Formal insurance and risk-sharing networks

Tizié Bene

November 28, 2023

For the latest version, please click here

Abstract

This paper examines the influence of formal insurance on the configuration of risk-sharing networks. When facing idiosyncratic risks, agents can choose between forming costly risk-sharing links and purchasing formal insurance. I characterize the equilibrium of the insurance game and show that an agent's equilibrium demand for formal insurance decreases with the number of agents he can rely on. I find that formal insurance and risk-sharing networks are substitutes. Moreover, the price of formal insurance determines how agents' incentives to form links vary with the number of agents they are connected to. For some price levels, there is multiplicity in the structure of stable networks. When the linking cost is high, the unraveling of the risk-sharing network is gradual, and the number of components in a stable network decreases weakly and progressively as the price of insurance decreases. When the linking cost is low, the unraveling is abrupt and agents go from being connected to the entire community to relying only on the formal insurance.

Keywords: Social networks, Formal Insurance, Informal risk sharing

JEL: C72, D85

1 Introduction

In poor regions, individuals are frequently confronted with a myriad of risks, ranging from health emergencies to economic instability, see Banerjee and Duflo (2011). Traditionally, these populations have been underserved by conventional insurance products, often due to a mismatch between the offerings and their specific needs. This disconnect opens the door to innovative market opportunities. Microinsurance emerges as a response to this challenge, providing tailored insurance solutions aimed at the unique circumstances of people in low-income countries. The objective is twofold: to address the unmet needs of these communities and to achieve profitability, see Merry and Rozo Calderon (2022), LLoyd's (2009). However, in general, a market for insurance does not emerge in a vacuum since in the absence of accessible formal insurance, individuals often depend on social networks as a safety net (Udry (1994), Fafchamps and Lund (2003)). These networks, rooted in social ties, function as informal risk-sharing mechanisms. Interestingly, they have been found to react to financial incentives and market changes, see Banerjee et al. (2021) and Binzel et al. (2015). This leads to the question of this study: How does formal insurance affect the structure of risk-sharing networks? This paper delves into this inquiry, exploring the interplay between formal insurance mechanisms and traditional social safety nets.

I propose a model where to insure against an idiosyncratic risk, individuals have the discretion to both form risk-sharing links and purchase formal insurance in the market. After shocks and insurance payouts, agents support those in need through private transfers. The study finds that the insurance market significantly influences social network structure. As the (exogenous) price of the formal insurance decreases, the incentive to form links also decreases, confirming the intuition that risk-sharing networks and formal insurance are substitutes. Interestingly, the price of insurance also determine how agents incentives to form links vary with their component size, i.e., the number of agents they can rely on. Furthermore, high linking costs lead to a gradual unraveling of the network as formal insurance prices drop. Conversely, low linking costs lead to an abrupt transition from a fully connected network to the empty network. I find that for the same level of prices, stable networks can differ in structure. For a society that sees the number of risk-sharing link drop after an intervention like insurance subsidies, the community can be stock on a Pareto dominated equilibrium, with less risk-sharing links after the program ends. Overall, this analysis gives vital insight for understanding how social network providing informal insurance react to formal insurance.

I consider a benchmark where first, agents form risk-sharing links. Second, they choose their formal insurance coverage. Third, shocks are realized and indemnities are paid. And fourth, agents make private transfers to help the unlucky ones. Moreover, Agents are symmetric. However, their positions in the network affects their incentives to maintain a link. Once a link is established, the pair of agents involved commit to a bilateral risk-sharing agreement which result in equal-sharing at the component level.[1][2] Note that the equal-sharing rule at the component level corresponds to the efficient level of insurance when agents have the same social weight. Therefore, any divergence between efficient networks and stable networks would result from the network formation process. For tractability, I use constant absolute risk aversion utilities for each agent and a normal distribution for all shocks that I consider to be independent from one another. The analysis is then developed in several stages

First, I characterize the equilibrium of the insurance game. I show that for a given price of formal insurance, an agent's demand is weakly decreasing with the number of agents she is connected to (directly or indirectly). This illustrate a substitution effect between formal insurance and risk sharing. I compute indirect utility and show that agents are better off in a larger component when the price of insurance is greater than the actuarial price. Otherwise, they are indifferent.

Second, for different price levels of formal insurance, I characterize stable networks by proposing a refinement of pairwise stability with utility transfer (PS^t) criteria as presented by Bloch and Jackson (2006).[3] I show that the price of formal insurance has a direct effect on risk sharing; the incentive to form links increases with the price of formal insurance, confirming the substitution effect between formal insurance and the risk-sharing network. This effect implies that the number of component in a stable network weakly increases when the price of formal insurance decreases.

[1] Bramoullé and Kranton (2007) and Ambrus and Elliott (2021) propose two ways of micro-founding this hypothesis

[2] There is no enforcement concern and no renegotiation.

[3] This approach guarantees the existence of stable networks and reduces the number of potential stable networks.

Third, I find that the price of formal insurance also has an indirect effect; it determines how agents' incentive to form links varies with the size of their component, i.e. the number of agents they are connected to. A direct consequence of this effect is that when the cost of a link is high, the unraveling of the risk-sharing network is gradual and the number of component in a stable network weakly increases progressively as the price of insurance decreases. In the opposite, when the cost of a link is low, the unraveling is abrupt and agent goes directly from being connected to the entire community (directly or indirectly) to only relying on the insurance market (formal insurance). Furthermore, I find that stable networks can exhibit varied structures for the same price of formal insurance. Banerjee et al. (2021) observed that social networks did not expand or form new connections after micro-credit programs were introduced and subsequently canceled. This model suggests that this could be explained in the context of formal insurance: individuals might remain confined in a stable network with smaller component and no possibilities the formation of new stable links even after the price of formal insurance has increased.

Fourth, I analyze the welfare and show that the Nash equilibrium of the insurance game is constrained Pareto efficient. Conditional on the structure of the social network, individual incentives to adopt formal insurance are aligned with social welfare. This surprising result is due to the fact the costs of an individual taking out insurance are shared among those in her component, as well as any payments made by the insurance company. It guarantees that private and social incentives are aligned. Moreover, when there is multiplicity in the structure of stable networks, the empty network, where agent totally rely on formal insurance, is always Pareto dominated. For a community large enough, when the price of formal insurance is relatively low, stable networks connect weakly fewer agents than efficient networks. When the price of formal insurance is high, stable networks can connect more agents than efficient networks. These findings provide insights into how the impact of the pricing of formal insurance on the structure of social networks impact community welfare.

This paper inserts in a huge literature on risk-sharing in communities.[4] This analy-

[4] A non-exhaustive list of paper include Rosenzweig (1988), Cochrane (1991), Townsend (1994), Fafchamps and Lund (2003), Fafchamps and Gubert (2007), Fafchamps (2011), Ligon and Schechter (2012), Mazzocco and Saini (2012), Belhaj et al. (2014) and Chiappori et al. (2014)

sis contributes, first, to the literature on formation and stability of risk-sharing networks.[5] Bramoullé and Kranton (2007) propose a model in which pairs of agents form risk-sharing links. They assume that each pair with a link commits to share money equally each time they meet. They show that when the round of meetings is large, there is equal sharing at the component level. In this context, they find that equilibrium risk-sharing networks always connect fewer agents than efficient networks. Ambrus and Elliott (2021) propose a model where agents can bargain over the surplus generated by the risk-sharing activity. Each pair of agents with a link make transfers that are pairwise efficient, i.e., that leave no gains from trading on the table between any two agents who have a risk-sharing link. It is shown that this leads to component-level efficiency and hence to an equal distribution of component income among the members of that component in all states of the world. They find that the most stable efficient network generates the most inequality. I build on these papers and propose a model where there is equal sharing at the component level. Bloch et al. (2008) consider a setting where risk-sharing links also serve to diffuse information and where informal transfers between agents obey a social norm. In contrast to the two papers mentioned above, they consider an exogenous network in which agents can renounce their say and fail to honor their ex-ante risk-sharing commitment ex-post. In this case, links serve as information conduits so that agents who deviate from the social norm can be punished. They find that thickly and thinly connected networks tend to be stable. In contrast, I consider a model in which agents form links and commit to sharing their monetary holdings equally, taking into account the price of formal insurance. Bene et al. (2021) consider a demand for formal insurance against an aggregate shock when agents are embedded in an altruism network. Incomes are subject to an aggregate shock and an individual shock, and an insurance company offers coverage against the aggregate shock. They find that the demand for formal insurance with altruism is higher than without altruism at low prices and lower at high prices. However, they consider the network as fixed. This paper consider that the risk-sharing network is endogenous and it react to the price of formal insurance.

This contributes, second, to the literature on the impact of markets on informal institu-

[5] One branch of this literature looks at informal transfers on networks, see, e.g., Ambrus et al. (2014), Bourlès et al. (2017); Bourlès et al. (2021), Ambrus et al. (2022)

tions. Gagnon and Goyal (2017) propose a model in which agents embedded in an exogenous network choose between a network and a market binary action. They assume that the two actions are either substitutes or complements and analyze equilibria, welfare, and inequality. In contrast, market actions in my setup are not binary, and whether the two actions are substitutes or complements is not assumed, but rather a main result of the analysis. Alger and Weibull (2010, 2007) propose a model of informal transfers motivated by altruism (or coerced altruism) between siblings and see how family ties respond to market incentives and how these ties affect economic outcomes. I consider a larger number of agents who can form and break ties. They do not have altruism towards each other, but they make utility transfers to form a new link or maintain an existing one. Overall, I focus on the interplay between formal insurance and endogenously formed risk-sharing networks.

This analysis contributes, third, to the literature on the interplay between informal transfers and formal insurance. One branch of this literature examines how the introduction of formal insurance affects existing informal arrangements. Attanasio and Ríos-Rull (2000) examines the effects of mandatory insurance against covariate risks on informal risk-sharing, particularly when the latter is constrained by limited commitment. The results suggest that formal insurance can crowd out informal risk sharing, potentially leading to welfare losses under certain conditions outlined in the study. Boucher, Delpierre, and Verheyden (2016) explores how index-based insurance and informal risk-sharing arrangements interact. They propose a model where moral hazard exists and agents are in an exogenous risk-sharing group. They find that formal insurance may crowd out informal insurance if the insurance contract is proposed to individuals. They also find that welfare can fall if the price of index insurance is high. Takahashi, Barrett, and Ikegami (2019) uses social network data on pastoralists in southern Ethiopia to study the impact of formal index insurance on informal risk-sharing. They find that index insurance does not crowd out informal risk sharing, and even find weak evidence of a crowding effect. In contrast, I consider a context where there is no a priori reason to suspect complementarity between formal insurance and informal risk-sharing (agents face independent idiosyncratic shocks). Despite this consideration, I find that isolated agents may engage in more risk-sharing activities after the introduction of formal insurance. Moreover, I consider a classical formal insurance product and assume that agents are free to purchase it. I also abstract from commitment and moral hazard issues to focus on the impact of the price of formal insurance on network structure.

Another branch of this literature looks at how the existence of informal institutions providing insurance affects the diffusion of formal ones. Arnott and Stiglitz (1991) showed early on that informal risk sharing can crowd out the demand for formal insurance. In their framework, informal risk-sharing takes place within pairs of symmetric agents and under moral hazard. In contrast, I consider risk-sharing networks linking heterogeneous agents and without moral hazard. In an empirical study of rural India, Rosenzweig (1988) finds that private transfers in networks of family and friends play a central role in risk sharing and often crowd out formal loans. These results are consistent with my theoretical results. An agent's demand for formal insurance decreases with the number of agents she is connected to (directly or indirectly). De Janvry, Dequiedt, and Sadoulet (2014) analyzes the demand for formal insurance against common shocks, when individual utility depends on individual and aggregate wealth. They highlight strategic interactions and free-riding in individual decisions to adopt formal insurance. In this paper, agents do not free-ride and consider the structure of the network rather than the decision of others when purchasing formal insurance. Overall, I provide an analysis of the demand for formal insurance when agents make informal transfers through networks.

The rest of the paper is organized as follows. In Section 2, I present the model. I characterize the insurance game and present an agent's expected utility given the size of her component in Section 3. I introduce the stability criteria in Section 4 and characterize stable networks for different price levels of formal insurance in Section 5.

2 Model

I consider a society composed of $n \geq 2$ agents in a set N. Incomes are stochastic and subject to an idiosyncratic shock. To mitigate this risk, agents can create costly links with others, generating an undirected risk-sharing network. There is also an external institution that sells formal insurance covering damages. Each agent decides ex-ante, how much formal insurance to buy. Once incomes and insurance claims are realized, linked agents make transfers to one another. I assume that decisions concerning insurance coverage are non-cooperative while the creation of a link between two agents requires mutual agreement. The model thus has 4 stages. In stage 1, agents form links to share risks. In stage 2, they choose their insurance coverage. In stage 3, income shocks and insurance claims are realized. In stage 4, agents make private transfers, conditional on realized incomes.

Stochastic incomes. Each agent i has an initial income y_i^0 and faces an idiosyncratic shock ϵ_i. For tractability, I assume that idiosyncratic shocks are independent and identically normally distributed, with expected value $\mu \geq 0$ and variance σ^2 for every agent.[6] This could represent health shocks on agents, affecting their productivity, or on livestock. The stochastic initial income for agent i is $y^0 - \epsilon_i$.

Insurance decisions. An external insurer offers an insurance contract against the idiosyncratic shock. This could represent health insurance for people or micro-insurance for livestock. For an agent i, with coverage $x_i \in [0, 1]$ the price of formal insurance is px_i with unit price $p > 0$ proposed exogenously by the insurer. Note that $\mathbf{x}$ is the vector of coverage of every agent in the network. From an ex-ante point of view, after agents have chosen their coverage and before transfers are made, agent i stochastic income is

$$y_i = y^0 - px_i - \epsilon_i(1 - x_i).$$

The market mechanisms determining the price of formal insurance will not be analysed. I focus on the demand for formal insurance in this analysis and take the price as given. I will later study the impact of a price variation on the structure of the network.

