \equiv \underset{x_2 \in V}{\operatorname{argmax}}\, U\left(\phi^2(x_2, \tilde{z}), \theta, b\right) < \bar{k} \text{ for all } \theta \in (\bar{\theta}, \theta')$$

From the continuity property of $U(.)$ and $\phi^i(.)$, when $z > x_1^c(\bar{\theta})$, then there is always a cutoff type θ' such that $x_2^c(\theta', z) = \bar{k}$. However, this implies that for all types in the interval $(\bar{\theta}, \theta')$ it must be that $x_2^c(\theta, z) < \bar{k}$. If this set exists, then A_1 is not maximizing its expected utility since it can always reduce actions and make A_2 contribute $\bar{k}$, due to Lemma 2. To see this, consider the following alternate action rule:

$$\forall t \in (\bar{\theta}, \theta') : x_1^c(t) = x_1^{inf}(t) \text{ such that } \phi^1(x_1^{inf}(t), \bar{k}) = \phi^1_{inf}(t) \in \mathcal{A}_t$$

$$\forall t' \in \bar{m}_p \setminus (\bar{\theta}, \theta') : x_1^c(t') = z$$

Clearly, on the interval subset $(\bar{\theta}, \theta')$, A_1 now achieves a greater expected utility since $\forall t \in (\bar{\theta}, \theta') : U\left(\phi^1(x_1^{inf}(t), \bar{k}), t\right) > U\left(\phi^1(z, x_2^c(t, z)), t\right)$. Further, this action rule is also incentive compatible in that A_2 cannot do better by misreporting. Therefore, there cannot be a single flat segment on $\bar{m}_p$. This proves Claim 1.

A.8.2 Proof of Claim 2

Consider, wlog, two pooling intervals $(\theta_1, \theta_1^{'})$ and $(\theta_2, \theta_2^{'})$ such that $\theta_1 < \theta_1^{'} < \theta_2 < \theta_2^{'}$. Suppose z_1 and z_2 are the two pooling actions associated with the pooling segments respectively. From Lemma 2 and Claim 1 the following is statements hold:

$$\forall \theta \in (\theta_i, \theta_i^{'}) : x_2^c(\theta, z_i) = \bar{k}, \qquad i \in \{1, 2\}$$

Pick a $\theta_1^{''} = \theta_1 + \epsilon_1 \in (\theta_1, \theta_1^{'})$. Clearly, $\phi^2(x_2^c(\theta_1^{''}, z_1), z_1) < \bar{\phi}^2_{\theta_1^{''}}$.
As a result, $U_1\left(\phi^2(x_2^c(\theta_1^{''}, z_1), z_1), \theta_1^{''}, b\right) > 0$. If $\theta_1^{''}$ reports to be in the higher pooling segment $(\theta_2, \theta_2^{'})$, then the deviation action $x_2^d(\theta_1^{''}, z_2)$ is such that,

$$x_2^d(\theta_1^{''}, z_2) \equiv \underset{x_2 \in V}{\text{argmax}}\, U\left(\phi^2(x_2^c(\theta_1^{''}, z_2), z_2), \theta_1^{''}, b\right)$$

If $x_2^d(.) < \bar{k}$ then $\phi^2(.) = \bar{\phi}^2_{\theta_1^{''}}$ and the deviation is clearly better for A_2, thereby violating the NC and IC constraints. If $x_2^d(.) = \bar{k}$ on the other hand, then,

$$\bar{\phi}^2_{\theta_1^{''}} \geq \phi^2(x_2^c(\theta_1^{''}, z_2), z_2) > \phi^2(x_2^c(\theta_1^{''}, z_1), z_1)$$

$$\implies U_1\left(\phi^2(x_2^c(\theta_1^{''}, z_2), z_2), \theta_1^{''}, b\right) > U_1\left(\phi^2(x_2^c(\theta_1^{''}, z_1), z_1), \theta_1^{''}, b\right)$$

This completes the proof of Claim 2.

A.8.3 Proof of Claim 3

Suppose, instead there exists a flat segment followed by a strictly increasing segment in $\bar{m}_p$. Say, wlog, the flat segment is on $(\theta_1, \theta_1^{'}]$ such that $\forall t \in (\theta_1, \theta_1^{'}] : x_1^c(t) = z$, and let the strictly increasing segment be on $(\theta_2, \theta_2^{'})$. From Lemma 2, it holds that A_2 must take an action $\bar{k}$ and the IC constraint must be satisfied on the separating interval in that $\forall t \in (\theta_2, \theta_2^{'}) : \phi^2(\bar{k}, x_1^{inf}(t)) = \bar{\phi}_t^2$. Similarly from Claims 1-2 it must be that for all $t \in (\theta_1, \theta_1^{'}) : x_2^c(t, z) \equiv \text{argmax}_{x_2 \in V} U\left(\phi^2(x_2, z), t, b\right) = \bar{k}$ and $\phi^2(\bar{k}, z) < \bar{\phi}_t^2$. This however implies that the pooling action is always lower than the separating region actions, i.e. $z < x_1^{inf}(t)$ for all $t \in (\theta_2, \theta_2^{'})$. Take any type $t \in (\theta_1, \theta_1^{'})$. For this type it must be that by deviating and reporting to be a higher type in $(\theta_2, \theta_2^{'})$ would increase its expected payoff. To see this, let us again consider the deviation action if $t \in (\theta_1, \theta_1^{'})$ reports to be $t^{'} \in (\theta_2, \theta_2^{'})$. The deviation action for A_2 type t is,

$$x_2^d(t, x_1^{inf}(t^{'})) \equiv \underset{x_2 \in V}{\text{argmax}}\, U\left(\phi^2(x_2, x_1^{inf}(t^{'})), t, b\right)$$

As before, if $x_2^d(.) < \bar{k}$, then it implies $\phi^2(x_2^d(.), x_1^{inf}(t^{'})) = \bar{\phi}_t^2$ and A_2 has an incentive to exaggerate and claim to be a higher type in the separating region. On the other hand, if $x_2^d(.) = \bar{k}$ then since $z < x_1^{inf}(t^{'})$ and $\bar{\phi}_t^2 < \bar{\phi}_{t^{'}}^2$,

$$\phi^2(\bar{k}, z) < \phi^2(\bar{k}, x_1^{inf}(t^{'})) \leq \bar{\phi}_t^2$$

$$\implies U\left(\phi^2(\bar{k}, x_1^{inf}(t^{'})), t, b\right) > U\left(\phi^2(\bar{k}, z), t, b\right)$$

The above violates IC constraint of the types in the pooling region. Therefore there can never be a separating region following a pooling region on $\bar{m}_p$. This proves Claim 3.