Definition of links and networks. In this society, the linking cost is $\kappa \geq 0$. This is the cost required for any link between two agents to be established. It can be seen as the effort required for a link to exist between two agents. However, I am agnostic about the way this cost is shared between them. It can be that one agent bears alone the cost needed for the formation of a link. It can also be that this cost is shared in half or another proportion.

[6] The assumption that shocks are normally distributed is common in the literature on incentives on the insurance market, see, e.g., Azevedo and Gottlieb (2017), Einav et al. (2013), Farinha Luz et al. (2023), Weyl and Veiga (2017), Veiga and Levy (2022)

Either way, as long as the cost is paid, the link is formed. It is assumed to be a non-monetary cost, so one can see it as the total disutility of forming a link. An undirected link between two agents i and j is denoted $g_{ij} = g_{ji} = 1$. The absence of a link is denoted $g_{ij} = g_{ji} = 0$. By convention, $g_{ii} = 0$. The network $g = \{(g_{ij})\}_{i,j \in N}$ is a formal description of the links between every pair of agents. There is a ***path*** between two individuals i and j in the graph g if there exists a sequence of individuals $i_1, ..., i_k$ such that $g_{ii_1} = g_{i_1 i_2} = ... = g_{i_k j} = 1$, and this path is a ***cycle*** if $g_{ii_1} = g_{i_1 i_2} = ... = g_{i_k i} = 1$. All individuals with whom agent i has a path define the component of i in g , which is denoted S^i. A ***tree*** is a graph with no cycles. A ***star*** is a graph where one agent is involved in all links. A ***line*** is a graph with no cycles where each agent is linked to two others except for the two end agents. A network is a ***forest*** when each of its components is a tree. A network is a ***star forest*** when each of its components is a star. A network is a ***line forest*** when each of its components is a line.

Informal transfers. In stage 4, once shocks and insurance claims are realized, agents make informal transfers to each other. I consider equal sharing at the component level. This assumption can be motivated by Bramoullé and Kranton (2007) where each pair of linked agents commit to share their income realisations equally and meet repeatedly enough to reach equal sharing at the component level once shocks and insurance claims are realized. If we give the same social weight to every agent, equal sharing at the component level can also be motivated by Ambrus and Elliott (2021), where each pair of linked agents chooses transfers that are pairwise efficient, i.e., that maximize the sum of their expected utilities. Agent i's private preferences are represented by a Constant Absolute Risk Aversion (CARA) utility function: $U_i(y) = a - e^{-\lambda y}$, with $a > \mathbb{E}U(\frac{\sum_{i=1}^{s} y_i}{s})$. For an undirected network g, the size of the component of an agent i is denoted by $s^i(g)$. For an agent i in a component of s agents, $s^i(g) = s$. Then, due to the CARA-Normal framework[7], each agent in a component of size s gets

$$\begin{aligned} u(s, \mathbf{x}) &= \mathbb{E}U(\frac{\sum_{i=1}^{s} y_i}{s}) \\ u(s, \mathbf{x}) &= a - \exp\left\{-\lambda\left(y^0 - \mu - (p - \mu)\sum_i \frac{x_i}{s} - \frac{\lambda\sigma^2}{2}\sum_i\left(\frac{1 - x_i}{s}\right)^2\right)\right\}. \end{aligned} \tag{1}$$

[7]see Sargent (1987)

In what follows, I first find the equilibrium coverage demand of an agent for a given network g. I then use this demand to compute the indirect expected utility and solve for stable networks given different levels of formal insurance price.

3 Insurance demand

In this section, I compute the equilibrium coverage of every agent for any given risk-sharing network. I find that if the insurance price is lower than or equal to the actuarial price, every agent chooses to be fully covered against the risk. Otherwise, agents choose partial coverage. In the latter case, the coverage of an agent decreases with s, the size of his component, i.e., the number of agents she can directly or indirectly rely on. I also compute the expected indirect utility. I show that when the price of insurance is lower than or equal to the actuarial price, it does not depend on the s. Otherwise, the expected indirect utility is a strictly increasing and strictly concave function of s.

Given the timing of the game, each agent anticipates her equilibrium coverage rate before creating links. Therefore I solve the insurance game by backward induction. Proposition 1 characterizes the equilibrium coverage rate.

Proposition 1. *A profile of insurance decision,* $\mathbf{x}^*$*, is a Nash equilibrium of the insurance game if and only if for any agent* i *in a component of size* s*,*

$$x_i^*(s) = min\left(max\left(0, 1 - s\frac{p-\mu}{\lambda\sigma^2}\right), 1\right).$$

The proof is relegated to the appendix ((*see proof*)). One first implication of CARA preferences is that an agent's demand does not depend on her wealth. Another property of this type of preference is that an agent's choices do not depend on others' shocks, since they are independent. As a consequence, the insurance game exhibits strategic independence and the best response of one agent does not depend on what others in her component do.

The comparative statics of insurance demand follow directly from Proposition 1. The individual demand for formal insurance x_i^* is non-increasing in price p and weakly increasing in the absolute risk aversion λ, mean μ and variance of the shock σ^2. When the insurance price is lower than or equal to the actuarial price ($p \leq \mu$), every agent is fully covered ($x_i^*(s) = 1$) whatever the size of her component. When the price is higher than the actuarial price ($p > \mu$), the equilibrium coverage rate of an agent is weakly decreasing in the number of agents in the components, it decreases until it reaches zero and then becomes flat. Overall, there is a substitution effect between formal insurance and informal risk-sharing networks since the coverage level chosen by an agent depends on the number of people he can rely on directly or indirectly.

In addition, there exists a threshold on the component size above which, the equilibrium coverage rate for each agent in the component is zero. In fact

$$x_i^*(s) = 0 \Leftrightarrow s \geq \frac{\lambda\sigma^2}{p - \mu}. \tag{2}$$

Define $\lceil x \rceil$ as the ceiling value of the real number x, i.e., the smallest integer larger than or equal to x. I derive from the previous result that

Corollary 1. *For $p > \mu$ there exists $\tilde{s}$ such that if $s \geq \lceil \tilde{s} \rceil$ the equilibrium coverage rate $x_i^*(s) = 0$ and if $s < \lceil \tilde{s} \rceil$ the equilibrium coverage rate $x_i^*(s) \in (0, 1)$. This threshold is equal to*

$$\tilde{s} = \frac{\lambda\sigma^2}{p - \mu}.$$

Figure 1 illustrates how an agent's demand for formal insurance varies with the size of her component. I consider a community of $n = 8$ agents with $\mu = 0.64$, $\sigma^2 = 2.055$, $\lambda = 2.64$. The solid line represents the demand when the price of formal insurance is lower than or equal to $\mu = 0.64$. The dash-dotted line represents the demand when $p = 1.855$. In this case, the component's size at which the demand for formal insurance equals zero is $\lceil \tilde{s} = 4.465 \rceil = 5$

The value of $\tilde{s}$ depends on the price p. When the formal insurance price is too high, nobody ever subscribes. In fact, $\lim_{p \to \infty} \tilde{s} = 0$ therefore the equilibrium coverage rate equals zero since a component contains at least one individual. The same logic prevails when the price converges to μ, every agent takes out full coverage for formal insurance. The next result gives the indirect expected utility.

Corollary 2. *If $p \leq \mu$, the expected indirect utility function does not vary with s. Otherwise, it is a smooth, strictly increasing and strictly concave function of s, The expected indirect*

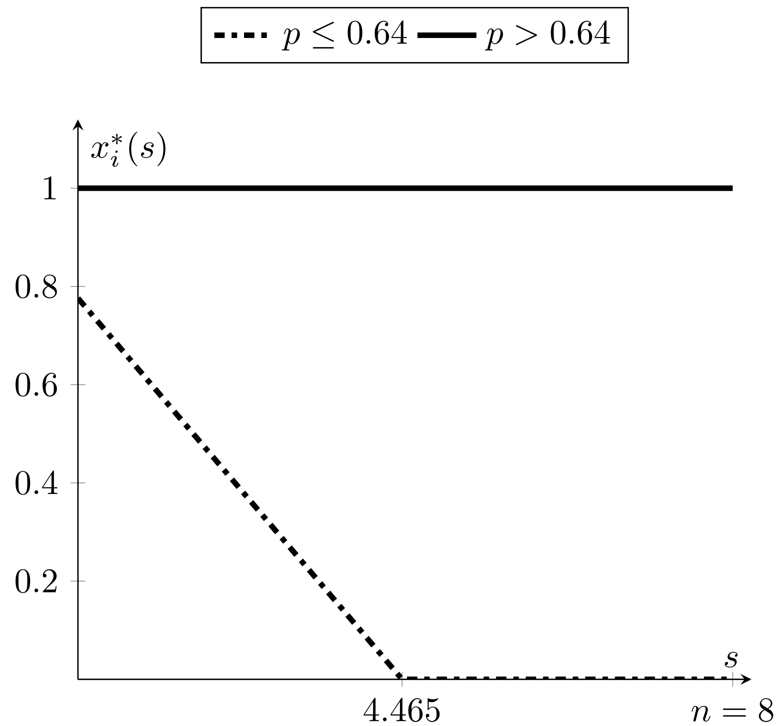

Figure 1: Individual demands for formal insurance as a function of component size

utility is equal to

$$v(s) = a - \exp\left\{-\lambda\left(y^0 - px_i^*(s) - \mu(1 - x_i^*(s)) - \frac{\lambda\sigma^2}{2s}(1 - x_i^*(s))^2\right)\right\}.$$

The proof of this result is in the appendix (*proof*). Links formed ex-ante determine the equilibrium coverage rate. If the price of insurance is lower than or equal to the actuarial price (μ), agents get fully insured by the formal insurance and face no more risk. Thus, the network has no motive to be. By contrast, if it is higher than the actuarial price, agents face some residual risk. In this case, for a fixed price of insurance, as s increases, there are two effects: First, being in a bigger component has a positive impact on agents' utilities due to better diversification. Second, it also decreases their demand for formal insurance. This implies on one hand that agents spend less on formal insurance, which increases their utility. On the other hand, they may end up more exposed to risks, which reduces their utility. Corollary 2 shows that the first effect dominates. Therefore, agents are better off in larger components.

4 Link formation and stable networks

In this section, I study the formation of risk-sharing networks. What structures will emerge when links are formed by pairs of agents, but agents cannot coordinate link formation across the whole population? To answer this question, I propose a refinement of the concept of pairwise stability with transfers[8] presented by Bloch and Jackson (2006). Like pairwise stability, pairwise stability with transfers allows link severance by individuals, and link formation by pairs. Moreover, agents make utility transfers to form new links but also to maintain existing ones. This entails that establishing a new link necessitates the exchange of utility, and the continuity of a link is upheld as the involved agents engage in reciprocal utility transfers. To refine this concept, I allow one and only one agent at a time to simultaneously cut a link and form another. Refer to this action as link switching. This approach is motivated by the fact that risk-sharing happens among family members or friends. Therefore it seems plausible to assume that there are utility transfers when two agents have or decide to form a risk-sharing link. Since utility transfers are allowed, I will not focus on each agent's private net utility on a link (their utility minus the linking cost formation each of them has to bear), but rather on the relative benefit of a link on a pair.

Assume an increasing and concave utility function v. The size of the component of any agent i is noted $s^i(g)$. Since agents i and j are in the same component, $s^i(g) = s^j(g)$ and $s^i(g - ij) + s^j(g - ij) = s^i(g)$. For any pair of agents $i \neq j$ with a risk-sharing link ij, the relative benefit of link ij in the network g is defined as

$$b_{ij}(g) = v(s^i(g)) - v(s^i(g - ij)) + v(s^j(g)) - v(s^j(g - ij)).$$

Let $\kappa \geq 0$ be the linking cost between two agents.

Definition 1. *A network g is pairwise-stable with transfers (PS^t) if and only if:*

(1) $\forall ij$ s.t. $g_{ij} = 0$, $b_{ij}(g + ij) < \kappa$.

(2) $\forall ij$ s.t. $g_{ij} = 1$, $b_{ij}(g) \geq \kappa$.

[8] One problem with the use of pairwise stability in this context is that pairwise stable networks often fail to exist. (see Bramoullé and Kranton (2007)).

For any i, an improving switch is a link $ih \notin g$ such for another link $ij \in g$, $v(s^i(g - ij + ih)) + v(s^h(g - ij + ih)) > v(s^i(g)) + v(s^j(g))$

Refinement. *A PS^t network g is switch-resistant iff, for any i and any $ij \in g$, there exists no improving switch ih such that $b_{ih}(g - ij + ih) > \kappa$*

Two agents will form a link if the relative benefit of forming the link is equal to or greater than the linking cost. Otherwise, they will not form it. Moreover, this link will be switch-resistant if and only if there is no stable improving switch. That is, even if there is more utility to be shared between two agents on this link, once it is formed its relative utility will be less than the link cost κ. In the current setting, all connected agents have the same increasing utility. Therefore,

Lemma 1. *A network g is switch-resistant pairwise-stable with transfers if and only if:*

(1) $\forall ij$ s.t. $g_{ij} = 0$, $b_{ij}(s^i(g + ij)) < \kappa$.

(2) $\forall ij$ s.t. $g_{ij} = 1$, $b_{ij}(s^i(g)) \geq \kappa$ and $\forall ih \neq ij$ s.t. $g_{ih} = 0$, if $s^h(g - ij + ih) \geq s^i(g)$ then $b_{ih}(s^i(g - ij + ih)) < \kappa$.

For ease of reading, I will use " stability" to designate "switch-resistant pairwise stability with transfers". Observe that when a link is removed, either this link is not a bridge and the components are unaffected, or this link is a bridge, and removing it cuts a component into two sub-components. The first implication is that:

Lemma 2. *If $\kappa > 0$, any stable network g is a forest. Equivalently, any component of a stable network is a tree.*

This property is shared by every network model where the benefits only depend on the size of the components.

Let i and j be to linked agents in a component of size s in the network g. Then $s = s^i(g - ij) + s^j(g - ij)$. The relative benefit of the link ij is written as follows:

$$b_{ij}(s, s^j(g - ij)) = 2v(s) - v(s - s^j(g - ij)) - v(s^j(g - ij)).$$

For a given component, define a *loose-end* as an agent that has one and only one link. If a component is a tree, then it has at least two loose ends. A loose-end link will be called a *loose-end link.* On a tree, if a link ij is a loose-end link, then the size difference between the two components obtained by removing ij is equal to $|s^i(g-ij)-s^j(g-ij)| = s-2$. Also, let define a *middle link* as a link such that after its removal creates two new components of sizes that differ by at most one agent. Formally, for a link ij in the component of size s in the network g, $|s^i(g-ij)-s^j(g-ij)| \in \{0,1\}$. Note that not every tree have a middle link (the star has none while the line has at least one). Note also that there is no difference between a loose-end link and a middle link up to components of three agents. For convenience, I will use the term "loose-end link" to refer to this type of link when the distinction is not clear. Therefore, a middle link can exist for components of four or more agents.

Consider a component of size s in a network g and two agents i, j with a link in this component. Ranking the relative benefit generated on a tree is equivalent to studying how $s^j(g-ij)$ affects $b_{ij}(s, s^j(g-ij))$.

$$\frac{\partial b_{ij}(s, s^j(g-ij))}{\partial s^j(g-ij)} = 0 \Leftrightarrow \frac{\partial v(s-s^j(g-ij))}{\partial(s-s^j(g-ij))} = \frac{\partial v(s^j(g-ij))}{\partial(s^j(g-ij))}$$
$$\frac{\partial b_{ij}(s, s^j(g-ij))}{\partial s^j(g-ij)} = 0 \Leftrightarrow s^j(g-ij)) = \frac{s}{2}$$
$$-\frac{\partial^2 v(s-s^j(g-ij))}{\partial\left(s-s^j(g-ij)\right)^2} - \frac{\partial^2 v(s^j(g-ij))}{\partial(s^j(g-ij))^2} \geq 0.$$

thus, the relative benefit of a link ij in a component of size s is the smallest when $s^i(g-ij) = s^j(g-ij) = s/2$. This also implies that the relative benefit of a link between two agents i, j, decreases with the difference between sizes of the two trees obtained by cutting the link ij, $|s^i(g-ij)-s^j(g-ij)|$. Therefore, on a given tree, the maximum relative benefit is generated on a loose-end link. While, if it has on, the middle link bring has the smallest relative benefit. Denote $b(s)$ and $\beta(s)$ respectively as the relative benefit of a loose-end link and the relative benefit of a middle link on a component of size s. Thus

$$b(s) = 2v(s) - v(s-1) - v(1) \text{ and } \beta(s) = 2\left(v(s) - v(\frac{s}{2})\right)$$

Lemma 3. *For any agents i, j with a link in a component of size $s \in [1, n]$ in a network g, the* *relative benefit* $b_{ij}(s)$ *decreases with* $|s^i(g-ij) - s^j(g-ij)|$. *Therefore,* $b(s) \geq b_{ij}(s) \geq \beta(s)$.

Lemma 3 has important implications. First, it implies that if a tree of size n with a middle link is stable, then any tree of size n is stable. Furthermore, if a forest in which every tree has a middle link is stable, then any network obtained by replacing every tree in that forest with any other tree of the same size is stable. Suppose that there is a stable tree of size n. Then, since each link on a star is a loose-end link, the star of size n is also stable. Suppose that there is a non-empty stable forest. Then, by the same logic, the network obtained by replacing each tree in the forest with a star of the same size is stable. This is because, for a given number of agents, the star is the tree for which the relative benefit of each link is maximized. Thus, knowing how b and β vary with s helps to characterize stable networks. Let us define the following two network structures. A network is a *s-quasi-uniform forest* if all but one of its components is a tree connecting s individuals, and the remaining component is either a singleton or a tree connecting s or fewer individuals. A network is an *s-quasi-uniform star forest*, if all but one of its components is a star connecting s individuals, and the remaining component is either a singleton or a star connecting s or fewer individuals. Also, define $\lfloor x \rfloor$ as the floor value of the real number x, i.e., the largest integer less than or equal to x. I define the component sizes $\overline{s}$ and $\overline{\overline{s}}$ as follows:

$$\overline{s} := max\{\lfloor s \rfloor : b(s) = \kappa\} \text{ and } \overline{\overline{s}} := max\{\lfloor s \rfloor : \beta(s) = \kappa\}.$$

They each represent the floor value of the maximum component size for which the relative benefit of a loose-end link $b(s)$ and the relative benefit of a middle link $\beta(s)$ are equal to the linking cost κ.[9] I show that key features of stable networks depend on how the functions b and β vary with s. Since I use a refinement of the PS^t criteria, I start by presenting the necessary condition a PS^t network needs to satisfy for every possible value of the linking cost κ.

Let us first focus on the function $b(s)$. Note that this function also expresses the relative benefit of each link in a star of size s. For a community of n agents, let us assume that $b(s)$ is non-increasing in s, which also implies that $\beta(s)$ is non-increasing in s. First, note that

[9] $\overline{s}$ depends on κ. For the sake of simplicity, this is mentioned

$\kappa > b(2)$. Then the empty lattice is the unique PS^t network. Now consider the case where $\kappa < b(n)$ then, the star is PS^t. The set of PS^t consists only of forests and trees, and the empty network is never PS^t in this case. Now suppose $\kappa \leq b(2)$ and $\kappa \geq b(n)$. Then there exists $\overline{s} \geq 2$ such that the $\overline{s}$-quasi-uniform star forest is PS^t. Furthermore, a PS^t network is a forest where the size of each tree is less than or equal to $\overline{s}$. If $\kappa < b(n)$, the star is PS^t. Also, a PS^t network is a forest. I now check if these PS^t network are switch resistant. This is equivalent to finding PS^t networks where there are no stable improving switches. It is worth noting that in this setting, a switch is considered to be improving for an agent i if he or she ends up in a larger component. Let us start with the case where $\kappa < b(n)$ and the star is PS^t. I claim that a stable network is a tree of size n. If instead there is more than one tree, then there exists a stable improving switch (a loose end in one tree can cut its link and form a new one in a larger tree). Furthermore, the star of size n is always stable. If $\overline{\overline{s}}$ exists and is smaller than n, then no stable tree has a middle link (a middle link can not be stable in a component that big, thus the line of size n is not stable). If $\overline{\overline{s}}$ is equal to n or does not exist, then every tree is stable. If $\kappa \leq b(2)$ and $\kappa \geq b(n)$, then a stable network has more than one component and the size of each component is less than or equal to $\overline{s}$. Suppose that the stable network g has at least two components. Let agents i and h be in two different components such that $s^i(g) \leq s^h(g) \leq \overline{s}$. If $s^h(g) \leq \overline{s} - 1$, then for any loose-end in component $s^i(g)$, there is a stable improving switch (delete the link in the initial component and form a link with any agent in a component of size $s^h(g)$). This is a contradiction. Thus, if g is stable, for any pair of components, the size of the largest component is $\overline{s}$. Only networks such that the size of all but one component is $\overline{s}$, and the size of the remaining component is less than or equal to $\overline{s}$, satisfy this condition ($\overline{s}$-forests). Thus, there is always a stable network and it is such that every component except one is a star of size $\overline{s}$ and the size of the remaining component is either a singleton or a star of size less than or equal to $\overline{s}$ ($\overline{s}$-quasi-uniform star forests). If $\kappa \leq b(2)$ and $\kappa \geq b(n)$, then $\kappa \leq \beta(2)$ and $\kappa > \beta(n)$. So $\overline{\overline{s}}$ exists and if $\overline{\overline{s}} \leq \overline{s} - 1$, then there is no middle link in any component in a stable $\overline{s}$ forest (thus no network with a line as component is stable). If $\overline{\overline{s}} = \overline{s}$, then every $\overline{s}$ forest is stable. Finally, for $\kappa > b(2)$, there are no improving switches, and the empty network is the unique stable network.

Now, suppose that $b(s)$ is non-decreasing in s. First, consider that $\kappa > b(n)$. Then the empty network is the unique PS^t network. And for $\kappa < b(2)$, the star is PS^t. Moreover, a PS^t network is a forest. Now suppose $\kappa \geq b(2)$ and $\kappa \leq b(n)$, then the star is PS^t. Also, a PS^t network is a forest where each tree is of size $s \geq \overline{s}$. Using the same strategy as before helps showing that for $\kappa > b(n)$, the empty network is the unique stable network. For $b(n) \geq \kappa \geq b(2)$, both the empty network and the star are stable. Moreover, a non-empty stable network is a tree. If $\beta(s)$ is non-increasing, since middle links exist for components of more than three agents and $\kappa \geq b(2)$, $\overline{\overline{s}}$ may exist. If it is the case and $\overline{\overline{s}} \leq n-1$, then no stable tree has a middle link and the line of size n is not stable. If $\overline{\overline{s}} = n$, then every tree is stable. If $\overline{\overline{s}}$ does not exist and $\beta(n) < \kappa$, then no stable tree has a middle link. If $\overline{\overline{s}}$ does not exist and $\beta(n) > \kappa$, then every tree is stable. If $\beta(s)$ is non-decreasing and $\overline{\overline{s}}$ exists, then every tree is stable (since every non-empty stable is a tree and the line of size $n \geq \overline{\overline{s}}$ is stable). If $\overline{\overline{s}}$ does not exists, then a stable network is a tree without middle link. For $\kappa < b(2)$, the star is stable and a stable network is a tree. In this case, if $\overline{\overline{s}}$ exists, then no tree with a middle link is stable. If it does not exist, then every tree is stable. In the next result, I focus on $b(s)$ since it is enough to have a proper characterization of stable network.

Proposition 2. *Consider a society of n agents and a linking cost $\kappa > 0$.*

(1) Suppose that $b(s)$ is non-decreasing.

- *(a) For $\kappa > b(n)$, the empty network is the unique stable network.*
- *(b) For $b(n) \geq \kappa \geq b(2)$, both the empty network and the star are stable. Moreover, a non-empty stable network is a tree.*
- *(c) For $\kappa < b(2)$, the star is stable and a stable network is a tree.*

(2) Suppose that $b(s)$ is non-increasing.

- *(a) for $\kappa > b(2)$, the empty network is the unique stable network.*
- *(b) For $b(2) \geq \kappa \geq b(n)$, the $\overline{s}$-quasi-uniform star forest is stable and a stable network is $\overline{s}$-forest.*
- *(c) For $\kappa < b(n)$, the star is stable and a stable network is a tree.*

All the results presented in this section hold for any increasing and concave utility function v.

Notice that the relative benefit of a link is the sum of the utility gains of both agents involved. In the case of a loose-end link, the relative benefit can be decomposed as follows

$$b(s) = v(s) - v(1) + v(s) - v(s-1).$$

It is the sum between the utility gain of a loose-end $(v(s) - v(1))$ which always increases with s and the utility gain or her neighbour $(v(s) - v(s-1))$ which always decreases with s. In the next section, a key step to characterize stable networks will be to determine whether the utility gain of the loose-end grows faster with s than the utility gains of her neighbour decreases with s.

5 Stable networks and formal insurance price

In this section, I characterize stable networks for different levels of formal insurance prices. When the price of insurance is lower than or equal to the actuarial price, the empty network is the unique stable network. When the price of insurance is larger than the actuarial price, the effect of the price on risk-sharing networks is non-monotonic. In particular, I find that stable networks may have more links when the insurance price is lower.

Let us rewrite Corollary 1 written in terms of price. Note that the threshold size $\tilde{s} \in (0, +\infty)$ and decreases in p. Therefore, for any component of size s, there is a unique price p_s such that $\tilde{s}(p_s) = s$.

Corollary 3. *Consider a component of size s. For $p > \mu$ there exists a price p_s such that if $p \geq p_s$ the equilibrium coverage rate $x_i^*(s) = 0$ and if $p < p_s$ the equilibrium coverage rate $x_i^*(s) > 0$. This price is*

$$p_s = \mu + \frac{\lambda\sigma^2}{s},$$

and it decreases with s.

The price p_s is the reservation price of an agent in a component of size s, i.e., the price beyond which an agent in a component of size s will prefer not to take out formal insurance.

This threshold price decreases with the size of a component. This implies that for a component large enough, the reservation price tends towards the actuarial price μ. Moreover, $s < \tilde{s}(p)$ is equivalent to $p \in (\mu, p_s)$, and $s \geq \tilde{s}(p)$ is equivalent to $p \geq p_s$. This formulation is better suited in the present context where the network is endogenous.

In what follows, I will study stable networks for four sets of pricing covering all the possible cases. Refer to the first one as the case of *high* prices (it is equivalent to having $p \geq p_1$). This situation represents the common hypothesis used in the current literature on risk-sharing networks. Under this assumption, no agent regardless of the size of her component takes out formal insurance. Denote the second case as the *low* prices case (when $p \leq \mu$) and the third one as the *relatively low* prices case (when $p \in (\mu, p_n)$). With this assumption, every agent regardless of the size of her component, takes out insurance. I will refer to the fourth case as the *relatively high* prices case (when $p \in (p_n, p_1)$). With prices in this set, agents in smaller components take out formal insurance while those in bigger components do not subscribe.

5.1 Stable networks with high formal insurance prices

In this context, the price is so high ($p \geq p_1$) that no agent takes out formal insurance. This is equivalent to the case without formal insurance and agents can only rely on informal risk-sharing. This is the most common assumption in the existing literature on risk-sharing in networks. However, it is the first time that pairwise stability with utility transfer is used as a stability criterion.