A.8.4 Proof of Claim 4

The claim directly follows from Claim 3. That is, if there exists two separating intervals on $\bar{m}_p$, then it cannot be that there is a pooling region in between the two separating intervals. If there are two adjacent intervals $(\theta_1, \theta_1^{'})$ and $(\theta_1^{'}, \theta_2)$ such that there is a discontinuous jump in A_1's action at $\theta_1^{'}$, then say $x_1^L(\theta_1^{'})$ and $x_1^H(\theta_1^{'})$ are such that $x_1^L(\theta_1^{'}) < x_1^H(\theta_1^{'})$. It immediately implies that $x_1^L(\theta_1^{'}) = x_1^{inf}(\theta_1^{'})$ and the higher separating action $x_1^H(\theta_1^{'})$ must therefore induce an action less than $\bar{k}$ from A_2, i.e.,

$$x_2^c(\theta_1^{'}, x_1^H(\theta_1^{'})) \equiv \underset{x_2 \in V}{\operatorname{argmax}}\, U\left(\phi^2(x_2, x_1^H(\theta_1^{'})), \theta_1^{'}, b\right) < \bar{k}$$

This cannot be optimal for A_1 since this action does not minimize the miscoordination (refer to Lemma 2). This proves Claim 4.

A.8.5 Proof of Claim 5

Claim 1-Claim 4 imply that the optimal commitment problem can be reformulated as the following on the pooling interval,

$$\mathcal{W}_1^c = \underset{x_1^c(\theta) \in V}{\operatorname{argmax}} \int_{\bar{\theta}}^{t} U\left(\phi^1\left(x_1^c(\theta), \bar{k}\right), \theta\right) dF + \int_{t}^{\tilde{\theta}} U\left(\phi^1\left(x_1^c(t), \bar{k}\right), \theta\right) dF$$
$$\text{such that} \quad x_1^c(\theta) \equiv \underset{x_1 \in V}{\operatorname{argmax}}\, U\left(\phi^2(\bar{k}, x_1), \theta, b\right), \quad \phi^2\left(\bar{k}, x_1^c(t)\right) = \bar{\phi}_t^2$$

The reformulation basically reduces the commitment problem to an optimal control problem with a *modified* IC constraint such that there is a cutoff state t up to which A_2 gets first best. Correspondingly, t is the state variable and $x_1^c(\theta)$ is the control variable. Given that the utility function and the coordination function are twice continuously differentiable in x_1, the above problem is equivalent to choosing an optimal cutoff t. The first order condition is given by,

$$\frac{d\mathcal{W}_1^c}{dt} = \int_t^{\tilde{\theta}} U_1\left(\phi^1\left(x_1^c(t), \bar{k}\right), \theta\right) . \left[\frac{d\phi^1\left(x_1^c(t), \bar{k}\right)}{dx_1} . x_1^{c'}(t)\right] dF = 0 \tag{16}$$

The term $\left[U\left(\phi^1\left(x_1^c(t), \bar{k}\right), \theta\right) - U\left(\phi^1\left(x_1^c(t), \bar{k}\right), \theta\right)\right] f(t) = 0$ and therefore not been included in the FOC above. Since $\left[\frac{d\phi^1\left(x_1^c(t), \bar{k}\right)}{dx_1} . x_1^{c'}(t)\right] > 0$,

$$\left.\frac{d\mathcal{W}_1^c}{dt}\right|_{t\uparrow\tilde{\theta}} = U_1\left(\phi^1\left(x_1^c(\tilde{\theta}), \bar{k}\right), \tilde{\theta}\right) . \left[\left.\frac{d\phi^1\left(x_1^c(t), \bar{k}\right)}{dx_1}\right|_{t\uparrow\tilde{\theta}} . x_1^{c'}(\tilde{\theta})\right] f(\tilde{\theta}) < 0$$

This directly follows from noting that $x_1^c(\tilde{\theta}) \equiv \underset{x_1 \in V}{\operatorname{argmax}} U\left(\phi^2(\bar{k}, x_1), \tilde{\theta}, b\right)$ which implies from previous arguments (Lemma 2) that $\phi^1\left(x_1^c(\tilde{\theta}), \bar{k}\right) > \bar{\phi}^1_{\tilde{\theta}}$. Therefore $U_1 < 0$ and this concludes the proof of Claim 5.

Together, the five claims imply the following rules hold under the optimal commitment mechanism:

1. On the interval $[0, \bar{\theta}]$, the optimal ex ante action rule mimics the simultaneous protocol actions, $\bar{x}_1(\theta)$.

2. There is a cutoff $\bar{\theta}_c \in m_{pool}$ such that A_1's action rule is dependent on communication up to $\bar{\theta}_c$ and given by $x_1^c(\theta) = x_1^{inf}(\theta)$; A_2's action is $x_2^c(\theta) = \bar{k}$ such that $\phi^2(\bar{k}, x_1^{inf}(\theta)) = \bar{\phi}^2_\theta$ and $\phi^1(x_1^{inf}(\theta), \bar{k}) \in \mathcal{A}_\theta$.

3. Finally, Claim 5 $\implies$ $\bar{\theta}_c < \tilde{\theta}$ and on the interval $[\bar{\theta}_c, \tilde{\theta}]$, A_1's action is independent of communication and is equal to $x_1^c(\theta) = x_1^c(\bar{\theta}_c)$.

This completes the proof. **QED**

A.9 Proof of Proposition 4

Under simultaneous actions with no commitment, the agent A_1 plays an expected action $x_1^s(\bar{m}_p) \equiv x_1^s$ on the pooling interval $\bar{m}_p$. In response, the agent A_2 plays $x_2^s(\theta, \bar{m}_p)$ for every possible type. subsection A.4 provides the solution to A_1's problem of choosing x_1^s and can be rewritten for the most informative equilibrium ex ante as the solution of the following FOC,

$$\int_{\bar{\theta}}^{\bar{\theta}_s} U_1\left(\phi^1(x_1^s, x_2^s(\theta, \bar{m}_p)), \theta\right) \cdot \left.\frac{d\phi^1}{dx_1}\right|_{x_1 = x_1^s} dF + \int_{\bar{\theta}_s}^{\tilde{\theta}} U_1\left(\phi^1(x_1^s, \bar{k}), \theta\right) \cdot \left.\frac{d\phi^1}{dx_1}\right|_{x_1 = x_1^s} dF = 0 \quad (17)$$

At $\bar{\theta}_s$, it follows that $\phi^2(\bar{k}, x_1^s) = \bar{\phi}^2_{\bar{\theta}_s}$. A_2 therefore achieves first best levels of coordination on the interval $(\bar{\theta}, \bar{\theta}_s]$. Consider the following commitment protocol sequence of actions for A_1 on $\bar{m}_p$:

$$\forall \theta \in (\bar{\theta}, \bar{\theta}_s) : x_1^c(\theta) = x_1^{inf}(\theta)$$

$$\forall \theta \in [\bar{\theta}_s, \tilde{\theta}] : x_1^c(\theta) = x_1^s(\bar{m}_p) \equiv x_1^s$$

The above commitment rule replicates the simultaneous protocol cutoff $\bar{\theta}_s$ in that it provides A_2 first best joint coordination $\bar{\phi}^2_\theta$ on the interval $(\bar{\theta}, \bar{\theta}_s]$. Clearly, this action rule is IC for A_2 and provides the same expected ex ante welfare to A_2 compared to simultaneous protocol case. A_1's expected welfare is equal to that of simultaneous protocol only on the interval $[\bar{\theta}_s, \tilde{\theta}]$. However, $\forall t \in (\bar{\theta}, \bar{\theta}_s)$, A_1 actually does better since A_2's action is maximal $(\bar{k})$ on this interval and this minimizes the inefficiency from miscoordination. That is,

$$\forall \theta \in (\bar{\theta}, \bar{\theta}_s) : U\left(\phi^1\left(x_1^{inf}(\theta), \bar{k}\right), \theta\right) > U\left(\phi^1\left(x_1^s, x_2^s(\theta, \bar{m}_p)\right), \theta\right)$$

Therefore by following an IC commitment rule that is strictly increasing on $(\bar{\theta}, \bar{\theta}_s)$ and flat on $[\bar{\theta}_s, \tilde{\theta}]$, A_1 achieves a higher ex ante welfare while A_2 is indifferent compared to the case of simultaneous decision making. Now consider the sequence of actions

$\left\{x_1^{inf}(\theta)\right\}_{\theta\in(\bar{\theta},\bar{\theta}_s)}$ and checking the marginal utility of A_1 for each type in the interval,

$$U_1\left(\phi^1\left(x_1^s, x_2^s(\theta,\bar{m}_p)\right),\theta\right) < U_1\left(\phi^1\left(x_1^{inf}(\theta),\bar{k}\right),\theta\right) \tag{18}$$

Equation 18 follows from noting that utility of A_1 is decreasing in ϕ^1 on this interval and since $U_{11} < 0$ and $\phi^1\left(x_1^s, x_2^s(\theta,\bar{m}_p)\right) > \phi^1\left(x_1^{inf}(\theta),\bar{k}\right) = \phi^1_{inf}(\theta) > \bar{\phi}^1_{\theta}$. Now, on the interval $[\bar{\theta}_s,\tilde{\theta}]$, since $x_1^c(\theta) = x_1^s$, the ex ante commitment rule provides the same expected marginal utility as simultaneous protocol for A_1. Rewriting Equation 17 with the sequence $\left\{x_1^{inf}(\theta)\right\}_{\theta\in(\bar{\theta},\bar{\theta}_s)}$ and $\{x_1^s\}_{\theta\in[\bar{\theta}_s,\tilde{\theta}]}$, it is clear that,

$$\int_{\bar{\theta}}^{\bar{\theta}_s} U_1\left(\phi^1\left(x_1^{inf}(\theta),\bar{k}\right),\theta\right)\left.\frac{d\phi^1}{dx_1}\right|_{x_1=x_1^s} dF + \int_{\bar{\theta}_s}^{\tilde{\theta}} U_1\left(\phi^1\left(x_1^s,\bar{k}\right),\theta\right)\left.\frac{d\phi^1}{dx_1}\right|_{x_1=x_1^s} dF > 0 \tag{19}$$

$$\implies \int_{\bar{\theta}_s}^{\tilde{\theta}} U_1\left(\phi^1\left(x_1^s,\bar{k}\right),\theta\right)\left.\frac{d\phi^1}{dx_1}\right|_{x_1=x_1^s} dF > -\int_{\bar{\theta}}^{\bar{\theta}_s} U_1\left(\phi^1\left(x_1^{inf}(\theta),\bar{k}\right),\theta\right)\left.\frac{d\phi^1}{dx_1}\right|_{x_1=x_1^s} dF > 0 \tag{20}$$