When $x_i^*(s) = 0$, denote v_2 the expected indirect utility obtained by agents in a risk-sharing component of size s where

$$v_2(s) = a - \exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{2s}\right)\right\}.$$

The relative benefit of a loose-end link is then:

$$\begin{aligned} b(s) = &- 2\exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{2s}\right)\right\} + \exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{2(s-1)}\right)\right\} \\ &+ \exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{2}\right)\right\}, \end{aligned}$$

and when the size of the component goes to infinity

$$b(\infty) = -\exp\left\{-\lambda\left(y^0 - \mu\right)\right\} + \exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{2}\right)\right\}.$$

There exists a critical size $\hat{s} > 2$ such that for $s \leq \hat{s}$ the relative benefit $b(s)$ decreases with s and for $s > \hat{s}$, it increases with s. Additionally, it holds that $b(2) > b(\infty)$. I can then propose a characterization of stable network for high prices of formal insurance. For high prices ($p \geq p_1$), let define $\overline{s}_h := max\{\lfloor s \rfloor : s < \hat{s} \text{ and } b(s) = \kappa\}$. The next result follows from Proposition 2.

Proposition 3. *Suppose the price a formal insurance $p \geq p_1$. Then there exists a critical size $\hat{s}$ such that, $b(s)$ decreases with s for $s \leq \hat{s}$, and otherwise it decreases with s. Also, $b(2) > b(\infty)$. Consider a society with $n > \hat{s}$ agents and a linking cost $k > 0$.*

(1) For $\kappa > b(2)$, the empty network is the unique stable network.

(2) For $b(2) \geq \kappa \geq b(\hat{s})$, there exists $\overline{s}_h < \hat{s}$, such that

(a) if $b(2) \geq \kappa > b(n)$ the $\overline{s}_h$-quasi-uniform star forest is stable. Moreover, a stable network is an $\overline{s}$-forest.

(b) If $b(n) \geq \kappa > b(\hat{s})$, both the $\overline{s}_h$-quasi-uniform star forest and the star are stable. Moreover, a stable network is either a tree or an $\overline{s}$-forest.

(3) For $\kappa \geq b(\hat{s})$, the star is stable and a stable network is a tree.

The proof of this result is in the appendix (*proof*). For small components ($s < \hat{s}$), the relative benefit of a loose-end link decreases with size and then increases for $s > \hat{s}$. This is because when $s < \hat{s}$ the utility gain of the isolated agent $(v(s) - v(1))$ grows slower than the utility gain of her neighbour, $v(s) - v(s-1)$. In fact, the utility gain of the isolated agent grows with the size of the component. Therefore, for smaller component, the loose-end does not gain enough to compensate the decrease in utility gain of her neighbour. The opposite happens when $s > \hat{s}$. As the size of the component increases, the utility gain of the loose-end grows faster and compensate the reduction in utility gain of her neighbor. Since $v(s)-v(s-1)$

converges to zero, one might expected that the utility gain of the loose-end may grow high enough to compensate the decrease in utility gain of her neighbor. However, Proposition 3 shows that the limit of $v(s)$ with s is not high enough for this to happen. Thus, the relative benefit of a loose-end link is maximum for two agents. Moreover, it takes its minimum value when the size of the tree equals $\hat{s}$. This threshold is defined implicitly and varies in the same direction as the risk aversion λ and the variance of the shock σ^2. But it does not depend on the price of insurance.

Figure 2 illustrates how $b(s)$ varies with s for high prices of formal insurance. I consider an environment where $y^0 = 0.54$, $\mu = 0.64$, $\sigma^2 = 2.055$, $\lambda = 2.64$ and $p \geq p_1 = 5.335$

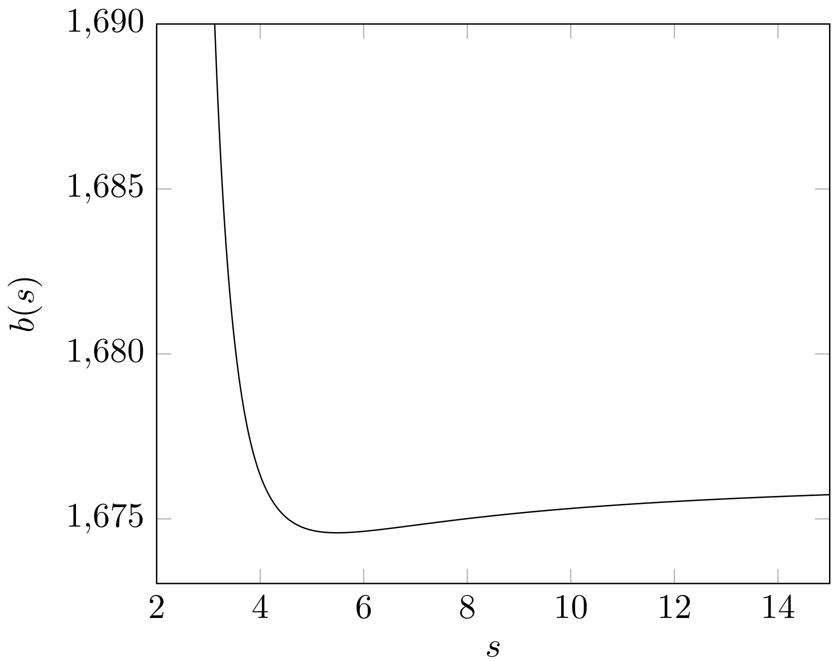

Figure 2: Relative benefit of a loose-end link when the price of formal insurance is high

5.2 Stable networks with low and relatively low formal insurance prices

When the price of formal insurance is low ($p \leq \mu$) or relatively low ($p \in (\mu, p_n)$), for any agent regardless of the size of her component, the equilibrium coverage demand is strictly positive. I first characterize stable networks in the low prices case. The indirect expected utility $v(s)$ is constant. Therefore, the relative benefit of a loose-end link $b(s)$ equals zero, and there are no risk-sharing links:

Proposition 4. *Consider a society with $n > 2$ agents, a price of formal insurance $p \leq \mu$ and a linking cost $k > 0$. Then, the empty network is the unique stable network.*

This result is a natural consequence of the fact that agents take full coverage when the price of formal insurance is lower than or equal to the actuarial price. There is no residual risk to cover and no reason to create risk-sharing connections.

Now, let us consider the case of a relatively low price. It is equivalent to having p between μ and p_n, the reservation price of an agent in a component of size n. In this context, from Corollary 2, we can express the expected indirected utility v_1 as follows

$$v_1(s) = a - \exp\left\{-\lambda(y^0 - p + \frac{(p-\mu)^2}{2\lambda\sigma^2}s)\right\}.$$

The relative benefit of a loose-end link is then equal to:

$$\begin{aligned} b(s) = &-2\exp\left\{-\lambda(y^0 - p + \frac{(p-\mu)^2}{2\lambda\sigma^2}s)\right\} + \exp\left\{-\lambda(y^0 - p + \frac{(p-\mu)^2}{2\lambda\sigma^2}(s-1))\right\} \\ &+ \exp\left\{-\lambda(y^0 - p + \frac{(p-\mu)^2}{2\lambda\sigma^2})\right\}. \end{aligned}$$

When the price of formal insurance is relatively low, denote by $\overline{s}_{rl} := max\{\lfloor s \rfloor : b(s) = \kappa\}$. The next result shows how b varies with the size of the components and with the price of formal insurance. It also shows how p affects $\overline{s}_{rl}$.

Lemma 4. *Consider a society with $n \geq 3$ agents and a price of formal insurance $p \in (\mu, p_n)$. Then $b(s)$ increases with p. Assume a linking cost $\kappa > 0$ and the threshold price $\hat{p} = \mu + \sigma\sqrt{2\ln 2}$.*

(1) If $p < \hat{p}$, then $b(s)$ increases in s. Therefore, (a) for $\kappa < b(2)$, the star of size is stable. (b) For $b(2) < \kappa < b(n)$ both the empty network and the star are stable. (c) For $b(n) < \kappa$, the empty network is the unique stable network.

(2) If $p > \hat{p}$, then $b(s)$ decreases in s. Therefore, (a) for $\kappa < b(n)$, the star is stable. (b) For $b(n) < \kappa < b(2)$, there exists a size $\overline{s}_{rl}$ such that the $\overline{s}_{rl}$-quasi-uniform star forest is

stable, and $\overline{s}_{rl}$ *increases with* p*. (c) For* $b(2) < \kappa$*, the empty network is the unique stable network.*

The proof is in the appendix (*proof*). When the price of formal insurance is relatively low, there are two effects on $b(s)$ as the price of formal insurance increases. First, the *relative benefit of links increases with the price of insurance*, which incentivizes the formation of links. As the price of insurance increases, the demand for formal insurance decreases in each component. This suggests an increase in the proportion of risk that is uninsured for each agent when they are isolated. Consequently, the relative benefit of each link within a component increases.

Second, the *price of insurance determines how b(s), the relative benefit of a link with a loose end, varies with a component size s.* To understand this effect, recall that the condition on how the relative benefit of a loose-end link varies with the size of a component is equivalent to determining whether the utility gain of a loose-end is increasing faster than the utility gain of its neighbor is decreasing. If this is the case, then the relative benefit of the connection increases with the size of the component. Otherwise, it decreases with s. When the price of insurance is high ($p \geq p_1$), the critical size $\hat{s}$ determines when this happens, as presented in Proposition 3. However, for relatively low prices of formal insurance ($p \in (\mu, p_n)$), this condition depends only on the value of p. This is due to the fact that for relatively low prices, the effect of the size of an agent's component on her consumption is linear and its intensity depends on p. The intuition is that when $p < \hat{p}$, formal insurance is more accessible. Thus, forming the first link is not as important as it is when there is no formal insurance ($p \geq p_1$). Therefore, for $s < \hat{s}$, the variation of $b(s)$ with s is not driven by the utility gain of the more connected agent at the loose end, but by the utility gain of the loose end. Moreover, increasing the size of a component has little effect on reducing the demand for formal insurance. Thus, the risk exposure induced by the reduction in demand is always offset by the diversification gain induced by increasing the size of the component. When $p > \hat{p}$, formal insurance is less accessible. The fraction of uninsured risk for an isolated agent is larger, and forming the first risk-sharing link brings high relative benefits. Thus, the variation of $b(s)$ with s is driven by the utility gain of the more connected agents. This is also the case for $s > \hat{s}$, where instead of seeing $b(s)$ increasing with s (as in the case of a high insurance price), we observe the opposite. This is due to the fact that increasing the size of a component has a greater effect on reducing the demand for formal insurance. Therefore, the risk exposure always dominates the diversification benefit. Thus, the increase in the loose end's utility gain does not compensate for the decrease in her neighbor's utility gain.

Figure 3 and Figure 4 respectively illustrate how $b(s)$ varies with s for relatively low prices of formal insurance when $p = 2.105 < \hat{p} \approx 2.33$ and when $p = 2.6 > \hat{p} \approx 2.33$. I consider the same environment as Figure 2, where $y^0 = 0.54$, $\mu = 0.64$, $\sigma^2 = 2.055$ and $\lambda = 2.64$. Only the price of formal insurance varies.

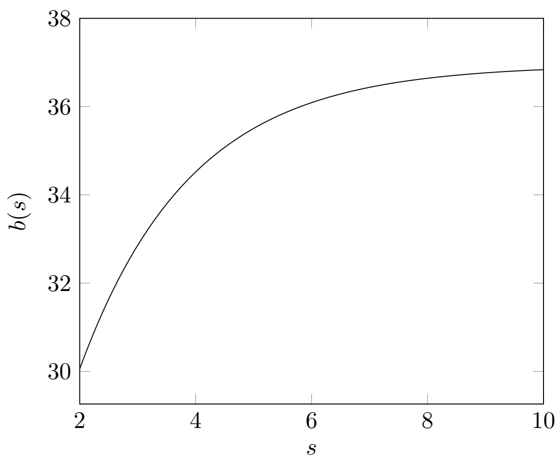

Figure 3: Relative benefit of a loose-end link when $p = 2.105 < \hat{p} \approx 2.33$

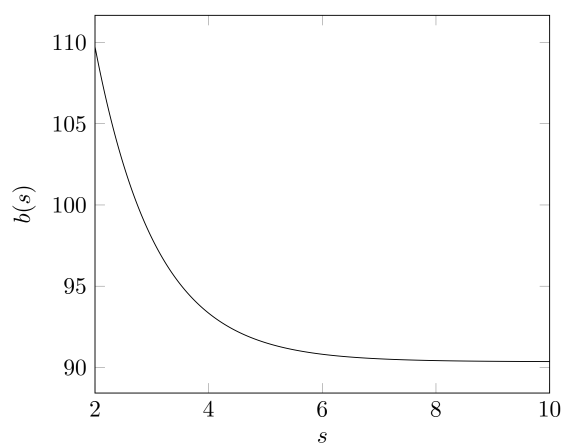

Figure 4: Relative benefit of a loose-end link when $p = 2.6 > \hat{p} \approx 2.33$

Therefore, we can characterize stable networks, as detailed in Lemma 4, for any pair price of formal insurance p and linking cost κ. Figure 5 illustrates this statement. I consider a community with $n = 6$ where $y^0 = 2.09$, $\mu = 0.444$, $\sigma^2 = 9.91$ and $\lambda = 2.48$.[10] The dotted line $b_2(p)$ represents how the relative benefit of a link between two isolated agents varies with the price of formal insurance. The solid line shows the relative benefit of a loose-end link in a component of size $s = 6$. We can then characterize stable networks for any couple (p, κ). For example, for any pair (p, κ) in region B, both the empty network and the star of community size $n = 6$ are stable.