From Equation 16, it is immediately clear that evaluating the equation at $t = \bar{\theta}_s$ and substituting $x_1^c(\bar{\theta}_s) = x_1^s$ gives,

$$\begin{aligned}\int_{\bar{\theta}_s}^{\tilde{\theta}} U_1\left(\phi^1\left(x_1^s,\bar{k}\right),\theta\right)\cdot\left.\frac{d\phi^1}{dx_1}\right|_{x_1=x_1^s} x_1^{c'}(\bar{\theta}_s)\quad dF \quad &> \\ &- \int_{\bar{\theta}}^{\bar{\theta}_s} U_1\left(\phi^1\left(x_1^{inf}(\theta),\bar{k}\right),\theta\right)\left.\frac{d\phi^1}{dx_1}\right|_{x_1=x_1^s} x_1^{c'}(\bar{\theta}_s)dF\end{aligned}$$

The above inequality follows from multiplying Equation 20 throughout by the term $x_1^{c'}(\bar{\theta}_s)$. Since $U_1\left(\phi^1\left(x_1^{inf}(\theta),\bar{k}\right),\theta\right) < 0$ everywhere on the interval $(\bar{\theta},\bar{\theta}_s)$, it follows that,

$$\int_{\bar{\theta}_s}^{\tilde{\theta}} U_1\left(\phi^1\left(x_1^s, \bar{k}\right), \theta\right) \cdot \left.\frac{d\phi^1}{dx_1}\right|_{x_1 = x_1^s} x_1^{c'}(\bar{\theta}_s) dF > 0$$

Therefore, the expected marginal utility of A_1 from choosing the cutoff $\bar{\theta}_s$ is increasing and this implies that $\bar{\theta}_c > \bar{\theta}_s$. Further, $x_1^c(\bar{\theta}_c) > x_1^s$.

A_1's welfare

By replacing the commitment protocol actions in the FOC of the simultaneous protocol (Equation 17), we get back Equation 19 which states that the commitment rule guarantees at least as much expected utility as the no commitment case since the marginal utility is increasing at the cutoff $\bar{\theta}_s$. Therefore, the expected utility from the commitment rule is greater compared to simultaneous protocol for A_1.

A_2's welfare

On the interval $[0, \bar{\theta}_c]$, A_2 achieves first best levels of joint coordination in that $\forall t \in [0, \bar{\theta}_c] : \phi^2(.) = \bar{\phi}_t^2$ under the optimal commitment rule. For all $t \in (\bar{\theta}_c, 1]$, $x_1^c(\bar{\theta}_c) > x_1^s$ and $x_1^c(\bar{\theta}_s) = x_1^s$ which implies $\bar{\phi}_t^2 > \phi^2(\bar{k}, x_1^c(\bar{\theta}_c)) > \phi^2(\bar{k}, x_1^s)$. Since there is under-provision (miscoordination) for A_2 on this interval,

$$\int_{\bar{\theta}_c}^{\tilde{\theta}} U\left(\phi^2(\bar{k}, x_1^c(\bar{\theta}_c)), \theta, b\right) dF > \int_{\bar{\theta}_c}^{\tilde{\theta}} U\left(\phi^2(\bar{k}, x_1^s), \theta, b\right) dF$$

The above inequality follows from noting that $U\left(\phi^2(\bar{k}, x_1^c(\bar{\theta}_c)), \theta, b\right) > U\left(\phi^2(\bar{k}, x_1^s), \theta, b\right)$ everywhere on the interval. Therefore, the overall expected ex ante welfare is greater under the optimal commitment mechanism. This completes the proof. **QED**

B Uniform Quadratic Example

Reconsider the example examined in Section 2. As in the example let the utility functions of the two agents be a quadratic loss function of the following form,

$$U^1 = -\left[\left(\frac{x_1 + \eta x_2}{1+\eta}\right) - \theta\right]^2$$

$$U^2 = -\left[\left(\frac{x_2 + \eta x_1}{1+\eta}\right) - \theta - b\right]^2$$

In accordance with the analysis of the previous sections let $\phi^1(x_1, x_2) = \left(\frac{x_1+\eta x_2}{1+\eta}\right)$ and $\phi^2(x_2, x_1) = \left(\frac{x_2+\eta x_1}{1+\eta}\right)$ be the respective joint coordination functions of the two agents. The parameter $\eta \in (0,1)$ measures the extent of interdependence between the two agents' actions. Let the action set of the two players be $V = [0, \bar{k}]$, where the lower bound is normalized, $\underline{k} = 0$[37]. As in Section 2, the state is uniformly distributed ($\theta \in \mathcal{U}[0,1]$) and perfectly observed only by A_2. Finally, miscoordination is simply interpreted as $\phi^1(x_1, x_2) >< \theta$ for A_1, and $\phi^2(x_2, x_1) >< (\theta + b)$ for A_2. The informed agent is able to convey information as long as revealing the type does not result in miscoordination in the action stage. Specifically, it is straightforward to observe that when the state is known then,

$$x_1^s(\theta) = \theta - \frac{\eta}{1-\eta}b; \qquad x_2^s(\theta) = \theta + \frac{1}{1-\eta}b \tag{21}$$

The most informative equilibrium $\bar{\theta}$ with communication (simultaneous protocol) can be immediately calculated by solving the equations,

$$x_1(\bar{\theta}) = (1+\eta)\bar{\theta} - \eta\bar{k}$$

$$\bar{k} = (1+\eta)(\bar{\theta} + b) - \eta x_1(\bar{\theta})$$

[37]This is without loss of generality since the lower bound does not affect the incentives of the informed player in the model.

All the subsequent results are represented as functions of the exogenous variables $(\eta, b, \bar{k})$. This way, the analysis of the uniform quadratic setting leads to interesting comparative statics with respect to these parameters. The following proposition characterizes all the cutoffs under both simultaneous and commitment protocols.