Using Lemma 4, stable networks can be characterized at relatively low prices of formal insurance. Denote $b_s(p)$, the function evaluating how the relative benefit of a loose-end link

[10] Figure 5 depicts the function $b_s(p)$ when the expected indirect utility function is $\frac{v_1(s)}{30\lambda}$.

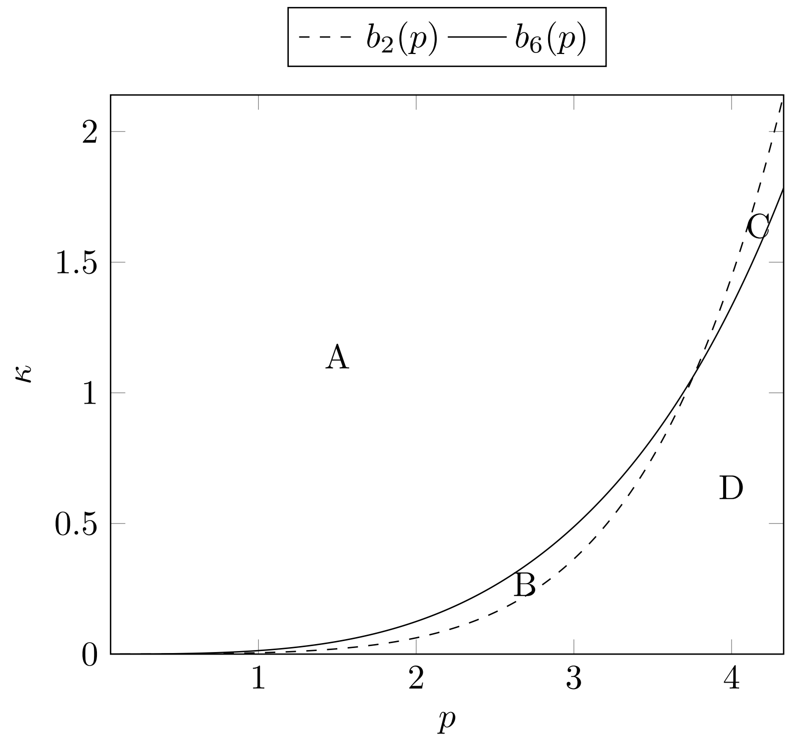

Figure 5: Stable networks for every pair (p, κ). A: *Empty network*, B: *Empty network and Star of size 6*, C: $\overline{s}_{rl}$*-quasi-uniform star forest*, D: *Star of size 6.*

in a component of size s varies with p. For low prices of formal insurance, define p^2_{rl} and p^n_{rl} as the prices where $b_2(p^2_{rl}) = \kappa$ and $b_n(p^n_{rl}) = \kappa$, respectively. It is suggested that the empty network is stable for $p < p^2_{rl}$, but unstable for $p \geq p^2_{rl}$. Additionally, the star of size n is stable for $p \geq p^n_{rl}$ but unstable for $p < p^n_{rl}$. For any linking cost κ, determining the ranking between p^2_{rl} and p^n_{rl} enables the characterization of stable networks in response to variations in the price of formal insurance p. This yields the following result.

Proposition 5. *Consider a society with $n \geq 3$ agents and a price of formal insurance p in $[\mu, p_n]$ such that $\hat{p} < p_n$. Then, $0 < b_2(\hat{p}) < b_n(p_n) < b_2(p_n)$. Assume a linking cost $\kappa > 0$.*

(1) For $\kappa < b_2(\hat{p})$, there exist two prices $p^n_{rl} < p^2_{rl}$ such that

(a) if $p < p^n_{rl}$ the empty network is the unique stable network.

(b) If $p^n_{rl} \leq p < p^2_{rl}$, then both the empty network and the star are stable. Moreover, a non-empty stable network is a tree.

(c) Finally, if $p \geq p^2_{rl}$, the star of size and a stable network is a tree.

(2) For $b_2(\hat{p}) < \kappa < b_n(p_n)$, there exist two prices $p_{rl}^2 > p_{rl}^n$ such that

(a) if $p < p_{rl}^2$ the empty network is the unique stable network.

(b) If $p_{rl}^2 \leq p < p_{rl}^n$, there exists a size $\overline{s}_{rl}$ such that the $\overline{s}_{rl}$-quasi-uniform star forest is stable. Moreover, a stable is an $\overline{s}_{rl}$-forest and $\overline{s}_{rl}$ increases with p.

(c) Finally, if $p \geq p_{rl}^n$, the star is stable and a stable network is a tree.

(3) For $\kappa > b_2(p_n)$, the empty network is the unique stable network.

Proposition 5 allows, for relatively low prices of formal insurance, to evaluate the impact of a price reduction on the structure of stable networks. If the reduction in the price of formal insurance is large, leading to $p \leq \mu$, then by Proposition 5, the only stable network is empty. A large price reduction leads to an extreme form of crowding out: agents buy enough formal insurance to be fully insured and do not form any costly risk-sharing relationships.

The interactions between formal and informal insurance become more intricate with smaller price reductions. If κ lies between $b_2(\hat{p})$ and $b_n(p_n)$, then $\hat{p} < p_2^{rl} < p_n^{rl}$. Thus, a price reduction such that p lies between p_n^{rl} and p_n does not crowd out stable networks: although agents will purchase more formal insurance, the relative benefits of loose-end links are still high.[11] Thus, the price reduction is insufficient to encourage them to abandon their risk-sharing connections. For a price reduction such that p lies between p_2^{rl} and p_n^{rl}, the size of the largest component progressively decreases from n to 2 as the price of formal insurance decreases. Agents buy more formal insurance, which reduces the appeal of risk-sharing arrangements. Since $\hat{p} < p_2^{rl} < p_n^{rl}$, the relative benefit of a risk-sharing link decreases with the size of component s.[12] Therefore, if the price falls between p_2^{rl} and p_n^{rl}, agents decrease the number of links they have, resulting in smaller components in stable networks. Thus, formal insurance partially crowd-out the risk-sharing network. For a price reduction such that p is less than p_2^{rl}, formal insurance completely crowds out the risk-sharing network. Since κ exceeds $b_2(\hat{p})$, the relative benefit of a risk-sharing link is insufficient to encourage link formation. As a result, agents rely solely on formal insurance.

[11] Links other than loose-end links may become unstable following a price reduction where $p_n^{rl} < p < p_n$. Nonetheless, agents have the ability to make stable improving switches, resulting in a stable star structure. See Proposition 2.

[12] According to Lemma 3, loose-end links provide the highest relative benefit on a tree. Therefore, as their relative benefit diminishes, so does the benefit of all other links.

When the value of κ drops below $b_2(\hat{p})$, it holds that $p_n^{rl} < p_2^{rl} < \hat{p}$. When the price exceeds $\hat{p}$, the relative benefit of a loose-end link decreases as the component size, s , increases. Nevertheless, since the linking cost, κ, is sufficiently low, a price reduction such that p lies between $\hat{p}$ and p_n, leave the structure of stable networks unchanged. Furthermore, larger reductions where p falls between p_n^{rl} and p_2^{rl} may still not lead to crowding out in the risk-sharing network. Remember that for $p < \hat{p}$, the relative benefit of a loose-end link, $b(s)$, increases with the size of a component, s. Therefore, for a price larger than p_{rl}^n and lower than p_{rl}^2, it comes from Proposition 2 that both the empty network and the star are stable.

I illustrate in Figure 6 how the geometry of stable networks may vary with insurance price. Take the same parameters as figure 5 , $n = 6$, $y^0 = 2.09$, $\mu = 0.444$, $\sigma^2 = 9.91$ and $\lambda = 2.48$. Assume linking costs lower than or equal $\kappa < b_2(p_6 = 4.54) = 2.87$. The stable risk-sharing networks are impacted by a decrease in the price of formal insurance, as seen through the reduction of p from $p_6 = 4.54$ to $\mu = 0.444$. When p falls slightly below $p_6 = 4.54$, the star of size 6 remains stable. If the linking cost κ is greater than $b_2(\hat{p}) = 1.37$, then the size of $\overline{s}_{rl}$ reduces with p, and the $\overline{s}_{rl}$-quasi-uniform star forest gradually becomes an empty network. However, a price reduction of the same amount will not have the same effect if κ is less than $b_2(\hat{p}) = 1.37$. Only a significant reduction will result in the crowding out of the network. As shown in Figure 6, an agent's tone lightens as the size of her component decreases.

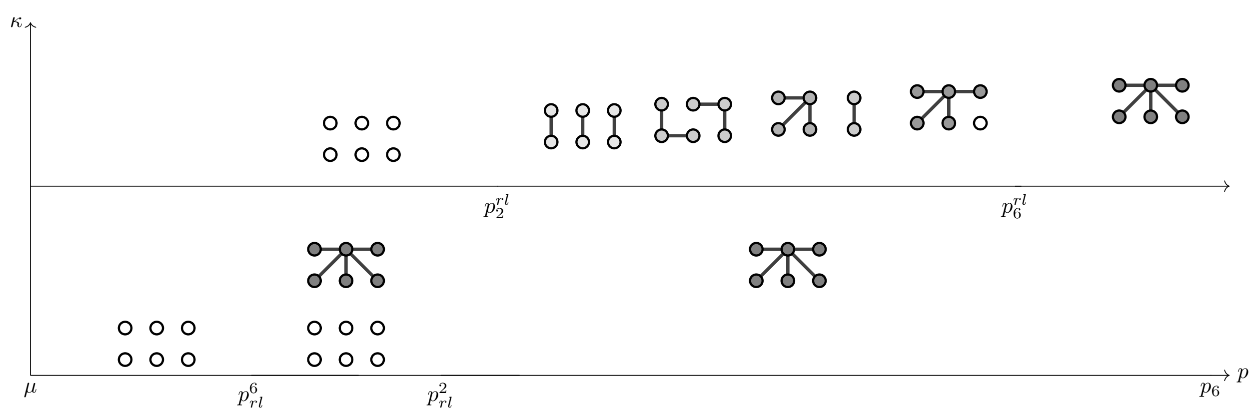

Figure 6: Stable network for different levels of prices

The region with relatively low prices of formal insurance is relevant for insurers. Indeed, they can have the community take out formal insurance while making a positive profit.

5.3 Stable networks with relatively high formal insurance prices

In the two previous sections, I characterized stable networks when, regardless of the size of their component, either all agents take out formal insurance or none of them do. What happens when the price is such that only agents in smaller components have a positive demand for formal insurance? In this section, I characterize stable networks in such a context. Remember that

$$v_1(s) = a - \exp\left\{-\lambda(y^0 - p + \frac{(p-\mu)^2}{2\lambda\sigma^2}s)\right\} \text{ and } v_2(s) = a - \exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{2s}\right)\right\}.$$

Formally, for a given price $p \in (p_n, p_1)$, the relative benefit of links in a star of size s is:

$$b(s) = \begin{cases} b_{rl}(s) = 2v_1(s) - v_1(s-1) - v_1(1) & \text{for } s \leq \tilde{s} \\ b_{rh_1}(s) = 2v_2(s) - v_1(s-1) - v_1(1) & \text{for } \tilde{s} \leq s \leq \tilde{s}+1 \\ b_{rh_2}(s) = 2v_2(s) - v_2(s-1) - v_1(1) & \text{for } s \geq \tilde{s}+1 \end{cases}$$

It follows from Lemma 4 that $b_{rl}(s)$ increases with s for $p < \hat{p}$ and decreases with s otherwise. Since $v_1(1)$ does not vary with s, the variation of $b_{rh_2}(s)$ with s is given by Lemma 3. Using these results, one can determine how $b_{rh_1}(s)$ varies with s, and by extension, how $b(s)$ varies with s. Note also that $v_1(s)$ decreases with p (since an increase in the price of formal insurance implies more risk exposure). Using Lemma 4 to complete the previous observation helps to determine how the relative benefit of a loose-end b varies with the price of formal insurance p. Note that since the variation of $b_{rh_2}(s)$ with s is given by Lemma 3, for any formal insurance price p, the value of $b_{rh_2}(s)$ is minimized at $s = \hat{s}$. Remember that $\hat{s} > 2$ and that $\hat{p} = \mu + \sigma\sqrt{2\ln 2}$.

Lemma 5. *Consider a society with $n > \hat{s}$ agents and a price of formal insurance p in (p_n, p_1). Then $b(s)$ increases with p. Moreover, there exists $\hat{p}_1 < \hat{p}$ such that:*

(1) If $p \leq \hat{p}_1$, $b(s)$ increases with s. (2) If $p > \hat{p}$, $b(s)$ decreases for $s \leq \hat{s}$ and then increases. (3) For $\hat{p}_1 < p \leq \hat{p}$, $b(s)$ reaches a local maximum at $\hat{s}_1$ in $[\tilde{s}, \tilde{s}+1)$ and a local minimum at $\hat{s} \geq \tilde{s}+1$ such that $\hat{s}_1$ converges to $\hat{s}$ as p tends to $\hat{p}_1$.

The proof of this result is relegated to the appendix *(proof)*. Note that $\hat{s}_1$ is implicitly defined. Lemma 5 shows the persistence of the two effects of the price of formal insurance p, on the relative benefit of loose-end link $b(s)$, and thus on the incentive to form risk-sharing links. First, the *relative benefit of links increases with the price of formal insurance*, as shown by Lemma 4. This is the direct effect of formal insurance on the incentives to form risk-sharing links. As the price of insurance decreases, the portion of uninsured risk for each agent if they were isolated decreases. There will be less risk to be mitigated by informal insurance institutions. This reduces the incentive to form risk-sharing link.

Second, the *price of insurance determines how the relative benefit of a loose-end link $b(s)$, varies with the size of a component, s*. Recall that $\hat{p}$ is the critical price above which $b(s)$ in a small component (where everyone has a positive demand for formal insurance) is driven by the utility gain of the more connected agent on a loose-end link. This price is higher than $\hat{p}_1$. Therefore, if the price of formal insurance is between p_n and $\hat{p}_1$, it reduces the need to create the first link for an isolated agent. Thus, the variation of $b(s)$ with s is driven by the utility gain of the loose-end.

If the price of formal insurance is between $\hat{p}_1$ and $\hat{p}$, Lemma 5 states that the value of $\hat{s}_1$ converges to $\hat{s}$ as the price of formal insurance tends to $\hat{p}_1$. The two values are the same when the price of insurance is equal to $\hat{p}_1$. However, since the value of $\hat{s}$ remains constant with p, it follows that as the insurance price approaches $\hat{p}$, the variation of $b(s)$ with the size s of the component mirrors its variation with s in the high-price insurance scenario ($p \geq p_1$). For this price level, the first link does not generate a high enough relative benefit, and the variation of $b(s)$ is driven by the utility gain of the loose end for a component size less than $\hat{s}_1$. However, since $\hat{s}_1$ is larger than $\overline{s}$, the component size above which the demand for formal insurance is zero, the diversification effect is outweighed by the increase in risk exposure. Therefore, the utility gain of the more connected agent on a loose-end link drives the variation of $b(s)$ with $s \in (\hat{s}_1, \hat{s})$. For component sizes above $s \geq \hat{s}$, the increase in utility gain of the loose-end is high enough to compensate for the decrease in utility gain of her neighbor.

In the last region, the price of formal insurance price p, is above the critical price $\hat{p}$ and below the reservation price of an isolated agent p_1. The variation of $b(s)$ with s is then driven by the utility gain of the more connected agent on a loose-end link for s less than $\hat{s}$. The first link an agent creates generates the highest relative benefit despite the fact that she can buy for insurance. Since the price of insurance is relatively high, the uninsured portion of her risk, after she would purchase formal insurance, is high enough to increases the incentive to create the first link. For a component of size above $\hat{s}$, we are in the case without formal insurance and the variation of $b(s)$ is driven by the utility gain of the loose-end.

Using Lemma 5, I provide a detailed characterization of stable networks for each level of linking cost κ in Appendix (see Lemma 6). A stable network is either the empty network, a tree or an $\overline{s}_{rh}$-forest with $\overline{s}_{rh}$.

I can then propose a characterization of stable networks for different levels of formal insurance price p. Remember that p_s is the reservation price of an agent in a component of size s. Remember also that $b_s(p)$ is the function that evaluates how the relative benefit of a loose-end link in a component of size s varies with p. For relatively high prices, define p^2_{rh}, $p^{\hat{s}_1}_{rh}$, $p^{\hat{s}}_{rh}$ and p^n_{rh} respectively as the prices for which $b_2(p)$, $b_{\hat{s}_1}(p)$, $b_{\hat{s}}(p)$ and $b_n(p)$ are equal to κ. When $p < p^2_{rh}$, the empty network is stable. Otherwise it is not. When $p < p^n_{rh}$, the star is stable. Otherwise it is not. When $p > p^{\hat{s}_1}_{rh}$, the $\overline{s}_{rh}$-quasi-uniform star forest is stable. Otherwise, it is not. Finally, when $p < p^{\hat{s}}_{rh}$, the $\overline{s}_{rh}$-quasi-uniform star forest is stable. Otherwise, it is not. For any linking cost κ, determining the ranking between p^2_{rh}, $p^{\hat{s}_1}_{rh}$, $p^{\hat{s}}_{rh}$ and p^n_{rh} allows to characterize stable networks in response to the variations in the price of formal insurance p. It follows from Lemma 5 that $\hat{s} = \hat{s}_1$ at an insurance price equal to $\hat{p}_1$, and that $b_2(p_n) < b_{\hat{s}}(\hat{p}_1) < b_n(p_n)$. It also follows that $b_{\hat{s}}(p_1) < b_n(p_1) < b_2(p_1)$. I define an *s-uniform tree-singleton forest* as a forest where every network is either a tree of size s or a singleton. Also define $\overline{s}_{rh} := max\{\lfloor s \rfloor : s < \hat{s} \text{ and } b(s) = \kappa\}$ and $\underline{s}_{rh} := min\{\lfloor s \rfloor : s < \hat{s} \text{ and } b(s) = \kappa\}$

Proposition 6. *Consider a society with $n > \hat{s}$ agents, a price of formal insurance p in $[p_n, p_1]$ and a linking cost $\kappa > 0$. Assume that $b_2(p_n) < b_{\hat{s}}(\hat{p}_1) < b_n(p_n) < b_{\hat{s}}(p_1) < b_n(p_1) < b_{\hat{s}_1}(\hat{p}) < b_2(p_1)$*

(1) For $b_n(p_n) < \kappa < b_{\hat{s}}(p_1)$, there exists four prices $p^n_{rl} < p^2_{rl} < p^{\hat{s}_1}_{rl} < p^{\hat{s}}_{rl}$ such that

(a) if $p < p^n_{rl}$, the empty network is the unique stable network.

(b) If $p^n_{rl} < p < p^2_{rl}$, both the empty network and the star are stable. Moreover, a non-empty stable network is a tree.

(c) *If* $p_{rl}^{2} < p < p_{rl}^{\hat{s}_1}$*, the empty network, the star are stable and any* $\overline{s}_{rh}$*-uniform star-singleton forest is stable. An* $\overline{s}_{rh}$*-quasi-uniform star forest is stable if and only if the size of the smallest component is larger than or equal to* $\underline{s}_{rh}$*. Moreover, a non-empty stable network is either a tree, an* s*-uniform tree-singleton forest or an* $\overline{s}_{rh}$*-forest, if and only if the size of the smallest component is larger than or equal to* $\underline{s}_{rh}$*.*

(d) *If* $p_{rl}^{\hat{s}_1} < p < p_{rl}^{\hat{s}}$*, both the star and the* $\overline{s}_{rh}$*-quasi-uniform star forest are stable. Moreover, a stable network is either a tree or an* $\overline{s}_{rh}$*-forest.*

(e) *If* $p \geq p_{rl}^{\hat{s}}$*, the star is stable and a stable network is a tree.*

Proposition 6 complements Proposition 5, providing an overall analysis of the impact of insurance price reductions on the structure of stable networks. It presents the cases for which there is multiplicity in the size of components in stable networks. When κ lies between $b_n(p_n)$ and $b_{\hat{s}}(p_1)$, if the price of formal insurance is set between $p_{rl}^{\hat{s}}$ and p_1, the size of the stable risk-sharing network stay unchanged compare to the case with high prices. Every pair of agents in the community are path connected. However, since the relative benefit of a loose-end link $b(s)$, decreases with the price of formal insurance p, the average length of a path between every pair of agents in a component of a stable network reduces as p decreases in this range.

If the price of insurance is set between $p_{rl}^{\hat{s}_1}$ and $p_{rl}^{\hat{s}}$, there is multiplicity in the size of components at the equilibrium. A stable network can be a tree connecting the entire community or an $\overline{s}_{rh}$-forest. For instance, if the initial network is a line, then after the introduction of formal insurance, a stable network can be an $\overline{s}_{rh}$-forest. The average path length between two agents is too large for the structure to survive the introduction of formal insurance at this level of price. However, if the initial network is a star, it will still be stable after an introduction of formal insurance at this level of price. Thus, for the same price of formal insurance, we will observe on average over the population, two different level of demand for formal insurance, solely due to the difference in the structure in the initial networks. Agents in the star will purchase less formal insurance compare to the one in an $\overline{s}_{rh}$-forest since they can rely on the entire community.

If the price of insurance is set between p_{rl}^{2} and $p_{rl}^{\hat{s}_1}$, there is multiplicity in the size of components at the equilibrium and more possible stable networks. Therefore, for the same price of formal insurance, one can either observe a small demand (in the case agent are in a tree), or a higher demand formal insurance (in the case the network is totally crowd-out). Also, for stable network that is an s-uniform star-singleton forest or an $\overline{s}_{rh}$-forest, agents in the same society will purchase different level of formal insurance. Those in bigger component will purchase none or less formal insurance than agents in smaller components. For this range of prices, even if a price reduction that does not affect the size of the largest component in a stable network, the average length path decreases. The analysis holds for p between p_{rl}^{2} and p_{rl}^{n}. However, there are less possibilities. The stable network is empty or minimally connected. And as the price decreases in this region of prices, the average path length between two agent decreases.

By Lemma 4 and Lemma 5, we know that the reduction of the price of formal insurance reduces the relative benefit of any link in a risk-sharing network. A *diameter* of component is the longest shortest path between any two agent in this component. Reducing the price of insurance will either only reduce the diameter of a the largest component in a stable network without affecting its size, or it will also reduce it. Thus, the closer the shape of the initial network is from the start, the more robust it will be to the introduction of formal insurance. On the other side, the closer this shape is from the line, the more it will be vulnerable to the introduction of formal insurance. Moreover, the size of the smallest component in a stable network is non-monotonic with the price of formal insurance, it increases then decreases.

Corollary 4. *The size of the largest component in a stable network is non-decreasing with the price of formal insurance p. The size of the smallest component increases then decreases with p. The smaller, the diameter of the largest component in an initial stable network, the more robust it is to the introduction of formal insurance.*

6 Conclusion

In conclusion, this study has delved into the dynamics of risk-sharing networks in impoverished regions, shedding light on the intricate relationship between formal insurance mechanisms and traditional social safety nets. The analysis has demonstrated that formal insurance significantly influences the structure of social networks, revealing several key insights.

Firstly, as the price of formal insurance decreases, the incentive for individuals to form risk-sharing links diminishes, confirming the notion that risk-sharing networks and formal insurance are substitutes. Interestingly, the price of insurance also plays a crucial role in shaping agents' incentives to form links, particularly in relation to the size of their network components.

Secondly, the study has shown that high linking costs lead to a gradual unraveling of the risk-sharing network as formal insurance prices drop. Conversely, low linking costs result in a rapid transition from a fully connected network to an empty network. Furthermore, for the same price levels of formal insurance, stable networks can exhibit varying structures.

Thirdly, the analysis has extended to welfare considerations, revealing that the Nash equilibrium of the insurance game is constrained Pareto efficient. Individual incentives to adopt formal insurance align with social welfare due to the shared costs among members of a component and any payments made by the insurance company. This alignment has important implications for community welfare.

Overall, this research provides critical insights into how social networks, serving as informal insurance mechanisms, respond to the introduction of formal insurance. The findings offer valuable contributions to the literature on risk-sharing networks, the impact of markets on informal institutions, and the interplay between informal transfers and formal insurance. In particular, the study highlights the complexity of these interactions and the significance of factors such as the price of formal insurance and linking costs in shaping network structures and community welfare.

A Appendix

A.1 Proof of proposition 1

$$\max_{x_{i_1}} \ u(s,\mathbf{x}) = -\exp\left\{-\lambda\left(y^0 - px_i^*(s) - \mu(1-x_i^*(s)) - \frac{\lambda\sigma^2}{2s}(1-x_i^*(s))^2\right)\right\} \tag{3}$$

Agent i expected utility increases with his coverage rate imply that

$$\begin{aligned} \frac{\partial u(s,\mathbf{C})}{\partial C_{i_1}} \geq 0 \Leftrightarrow -\lambda u(s,\mathbf{x})\left(-\frac{p-\mu}{s} + \frac{\lambda\sigma^2}{s^2}(1-x_i)\right) \geq 0 \\ \Leftrightarrow x_i \leq 1 - \frac{s}{\lambda\sigma^2}(p-\mu). \end{aligned} \tag{4}$$

Thus for any agent in a component of size s, the unique coverage rate that maximizes the expected utility is $x^*(s) = 1 - \frac{s}{\lambda\sigma^2}(p-\mu)$. To complete the proof, note that the coverage rates lie between $(0,1)$.

A.2 Proof of corollary 2

$$v(s) = -exp\left\{-\lambda\left(y^0 - (px_i^*(s) + \mu(1-x_i^*(s))) - \frac{\lambda\sigma^2}{2s}(1-x_i^*(s))^2\right)\right\}$$

$$v'(s) = -\lambda v(s)\left(-\frac{\partial x_i^*(s)}{\partial s}(p-\mu) + \frac{\lambda\sigma^2}{2s^2}(1-x_i^*(s))^2 + \frac{\lambda\sigma^2}{s}\frac{\partial x_i^*(s)}{\partial s}(1-x_i^*(s))\right).$$

If $x_i^*(s) \leq 0$,

$$v'(s) = -\lambda v(s)\left(\frac{\lambda\sigma^2}{2s^2}\right) > 0,$$

and

$$v"(s) = -\lambda v'(s)\left(\frac{\lambda\sigma^2}{2s^2}\right) + \lambda v(s)\left(\frac{\lambda\sigma^2}{s^3}\right) < 0.$$

If $0 < x_i^*(s) < 1$ then $-\frac{1-x_i^*(s)}{s} = \frac{\partial x_i^*(s)}{\partial s} = -\frac{p-\mu}{\lambda\sigma^2}$, thus the third term in v' equals

$$\frac{(p-\mu)^2}{\lambda\sigma^2} + \frac{\lambda\sigma^2}{2}\left(\frac{p-\mu}{\lambda\sigma^2}\right)^2 - \lambda\sigma^2\left(\frac{p-\mu}{\lambda\sigma^2}\right)^2 = \frac{(p-\mu)^2}{2\lambda\sigma^2}$$

and

$$v'(s) = -v(s)\frac{(p-\mu)^2}{2\sigma^2} > 0$$

. Therefore v is strictly increasing. Moreover,

$$v"(s) = -\, v'(s)\frac{(p-\mu)^2}{2\sigma^2} < 0,$$

thus $v"(s) < 0$ and $v(s)$ is strictly concave. The smoothness comes from the fact that at $\overline{s}$ the left-hand derivative equals the right-hand one.