Proposition 5. *In the uniform quadratic setting,*

(a) There is no fully revealing equilibria if $\bar{k} \in \left((1+\eta)b, 1+\frac{1}{1-\eta}b\right)$*; only PRTE exists.*

(b) The most informative threshold is given by $\bar{\theta} = \bar{k} - \frac{1}{1-\eta}b$ *and* $\bar{m}_p = (\bar{\theta}, 1]$*. The actions of the two agents for all* $\theta \in [0, \bar{\theta}]$ *is given by* $x_1^s(\theta) = x_1^c(\theta) = \theta - \frac{\eta}{1-\eta}b$ *and* $x_2^s(\theta) = x_2^c(\theta, x_1^c(\theta)) = \theta + \frac{1}{1-\eta}b$*.*

(c) On the interval $\bar{m}_p$ *without commitment (simultaneous protocol):*

$$\bar{\theta}_s = \frac{(4+\eta)(1-\eta)}{(4+\eta)(1-\eta)+\eta}\bar{k} - \frac{(4+\eta)}{(4+\eta)(1-\eta)+\eta}b + \frac{\eta}{(4+\eta)(1-\eta)+\eta}$$

$$x_1^s(\bar{m}_p) = \frac{((4+\eta)(1-\eta)-1)}{(4+\eta)(1-\eta)+\eta}\bar{k} - \frac{(3+\eta)(1+\eta)}{(4+\eta)(1-\eta)+\eta}b + \frac{(1+\eta)}{(4+\eta)(1-\eta)+\eta}$$

$$x_2^s(\theta, \bar{m}_p) = \min\left\{\bar{k} \quad , \quad \bar{k} - (1+\eta)(\bar{\theta}_s - \bar{\theta})\right\}$$

(d) On the interval $\bar{m}_p$ *under commitment protocol:*

$$\bar{\theta}_c = \frac{2(1-\eta)}{(2-\eta)}\bar{k} - \frac{2}{(2-\eta)}b + \frac{\eta}{(2-\eta)}$$

$$\forall\theta \in (\bar{\theta}, \bar{\theta}_c) : x_1^c(\theta) = \frac{1+\eta}{\eta}(\theta + b) - \frac{1}{\eta}\bar{k}$$

$$\forall\theta \in [\bar{\theta}_c, 1] : x_1^c(\theta) = \frac{1+\eta}{\eta}(\bar{\theta}_c + b) - \frac{1}{\eta}\bar{k}$$

$$\forall\theta \in \bar{m}_p : x_2^c(\theta) = \bar{k}$$

(e) Let $\bar{\theta}_d = \bar{\theta}_c - \bar{\theta}_s$*. If* $\bar{k} \in \left((1+\eta)b, 1+\frac{1}{1-\eta}b\right)$*, then* $\bar{\theta}_d > 0$*.*

Proof. (a) To ensure partial information revelation, but not full or no informative communication, the following boundary conditions for truth-telling have to be satisfied. Specifically, it is straightforward to observe that when the state is known then $\bar{x}_1(\theta) = \theta - \frac{\eta}{1-\eta}b$ and $\bar{x}_2(\theta) = \theta + \frac{1}{1-\eta}b$. This implies that the boundary actions for the agents are,

$$(\bar{x}_1(0), \bar{x}_2(0)) = (0, (1+\eta)b)$$

$$(\bar{x}_1(1), \bar{x}_2(1)) = \left(1 - \frac{\eta}{1-\eta}b, 1 + \frac{1}{1-\eta}b\right)$$

Therefore for partial information revelation, $\bar{k} > (1+\eta)b$, $\bar{k} < 1 + \frac{1}{1-\eta}b$, and $(1+\eta)b < 1 + \frac{1}{1-\eta}b$. This proves the first part of the proposition.

(b) Under the *simultaneous protocol*, the action of agent A_1 given the pooling message $\bar{m}_p$, $x_1^s(\bar{m}_p)$, can be computed from the following equality,

$$\frac{\bar{k} + \eta x_1}{1+\eta} = (\bar{\theta}_s + b)$$

The above equation states that there exists a state $\bar{\theta}_s \in \bar{m}_p$ and an action of A_1, x_1^s, such that the informed agent's action is exactly equal to $\bar{k}$ (follows from Theorem 3 and the associated lemma). Rewriting the above gives,

$$x_1 = \frac{1+\eta}{\eta}(\bar{\theta}_s + b) - \frac{1}{\eta}\bar{k} \tag{22}$$

For A_2 the actions are best responses to x_1 in equilibrium,

$$\forall \theta \in (\bar{\theta}, \bar{\theta}_s) : x_2^s(\theta, \bar{m}_p) = (1+\eta)(\theta + b) - \eta x_1$$

Substituting for $x_1^s(\bar{m}_p)$ from Equation 22 and simplifying yields,

$$x_2^s(\theta, \bar{m}_p) = \bar{k} - (1+\eta)(\bar{\theta}_s - \theta) \tag{23}$$

Clearly, when $\theta > \bar{\theta}_s$, the agent's action is bounded by $\bar{k}$. This characterizes A_2's actions on the pooling interval. To solve for $x_1^s(\bar{m}_p)$, I use Equation 8 in the proof of Theorem 3. The maximization problem for the agent A_1 on the pooling interval can be stated as,

$$x_1^s(\bar{m}_p) \equiv \underset{x_1 \in [0,\bar{k}]}{\operatorname{argmax}} - \int_{\bar{\theta}}^{\bar{\theta}_s} \left(\frac{x_1 + \eta x_2^s(\theta, \bar{m}_p)}{1+\eta} - \theta \right)^2 f(\theta) d\theta - \int_{\bar{\theta}_s}^{1} \left(\frac{x_1 + \eta \bar{k}}{1+\eta} - \theta \right)^2 f(\theta) d\theta \tag{24}$$

The FOC yields,

$$- \int_{\bar{\theta}}^{\bar{\theta}_s} \left(\frac{x_1 + \eta x_2^s(\theta, \bar{m}_p)}{1+\eta} - \theta \right) f(\theta) d\theta - \int_{\bar{\theta}_s}^{1} \left(\frac{x_1 + \eta \bar{k}}{1+\eta} - \theta \right) f(\theta) d\theta = 0$$