A.3 Proof of lemma 3

$$b(s) = -2\exp\left\{-\lambda\left(y^0-\mu-\frac{\lambda\sigma^2}{2s}\right)\right\} + \exp\left\{-\lambda\left(y^0-\mu-\frac{\lambda\sigma^2}{2(s-1)}\right)\right\} + \exp\left\{-\lambda\left(y^0-\mu-\frac{\lambda\sigma^2}{2}\right)\right\}$$

$$b'(s) = -v_2(s)\left(\frac{\lambda\sigma}{s}\right)^2 + \frac{v_2(s-1)}{2}\left(\frac{\lambda\sigma}{s-1}\right)^2$$

$$b'(s) \geq 0 \Leftrightarrow -v_2(s)\left(\frac{\lambda\sigma}{s}\right)^2 \geq -\frac{v_2(s-1)}{2}\left(\frac{\lambda\sigma}{s-1}\right)^2$$
$$\Leftrightarrow 2\underbrace{\left(\frac{s-1}{s}\right)^2 \exp\left\{\frac{-(\lambda\sigma)^2}{2s(s-1)}\right\}}_{h(s)} \geq 1$$

$$h'(s) = \frac{2(s-1)}{s^3}\exp\left\{\frac{-(\lambda\sigma)^2}{2s(s-1)}\right\} + \frac{2s-1}{s^4}\frac{(\lambda\sigma)^2}{2}\exp\left\{\frac{-(\lambda\sigma)^2}{2s(s-1)}\right\} \geq 0$$

Moreover,

$$2h(2) = \frac{1}{2}\exp\left\{\frac{-(\lambda\sigma)^2}{4}\right\} < 1,$$

and

$$2h(\infty) = 2.$$

Thus there exists a size $\hat{s}$ such that for $s \leq \hat{s} \Rightarrow 2h(s) \leq 1$ and $b(s)$ is decreasing and for $s > \hat{s} \Rightarrow 2h(s) > 1$ and $b(s)$ is increasing.

$$b(2) = 2\exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{2}\right)\right\} - 2\exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{4}\right)\right\}$$

$$b(\infty) = \exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{2}\right)\right\} - \exp\left\{-\lambda\left(y^0 - \mu\right)\right\}$$

$$b(2) \geq b(\infty) \Leftrightarrow \exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{2}\right)\right\} - 2\exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{4}\right)\right\} \geq -\exp\left\{-\lambda\left(y^0 - \mu\right)\right\}$$

$$\Leftrightarrow \exp\left\{\frac{(\lambda\sigma)^2}{2}\right\} - 2\exp\left\{\frac{(\lambda\sigma)^2}{4}\right\} \geq -1$$

$$\Leftrightarrow \left(\exp\left\{\frac{(\lambda\sigma)^2}{4}\right\} - 1\right)^2 \geq 0$$

Since the last line is always true with strict inequality, $b(2) > b(\infty)$. QED

A.4 Proof of lemma 4

To prove the first part of the result I write

$$b(s) = v(s) - v(s-1) + v(s) - v(1).$$

Thus

$$\frac{\partial b(s)}{\partial p} = \frac{\partial}{\partial p}\left(v(s) - v(s-1)\right) + \frac{\partial}{\partial p}\left(v(s) - v(1)\right).$$

Remember that p lies between μ and p_n and that $v(s) = -exp\left\{-\lambda\left(y^0 - p + \frac{(p-\mu)^2}{2\lambda\sigma^2}s\right)\right\}$. The first term is positive if

$$\frac{\partial}{\partial p}\left(v(s) - v(s-1)\right) \geq 0 \Leftrightarrow -v(s)\left(\frac{p-\mu}{\lambda\sigma^2}s - 1\right) \geq -v(s-1)\left(\frac{p-\mu}{\lambda\sigma^2}(s-1) - 1\right). \quad (5)$$

Note that when p lies between μ and p_n, the coverage rate of any agent in any component is strictly positive. Equivalently, $s - 1 < s < \bar{s}$. Which implies that

$$s - 1 < s < \underbrace{\frac{\lambda\sigma^2}{p-\mu}}_{\bar{s}} \Leftrightarrow \frac{p-\mu}{\lambda\sigma^2}(s-1) - 1 < \frac{p-\mu}{\lambda\sigma^2}s - 1 < 0.$$

Therefore,

$$-v(s)\left(\frac{p-\mu}{\lambda\sigma^2}s-1\right) \geq -v(s-1)\left(\frac{p-\mu}{\lambda\sigma^2}s-1\right) \geq -v(s-1)\left(\frac{p-\mu}{\lambda\sigma^2}(s-1)-1\right).$$

Thus inequality 5 is verified. A similar argument can be used to show that $\frac{\partial}{\partial p}\left(v(s)-v(1)\right) \geq 0$ and $\underline{b}(s) \geq \frac{\partial}{\partial p}\left(v(s)-v(\frac{s}{2})\right) \geq 0$. Hence $b(s)$ increases with the price of insurance.

Now let us prove that the way b varies with s is determined by the price.

$$\begin{aligned}
\frac{\partial b(s)}{\partial s} \geq 0 &\Leftrightarrow -2\frac{(p-\mu)^2}{2\sigma^2}v(s) + \frac{(p-\mu)^2}{2\sigma^2}v(s-1) \geq 0 \\
&\Leftrightarrow 2 \geq \exp\left\{\frac{(p-\mu)^2}{2\sigma^2}\right\} \\
\frac{\partial b(s)}{\partial s} \geq 0 &\Leftrightarrow p \leq \underbrace{\mu + \sigma\sqrt{2\ln 2}}_{\hat{p}}
\end{aligned}$$

I conclude the proof by using the implicit function theorem to determine how s_{h_1} varies with p;

$$\frac{\partial s_{h_1}}{\partial p} = -\frac{\frac{\partial b(s)}{\partial p}}{\frac{\partial b(s)}{\partial s}}.$$

A.5 Proof of proposition 5

I start by proving the first affirmation. **Lemma** 4 established that if $p < \hat{p}$, b is increasing in s. let us assume that $b(2) < \kappa$ and $b(n) > \kappa$. Then, there exists $s_{h_1} \geq 3$ such that $n < \lceil s_{h_1} \rceil \Rightarrow b(n) < \kappa$ and the star with n agents is not stable and $n \geq \lceil s_{h_1} \rceil \Rightarrow b(2) \geq \kappa$ and the star with n agents is eligible for stability. Moreover, no non-empty network is stable for $n \leq \lceil s_{h_1} \rceil$ (If a non-empty network is stable, it has a stable component of size s and this implies that in the star with $s \leq n$ agents, each link generated a positive net relative benefit, which is not possible). If the number of agents n is larger or equal to $\lceil s_{h_1} \rceil$, for a network to be eligible for stability, it has to be either empty or non-empty with the size of each of its components larger than or equal to s_{h_1}. The last characteristic a network must have to be stable is the absence of stable improving switches for any agents. A link switch (or a switch) is said to be improving if the sum of utilities on the new link is strictly higher than the sum of utilities on the link being substituted. In a component, every agent has the same utility. This implies that, for three agents $i \neq j \neq h$ in a network g such that $g_{ij} = 1$ and $g_{ih} = g_{jh} = 0$, a switch from ij to ih is improving if and only if $s^i(g) < s^h(g - ij + ih)$.

let us assume that $n \geq s_{h_1}$ since otherwise only the empty network can be stable. Consider the set of non-empty stable networks. I claim that the set of non-empty stable networks is only composed of minimally connected networks. Assume there exists a stable network g with at least two components. Consider a loose-end i in a component $s^i(g)$ and an agent j such that $g_{ij} = 1$. Consider an agent $h \neq j$ such that $s_{h_1} \leq s^i(g) \leq s^h(g)$ (since each component exhibit internal stability). If $h \in s^i(g)$, then $s^h(g - ij + ih) = s^h(g) = s^i(g)$ and for agent i switching from ij to ih is not improving. If $h \notin s^i(g)$, then $s^h(g - ij + ih) = s^h(g) + 1 \geq s^i(g) + 1 > s^i(g)$ and for agent i, switching from ij to ih is improving. Consequently, for any leaf of the component $s^i(g)$, there exists an improving switch (delete the link in the initial component and form a link at the end of a tree, with any agent in the component of size $s^h(g)$). Moreover, those improving switches create links with positive net relative benefits, since $b(s^h(g - ij + ih)) > b(s^i(g)) \geq \kappa$. This contradicts the definition. Therefore, if a non-empty network is stable, it is minimally connected (a tree of size n). Moreover, if $b(2) > \kappa$, then $b(s) > \kappa$ for any $s \geq 2$ and if a network is stable, then it is a tree of size $n \geq 2$. If $b(n) < \kappa$, then the empty network is the only stable network for any $n \geq 2$. This concludes the proof of the first part.

Now let us prove the second part of the proposition. **Lemma 4** also established that if $p > \hat{p}$, b is decreasing in s. Suppose that $b(2) > \kappa$ and $b(n) < \kappa$. Then, the empty network is never stable as two isolated agents can always create a stable link. Since b is decreasing, there exists $s_{h_1} \geq 3$ such that $n > \lfloor s_{h_1} \rfloor \Rightarrow b(n) < \kappa$ and the star with n agents is not stable and $n \leq \lfloor s_{h_1} \rfloor \Rightarrow b(n) \geq \kappa$ and the star of size n is eligible for stability. Moreover, if a network is stable, the size of each of its components is lower than or equal to $\lfloor s_{h_1} \rfloor$ (If a network is stable, and it has a component of size $s > s_{h_1}$, then the network obtained by transforming this component into a star is also stable. There is a contradiction since no link in a star of size $s > s_{h_1}$ generates a positive π).

To complete the characterization, let us look for networks where any improving switch creates a link with a negative relative profit. Considering $n \leq s_{h_1}$, I claim that if a network is stable then it is a tree of size n. If instead, it has more than one tree, then there exists an improving switch (a loose-end in a tree can cut its link and form a new one in a larger tree), and this switch creates a network with a positive π. If $n \geq \lfloor s_{h_1} \rfloor$, a stable network has more than one component and the size of each component is lower than or equal to $\lfloor s_{h_1} \rfloor$. Consider the stable network g with at least two components. Consider agent i and h in two different components such that $s^i(g) \leq s^h(g) \leq s_{h_1}$. If $s^h(g) \leq \lfloor s_{h_1} \rfloor - 1$, then for any loose-end in component $s^i(g)$, there is a stable improving switch (delete the link in the initial component and form a link with any agent in a component of size $s^h(g)$). This is a contradiction. Thus, if g is stable, for any pair of components, the size of the largest component is $\lfloor s_{h_1} \rfloor$. Only networks such that, the size of every component except one is $\lfloor s_{h_1} \rfloor$, and the size of the remaining component is lower than or equal to $\lfloor s_{h_1} \rfloor$, satisfy this condition (forest of almost $\lfloor s_{h_1} \rfloor$). Thus, there is always a stable network and it is such that every component except one is a star of size $\lfloor s_{h_1} \rfloor$ and the size of the remaining star is lower than or equal to $\lfloor s_{h_1} \rfloor$ (quasi-uniform star forest of almost $\lfloor s_{h_1} \rfloor$). Other forests of almost $\lfloor s_{h_1} \rfloor$ can be stable. Furthermore, if $b(2) < \kappa$, then $b(s) < \kappa$ for any $s \geq 2$ and the empty network is the only stable network. If $b(n) > \kappa$, then a stable network is a tree of size $n \geq 2$.

A.6 Proof of lemma 5

Lemma 4 present how $b(s)$ varies with s for $p \in (\mu, p_n)$. Moreover, since $v_1(1)$ does not vary with s, Lemma 3 gives how $b_{rh_2}(s)$ varies with s. Therefore, I only focus on $b_{rh_1}(s)$. For $\overline{s} < s < \overline{s} + 1$, the relative benefit is then written as follows:

$$
\begin{aligned}
b_{rh_1}(s) =& 2v_2(s) - v_1(s-1) - v_1(1) \\
b_{rh_1}(s) =& -2\exp\left\{-\lambda\left(y^0 - \mu - \frac{\lambda\sigma^2}{2s}\right)\right\} + \exp\left\{-\lambda\left(y^0 - p + \frac{(p-\mu)^2}{2\lambda\sigma^2}(s-1)\right)\right\} \\
& + \exp\left\{-\lambda\left(y^0 - p + \frac{(p-\mu)^2}{2\lambda\sigma^2}\right)\right\}.
\end{aligned}
$$

The first order derivative is

$$b'_{rh_1}(s) = 2v'_2(s) - v'_1(s-1)).$$

$$\frac{\partial b_{rh_1}(s)}{\partial s} \geq 0 \Leftrightarrow 2\frac{\partial v_2(s)}{\partial s} \geq \frac{\partial v_1(s-1)}{\partial s}$$
$$\frac{\partial b_{rh_1}(s)}{\partial s} \geq 0 \Leftrightarrow -\frac{(\lambda\sigma)^2}{s^2}v_2(s) \geq -\frac{(p-\mu)^2}{2\sigma^2}v_1(s-1)$$
$$\frac{\partial b_{rh_1}(s)}{\partial s} \geq 0 \Leftrightarrow \left(\frac{\overline{s}(p)}{s}\right)^2 \geq \frac{o(s)}{2}$$

with

$$o(s) = \frac{v_1(s-1)}{v_2(s)} = \exp\left\{\lambda\left(\frac{(p-\mu)^2}{2\lambda\sigma^2}(s-1) - (p-\mu) + \frac{\lambda\sigma^2}{2s}\right)\right\}$$
$$o'(s) = \lambda\left(\frac{(p-\mu)^2}{2\lambda\sigma^2} - \frac{\lambda\sigma^2}{2s^2}\right)o(s).$$

Here $s-1 < \overline{s} \leq s$. Therefore, the term in $o'(s)$ is positive for $s \geq \overline{s}(p) = \frac{\lambda\sigma^2}{p-\mu}$. Thus $o'(s) \geq 0$ and $o(s)$ is increasing in s. Moreover, notice that $\frac{(p-\mu)^2}{2\lambda\sigma^2}(s-1) - \frac{p-\mu}{2} < 0$ since $s-1 < \overline{s}(p)$, and $\frac{\lambda\sigma^2}{2s} - \frac{p-\mu}{2} \leq 0$ since $s > \overline{s}(p)$, hence $o(s)/2 \leq 1/2$. The function $\frac{\overline{s}(p)}{s}$ is decreasing in s and lower than or equal to 1 since $s \geq \overline{s}(p)$ and is equal to 1 when $s = \overline{s}(p)$. Since the two functions are monotonous, they intersect at most once. Therefore, for $s \in [\overline{s}, \overline{s}+1]$, either $b_{rh_1}(s)$ is increasing in s or it increases then decreases in s.