Using equations 22 and 23, and rewriting the terms in the integral,

$$\frac{x_1 + \eta x_2^s(\theta, \bar{m}_p)}{1+\eta} = \frac{1}{\eta}(\bar{\theta}_s + b) - \frac{1-\eta}{\eta}\bar{k} - \eta(\bar{\theta}_s - \bar{\theta})$$

$$\frac{x_1 + \eta \bar{k}}{1+\eta} = \frac{1}{\eta}(\bar{\theta}_s + b) - \frac{1-\eta}{\eta}\bar{k}$$

Substituting them back into the FOC,

$$\int_{\bar{\theta}}^{\bar{\theta}_s} \left[(1-\eta^2)\bar{\theta}_s - (1-\eta)\bar{k} + b - \eta(1-\eta)\theta \right] f(\theta) d\theta + \int_{\bar{\theta}_s}^{1} \left[\bar{\theta}_s - (1-\eta)\bar{k} + b - \eta\theta \right] f(\theta) d\theta = 0$$

Applying the fact that the pdf of an uniform distribution over any interval (a, b) is given by $f(\theta) = \frac{1}{b-a}$, and solving the above equation for $\bar{\theta}_s$ gives:

$$\left[2 - \eta - \frac{\eta^2}{2} \right] \bar{\theta}_s = \frac{(4+\eta)(1-\eta)}{2}\bar{k} - \frac{(4+\eta)}{2} b + \frac{\eta}{2}$$

Simplifying the above gives,

$$\bar{\theta}_s = \frac{(4+\eta)(1-\eta)}{(4+\eta)(1-\eta)+\eta}\bar{k} - \frac{(4+\eta)}{(4+\eta)(1-\eta)+\eta}b + \frac{\eta}{(4+\eta)(1-\eta)+\eta}$$

Substituting this back into Equation 22 and simplifying gives,

$$x_1^s(\bar{m}_p) = \frac{1+\eta}{\eta}\left[\frac{(4+\eta)(1-\eta)}{(4+\eta)(1-\eta)+\eta}\bar{k} - \frac{\eta(3+\eta)}{(4+\eta)(1-\eta)+\eta}b + \frac{\eta}{(4+\eta)(1-\eta)+\eta}\right] - \frac{1}{\eta}\bar{k}$$

$$\implies x_1^s(\bar{m}_p) = \frac{((4+\eta)(1-\eta)-1)}{(4+\eta)(1-\eta)+\eta}\bar{k} - \frac{(3+\eta)(1+\eta)}{(4+\eta)(1-\eta)+\eta}b + \frac{(1+\eta)}{(4+\eta)(1-\eta)+\eta}$$

Together the set of actions $\left(x_1^s, x_2^s(\theta, \bar{m}_p)\right)_{\theta\in\bar{m}_p}$ completely describe the profile of actions for the agents when information is pooled under no commitment.

(c) With commitment, A_2's action is always $\bar{k}$ on the pooling interval. A_1 chooses instead a cutoff $\bar{\theta}_c$ and a stream of actions $x_1^c(\theta)$ for all $\theta \in (\bar{\theta}, \bar{\theta}_c)$ on $\bar{m}_p$. The stream of actions can be computed from the following,

$$\frac{\bar{k}+\eta x_1^c(\theta)}{1+\eta} = \theta + b$$

$$\implies x_1^c(\theta) = \frac{1+\eta}{\eta}(\theta+b) - \frac{1}{\eta}\bar{k}$$

Proposition 3 characterizes the optimal cutoff choice problem for A_1. Rewriting the equation in Proposition 3 for the parameterized case,

$$\bar{\theta}_c \equiv \underset{t\in\bar{m}_p}{\text{argmax}} - \int_{\bar{\theta}}^{t} \left(\frac{\frac{1+\eta}{\eta}(\theta+b) - \frac{1-\eta^2}{\eta}\bar{k}}{1+\eta} - \theta \right)^2 f(\theta)d\theta - \int_{t}^{1} \left(\frac{\frac{1+\eta}{\eta}(t+b) - \frac{1-\eta^2}{\eta}\bar{k}}{1+\eta} - \theta \right)^2 f(\theta)d\theta \tag{25}$$

To find the solution to the above equation, I can focus attention only to the second integral in the FOC. (The terms that differentiate the limits vanish in the FOC.)

$$\int_{t}^{1} \left(\frac{1}{\eta}(t+b-(1-\eta)\bar{k}) - \theta \right) f(\theta)d\theta = 0$$

$$\frac{1}{\eta}(t+b-(1-\eta)\bar{k}) - \frac{1+t}{2} = 0$$

Simplifying this equation gives the required expression for $\bar{\theta}_c$.

$$\bar{\theta}_c = \frac{2(1-\eta)}{(2-\eta)}\bar{k} - \frac{2}{(2-\eta)}b + \frac{\eta}{(2-\eta)}$$

(d) Subtracting the two thresholds yields,

$$\bar{\theta}_d = \frac{\eta(2+\eta)}{8-8\eta+\eta^3}b - \frac{\eta(1-\eta)(2+\eta)}{8-8\eta+\eta^3}\bar{k} + \frac{\eta(1-\eta)(2+\eta)}{8-8\eta+\eta^3}$$

Simplifying and collecting terms,

$$\bar{\theta}_d = \frac{\eta(2+\eta)}{8-8\eta+\eta^3}\left[b+(1-\eta)(1-\bar{k})\right]$$

Clearly $\bar{\theta}_d > 0$ as long as $\bar{k} < 1 + \frac{1}{1-\eta}b$, which is a sufficient condition for a PRTE to exist. This completes the proof. **QED**

□

The result provides an intuitive closed form characterization of the most informative equilibria. Parts (a) and (b) result from the fact that if the truth-telling condition holds for the lowest type but not for the highest type then a cutoff in between must exist up to which there is full revelation and beyond which there is full pooling. In other words, the result identifies the precise conditions under which HTIC breaks down. The informational thresholds $\bar{\theta}_s$ and $\bar{\theta}_c$ represent the cutoffs up to which the informed agent achieves first best levels of coordination in the no commitment and commitment cases respectively. The expressions for the two thresholds follow from equation 8 and Proposition 3 respectively. The final part says that in the case where only a PRTE exists, i.e. the conditions on $(\bar{k}, b, \eta)$ are according to part (a), the relation $\bar{\theta}_c > \bar{\theta}_s$ always holds.

Proposition 6. *In the uniform quadratic setting, the ex ante expected welfare of the agents are,*

(a) Under simultaneous protocol:

$$W_s^1 = -\frac{2}{3}\frac{(\eta^2+4\eta+5)(1-\eta)^2}{(4-2\eta-\eta^2)^2}\left(1-\bar{k}+\frac{b}{(1-\eta)}\right)^2$$

$$W_s^2 = -\frac{1}{3}\frac{(4+\eta)^2(1-\eta)^2}{(4-2\eta-\eta^2)^2}\left(1-\bar{k}+\frac{b}{(1-\eta)}\right)^2$$

(b) Under commitment protocol:

$$W_c^1 = -\frac{2}{3}\frac{(1-\eta)^2}{(2-\eta)^2}\left(1-\bar{k}+\frac{b}{(1-\eta)}\right)^2$$

$$W_c^2 = -\frac{4}{3}\frac{(1-\eta)^2}{(2-\eta)^2}\left(1-\bar{k}+\frac{b}{(1-\eta)}\right)^2$$

(c) The welfare of both agents is increasing in $\bar{k}$ and decreasing in b, under both protocols.

(d) *The welfare gains from commitment is positive for both agents, decreasing in $\bar{k}$, and increasing in b.*

Proof. The welfare of the agents are computed using the thresholds derived in the previous proposition.

A_2's Welfare:

A_2 achieves first best on the interval $[0, \bar{\theta}_s]$ under the simultaneous protocol and on $[0, \bar{\theta}_c]$ with commitment. As a result the welfare calculation is straightforward. Since $\bar{\phi}_{\bar{\theta}_s} = \bar{\theta}_s + b$, in the simultaneous protocol,

$$W_s^2 = -\int_{\bar{\theta}_s}^{1} \left(\bar{\theta}_s - \theta\right)^2 f(\theta) d\theta$$

Very simple algebra yields,

$$W_s^2 = \frac{2}{3}\bar{\theta}_s - \frac{1}{3}\bar{\theta}_s^2 - \frac{1}{3}$$

Substituting for $\bar{\theta}_s$ and simplifying,[38]

$$W_s^2 = -\frac{1}{3}\frac{(4+\eta)^2(1-\eta)^2}{(4-2\eta-\eta^2)^2}\left(1 - \bar{k} + \frac{b}{(1-\eta)}\right)^2$$

Similarly, with commitment $\bar{\phi}_{\bar{\theta}_c} = \bar{\theta}_c + b$ and therefore,

$$W_c^2 = \frac{2}{3}\bar{\theta}_c - \frac{1}{3}\bar{\theta}_c^2 - \frac{1}{3}$$

$$W_c^2 = -\frac{4}{3}\frac{(1-\eta)^2}{(2-\eta)^2}\left(1 - \bar{k} + \frac{b}{(1-\eta)}\right)^2$$

[38]The Mathematica code for all the computations in this Proposition are available upon request.

The welfare gains (value) of commitment for A_2 is just the difference between the two payoffs,

$$G^2 = W_c^2 - W_s^2 = \frac{1}{3}\frac{\eta(\eta+2)(1-\eta)^2(16-6\eta-3\eta^2)}{(2-\eta)^2(4-2\eta-\eta^2)^2}(1-\bar{\theta})^2 \tag{26}$$

A_1's Welfare

For agent A_1 the expected ex ante welfare without commitment can be written in terms of the cutoffs $(\bar{\theta}, \bar{\theta}_s)$ and the pooling action $x_1^s \equiv x_1^s(\bar{m}_p)$, and is given by,

$$W_s^1 = -\int_{\bar{\theta}}^{\bar{\theta}_s}\left(\frac{x_1^s + \eta x_2^s(\bar{m}_p)}{1+\eta} - \theta\right)^2 f(\theta)d\theta - \int_{\bar{\theta}_s}^{1}\left(\frac{x_1^s + \eta\bar{k}}{1+\eta} - \theta\right)^2 f(\theta)d\theta$$

$$\begin{aligned} W_s^1 = &-\int_{\bar{\theta}}^{\bar{\theta}_s}\left(\frac{(1-\eta)}{\eta}\left((1+\eta)\bar{\theta}_s - \bar{\theta}\right) - (1-\eta)\theta\right)^2 f(\theta)d\theta \\ &-\int_{\bar{\theta}_s}^{1}\left(\frac{(1-\eta)}{\eta}\left(\frac{1}{(1-\eta)}\bar{\theta}_s - \bar{\theta}\right) - \theta\right)^2 f(\theta)d\theta \end{aligned}$$

$$\begin{aligned} W_s^1 = &-\frac{(1-\eta)^2}{\eta^2}\left((1+\eta)\bar{\theta}_s - \bar{\theta}\right)^2 + \frac{(1-\eta)^2}{\eta}\left((1+\eta)\bar{\theta}_s - \bar{\theta}\right)\left(\bar{\theta}_s + \bar{\theta}\right) \\ &-\frac{(1-\eta)^2}{\eta^2}\left(\frac{1}{(1-\eta)}\bar{\theta}_s - \bar{\theta}\right)^2 + \frac{(1-\eta)}{\eta}\left(\frac{1}{(1-\eta)}\bar{\theta}_s - \bar{\theta}\right)(1+\bar{\theta}_s) \\ &-\frac{(1-\eta)^2}{3}(\bar{\theta}_s^2 + \bar{\theta}\bar{\theta}_s + \bar{\theta}^2) - \frac{1}{3}(\bar{\theta}_s^2 + \bar{\theta}_s + 1) \end{aligned}$$

Substituting for the expressions and simplification yields,

$$W_s^1 = -\frac{2}{3}\frac{(\eta^2+4\eta+5)(1-\eta)^2}{(4-2\eta-\eta^2)^2}\left(1-\bar{\theta}\right)^2 \tag{27}$$

With commitment,

$$W_c^1 = -\int_{\bar{\theta}}^{\bar{\theta}_c} \left(\frac{x_1^c(\theta) + \eta \bar{k}}{1+\eta} - \theta \right)^2 f(\theta) d\theta - \int_{\bar{\theta}_c}^{1} \left(\frac{x_1^c(\bar{\theta}_c) + \eta \bar{k}}{1+\eta} - \theta \right)^2 f(\theta) d\theta$$

$$W_c^1 = -\int_{\bar{\theta}}^{\bar{\theta}_c} \frac{(1-\eta)^2}{\eta^2} \left(\theta - \bar{\theta}\right)^2 f(\theta) d\theta - \int_{\bar{\theta}_c}^{1} \left(\frac{(1-\eta)}{\eta} \left(\frac{1}{(1-\eta)} \bar{\theta}_c - \bar{\theta} \right) - \theta \right)^2 f(\theta) d\theta$$

Simplifying this gives,

$$\begin{aligned} W_c^1 = & -\frac{(1-\eta)^2}{\eta^2} \bar{\theta}^2 - \frac{(1-\eta)^2}{\eta} \bar{\theta}(\bar{\theta}_c + \bar{\theta}) - \frac{(1-\eta)^2}{\eta^2} \left(\frac{1}{(1-\eta)} \bar{\theta}_c - \bar{\theta} \right)^2 \\ & + \frac{(1-\eta)}{\eta} \left(\frac{1}{(1-\eta)} \bar{\theta}_c - \bar{\theta} \right) (1 + \bar{\theta}_c) - \frac{(1-\eta)^2}{3\eta^2} (\bar{\theta}_c^2 + \bar{\theta}\bar{\theta}_c + \bar{\theta}^2) - \frac{1}{3} (\bar{\theta}_c^2 + \bar{\theta}_c + 1) \end{aligned}$$

Expanding the terms $(\bar{\theta}, \bar{\theta}_c)$ and simplifying gives,

$$W_c^1 = -\frac{2}{3} \frac{(1-\eta)^2}{(2-\eta)^2} \left(1 - \bar{\theta}\right)^2 \tag{28}$$

The gains from commitment for A_1 is,

$$G^1 = W_c^1 - W_s^1 = \frac{1}{3} \frac{(\eta+2)(1-\eta)^2 (2 + 5\eta - 4\eta^2)}{(2-\eta)^2 (4 - 2\eta - \eta^2)^2} (1 - \bar{\theta})^2 \tag{29}$$

Since $\frac{d\bar{\theta}}{d\bar{k}} > 0$ and $\frac{d\bar{\theta}}{db} < 0$, the statements in part (c) and (d) of the Proposition follow trivially from the expressions W_j^i and G^i. This completes the proof. **QED**

□

Proposition 6 shows that the welfare of agents – with or without commitment – is positively correlated with the information threshold $\bar{\theta} = \left(\bar{k} - \frac{b}{(1-\eta)} \right)$. That is, greater the threshold, higher the welfare of agents. As a consequence, any change in the exogenous parameters $(\bar{k}, b)$ also affects the welfare of agents by shifting the information threshold. Finally, the *value of commitment* is always positive as long as there is no full revelation

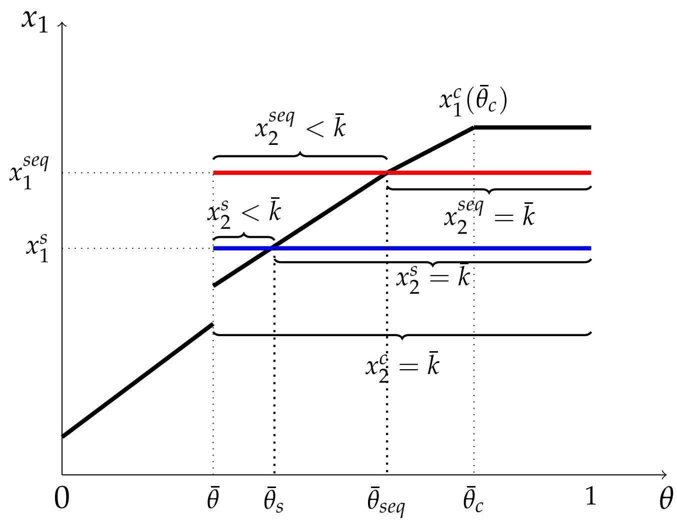

Figure 7: As the degree of commitment $j \in \{s, seq, c\}$ varies, the threshold on the pooling interval $\bar{\theta}_j$ also changes. Specifically it is increasing in the degree of commitment, $\bar{\theta}_c > \bar{\theta}_{seq} > \bar{\theta}_s$. The en-ante welfare of agents are also ordered accordingly, i.e. $W_c^j > W_{seq}^j > W_s^j$ for $j \in \{1, 2\}$.

of information. However, the gains from commitment diminish as $\bar{\theta}$ increases. If more information is conveyed in the absence of commitment, then the scope for welfare improvement is smaller. This decreases the value of commitment. Clearly, an increase in $\bar{k}$ or a decrease in the conflicts of interest b increases information threshold $\bar{\theta}$, and as a result increases the welfare of agents but reduces the value of commitment.

C Additional Results: Sequential Protocol

In sequential decision-making, the uninformed agent is the *stackelberg leader* in that A_1 takes an action first, followed by the informed A_2. The degree of commitment is in between the simultaneous and commitment protocols. To put it differently, the sequential protocol allows for ex-post commitment by the uninformed agent, i.e., A_1 commits to an action rule after communication. Figure 7 captures how a greater degree of commitment to the uninformed agent results in increased welfare for both agents. See the Online Appendix.