Now, let us rank $b_{rl}(s)$, $b_{rh_1}(s)$ and $b_{rh_2}(s)$. Since agents are risk-averse, for all sizes s of component $v_1(s) \geq v_2(s)$. This implies that $v_1(s-1) + v_1(1) \geq v_2(s-1) + v_1(1) \geq v_2(s-1) + v_2(1)$ which combined with the previous inequality, gives $b_{rl}(s) \geq b_{rh_1}(s)$ and $b_{rh_2}(s) \geq b_{rh_1}(s)$. The ranking between $b_{rl}(s)$ and $b_{rh_2}(s)$ is less trivial. First let us write $b_{rl}(s) - b_{rh_2}(s) = 2v_1(s) - v_1(s-1) - (2v_2(s) - v_2(s-1))$. Notice that $b_{rl}(\overline{s}) - b_{rh_2}(\overline{s}) = v_2(\overline{s}-1) - v_1(\overline{s}-1) < 0$, $b_{rl}(\infty) - b_{rh_2}(\infty) = -v_2(\infty) > 0$ and $b_{rl}(s)$ is monotonic. Thus, there exists a unique size such that $b_{rl}(s) - b_{rh_2}(s) = 0$ and before this size, $b_{rl}(s) < b_{rh_2}(s)$. Notice that $b_{rl}(s) - b_{rh_1}(s) = 2(v_1(s) - v_2(s))$, therefore, $b_{rl}(\overline{s}) - b_{rh_1}(\overline{s}) = b'_{rl}(\overline{s}) - b'_{rh_1}(\overline{s}) = 0$ and $b_{rh_2}(s) - b_{rh_1}(s) = v_1(s-1) - v_2(s-1)$, thus $b_{rh_2}(\overline{s}+1) - b_{rh_1}(\overline{s}+1) = b'_{rh_2}(\overline{s}+1) - b'_{rh_1}(\overline{s}+1) = 0$. Therefore, since $b_{rl}(s) \geq b_{rh_1}(s)$ and $b_{rh_2}(s) \geq b_{rh_1}(s)$, $b_{rh_2}(s)$ and $b_{rl}(s)$ intersect for a size in the interval $(\overline{s}, \overline{s}+1)$. Remember that there exists a threshold size $\hat{s}$ such that for $s \leq \hat{s}$, $b_{rh_2}(s)$ decreases and otherwise, it increases. The argument presented before induce that $\overline{s}+1 \leq \hat{s}$. When $\overline{s}+1 = \hat{s}$, $b'_{rh_2}(\overline{s}+1) = b'_{rh_1}(\overline{s}+1) = 0$. There is no more local extrema and $\hat{s} = \hat{s}_1$ is a saddle point.

When $p > \hat{p}$, $b_{rl}(s)$ decreases. Therefore, since $b_{rh_2}(s)$ and $b_{rl}(s)$ intersect for a size in the interval $(\overline{s}, \overline{s}+1)$, and on this interval either $b_{rh_1}(s)$ is increasing in s or it increases then decreases in s, for $p > \hat{p}$, $b_{rh_1}(s)$ can only decrease. If it was not the case, since $b'_{rl}(\overline{s}) > 0$, $b_{rh_1}(s)$ will first decrease, then increase to decrease again. This contradict the behavior of $b_{rh_1}(s)$ on $(\overline{s}, \overline{s}+1)$. Hence, $b'_3(\overline{s}+1) = b'_2(\overline{s}+1) < 0$ for $p > \hat{p}$. When $p < \hat{p}$, $b_{rl}(s)$ increases, then there are three cases. Either $b'_3(\overline{s}+1)$ is lower than zero, equal to zero or larger than zero. Using the same argument as before, it appears that when $b'_3(\overline{s}+1) \geq 0$, $b_{rh_1}(s)$ increases. As well, when $b'_3(\overline{s}+1) < 0$, then $b_{rh_1}(s)$ increases first, then decreases. Remark that,

$$\frac{v_2(\overline{s})}{v_2(\overline{s}+1)} = \exp\left\{\frac{\lambda(p-\mu)^2}{2\left(\lambda\sigma^2+p-\mu\right)}\right\} \text{ and } \frac{1}{\overline{s}} = \frac{p-\mu}{\lambda\sigma^2}.$$

Thus, for $p < \hat{p}$

$$\begin{aligned}
b'_3(\overline{s}+1) \geq 0 &\Leftrightarrow 2 \geq \frac{v_2(\overline{s})}{v_2(\overline{s}+1)}\left(1+\frac{1}{\overline{s}}\right)^2 \\
&\Leftrightarrow \ln 2 \geq \frac{\lambda(p-\mu)^2}{2\left(\lambda\sigma^2+p-\mu\right)} + 2\ln\left(1+\frac{p-\mu}{\lambda\sigma^2}\right) \\
b'_3(\overline{s}+1) \geq 0 &\Leftrightarrow \underbrace{\ln 2 - 2\ln\left(1+\frac{p-\mu}{\lambda\sigma^2}\right)}_{\alpha(p)} \geq \underbrace{\frac{\lambda(p-\mu)^2}{2\left(\lambda\sigma^2+p-\mu\right)}}_{\beta(p)}.
\end{aligned}$$

Moreover, $\alpha(\mu) = \ln(2) > \beta(\mu) = 0$,

$$\alpha'(p) = -\frac{2}{\lambda\sigma^2+p-\mu} < 0 \text{ and } \beta'(p) = \frac{\lambda(2(p-\mu)\lambda\sigma^2+(p-\mu)^2)}{2\left(\lambda\sigma^2+p-\mu\right)^2} > 0,$$

therefore, for $p < \hat{p}$ there exists a price $\hat{p}_1$

$$\begin{aligned}
b'_3(\overline{s}+1) \geq 0 &\Leftrightarrow p \leq \hat{p}_1 \\
b'_3(\overline{s}+1) < 0 &\Leftrightarrow p > \hat{p}_1.
\end{aligned}$$

Proving the existence of $\hat{p}_2, \hat{p}_3, \hat{p}_4$ is easier. Since b increases with s for $p < \hat{p}_1$ and decreases with s until $\hat{s}$ for $p > \hat{p}$, those thresholds necessarily exist.

Lemma 6. *Consider $p \in (p_n, p_1)$ and n large enough.*

- *If $p < \hat{p}_1$, then (a) for $\kappa < b(2)$, the star is stable and any other stable network is a* *tree. (b) For* $b(2) < \kappa < b(n)$ *both the empty network and the star are stable. Moreover, any other stable network is a tree. (c) For* $b(n) < \kappa$*, the empty network is the unique stable network.*

- *If* $\hat{p}_1 < p < \hat{p}_2$*, then (a) for* $\kappa < b(2)$*, the star is stable and any other stable network is a tree. (b) For* $b(2) < \kappa < b(\hat{s})$ *both the empty network and the star are stable. Moreover, any other stable network is a tree.(c) For* $b(\hat{s}) < \kappa < b(\hat{s}_1)$ *the empty network, the* s_{h_2}*-quasi-uniform star forest and the star are simultaneous stable. Moreover, a non-empty stable network is either a tree or a* s_{h_2}*-forest. (d) For* $b(\hat{s}_1) < \kappa < b(n)$ *both the empty network and the star are stable. Moreover, any other stable network is a tree. (e) For* $b(n) < \kappa$*, the empty network is the unique stable network.*

- *If* $\hat{p}_2 < p < \hat{p}_3$*, then (a) for* $\kappa < b(\hat{s})$*, the star is stable and any other stable network is a tree. (b) For* $b(\hat{s}) < \kappa < b(2)$*, both the star and the* s_{h_2}*-quasi-uniform star forest are stable. Moreover, a stable network is either a tree or a* s_{h_2}*-forest. (c) For* $b(2) < \kappa < b(\hat{s}_1)$*, the empty network, the* s_{h_2}*-quasi-uniform star forest and the star are simultaneous stable. Moreover, a non-empty stable network is either a tree or a* s_{h_2}*-forest. (d) For* $b(\hat{s}_1) < \kappa < b(n)$ *both the empty network and the star are stable. Moreover, any other stable network is a tree. (e) For* $b(n) < \kappa$*, the empty network is the unique stable network.*

- *If* $\hat{p}_3 < p < \hat{p}_4$*, then (a) for* $\kappa < b(\hat{s})$*, the star is stable and any other stable network is a tree. (b) For* $b(\hat{s}) < \kappa < b(2)$*, both the star and the* s_{h_2}*-quasi-uniform star forest are stable. Moreover, a stable network is either a tree or a* s_{h_2}*-forest. (c) For* $b(2) < \kappa < b(n)$*, the empty network, the* s_{h_2}*-quasi-uniform star forest and the star are simultaneous stable. (d) For* $b(n) < \kappa < b(\hat{s}_1)$*, both the empty network and the* s_{h_2}*-quasi-uniform star forest are stable. Moreover, a non-empty stable network is a* s_{h_2}*-forest. (e) For* $b(\hat{s}_1) < \kappa$*, the empty network is the unique stable network.*

- *If* $\hat{p}_4 < p < \hat{p}$*, then (a) for* $\kappa < b(\hat{s})$*, the star is stable and any other stable network is a tree. (b) For* $b(\hat{s}) < \kappa < b(n)$*, both the star and the* s_{h_2}*-quasi-uniform star forest are stable. Moreover, a stable network is either a tree or a* s_{h_2}*-forest. (c) For* $b(n) <$

$\kappa < b(2)$, the s_{h_2}-quasi-uniform star forest is stable. Moreover, a stable network is a s_{h_2}-forest. (d) For $b(2) < \kappa < b(\hat{s}_1)$, both the empty network and the s_{h_2}-quasi-uniform star forest are stable. Moreover, a non-empty stable network is a s_{h_2}-forest. (e) For $b(\hat{s}_1) < \kappa$, the empty network is the unique stable network.

- *If $p > \hat{p}$, then (a) for $\kappa < b(\hat{s})$, the star is stable and any other stable network is a tree. (b) For $b(\hat{s}) < \kappa < b(n)$, both the star and the s_{h_2}-quasi-uniform star forest are stable. Moreover, a stable network is either a tree or a s_{h_2}-forest. (c) For $b(n) < \kappa < b(2)$, the s_{h_2}-quasi-uniform star forest is stable. Moreover, a stable network is a s_{h_2}-forest. (d) For $b(\hat{s}_1) < \kappa$, the empty network is the unique stable network.*

back to the discussion

References

I. Alger and J. W. Weibull. Family ties, incentives and development: a model of coerced altruism. *SSE/EFI Working Paper Series in Economics and Finance*, (681):07–10, 2007.

I. Alger and J. W. Weibull. Kinship, incentives, and evolution. *American Economic Review*, 100(4):1725–58, 2010.

A. Ambrus and M. Elliott. Investments in social ties, risk sharing, and inequality. *The Review of Economic Studies*, 88(4):1624–1664, July 2021. ISSN 0034-6527. doi: 10.1093/restud/rdaa073. URL https://academic.oup.com/restud/article/88/4/1624/5956730.

A. Ambrus, M. Mobius, and A. Szeidl. Consumption risk-sharing in social networks. *American Economic Review*, 104(1):149–82, 2014.

A. Ambrus, W. Gao, and P. Milán. Informal risk sharing with local information. *The Review of Economic Studies*, 89(5):2329–2380, 2022.

R. Arnott and J. E. Stiglitz. Moral hazard and nonmarket institutions: Dysfunctional crowding out of peer monitoring? *American Economic Review*, 81(1):179–190, 1991.

O. Attanasio and J.-V. Ríos-Rull. Consumption smoothing in island economies: Can public insurance reduce welfare? *European economic review*, 44(7):1225–1258, 2000.

E. M. Azevedo and D. Gottlieb. Perfect Competition in Markets With Adverse Selection. *Econometrica*, 85(1):67–105, 2017. ISSN 1468-0262. doi: 10.3982/ECTA13434. URL http://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA13434. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA13434.

A. Banerjee, E. Breza, A. G. Chandrasekhar, E. Duflo, M. O. Jackson, and C. Kinnan. Changes in social network structure in response to exposure to formal credit markets. Technical report, National Bureau of Economic Research, 2021.

A. V. Banerjee and E. Duflo. *Poor economics: a radical rethinking on the way to fight global poverty*. Public affairs, 2011.

M. Belhaj, R. Bourlès, and F. Deroïan. Risk-Taking and Risk-Sharing Incentives under Moral Hazard. *American Economic Journal: Microeconomics*, 6(1):58–90, Feb. 2014. ISSN 1945-7669, 1945-7685. doi: 10.1257/mic.6.1.58. URL https://pubs.aeaweb.org/doi/10.1257/mic.6.1.58.

T. Bene, Y. Bramoullé, and F. Deroïan. Working papers/documents de travail. 2021.

C. Binzel, E. Field, and R. Pande. Does the arrival of a formal financial institution alter informal sharing arrangements? experimental evidence from india. Technical report, Mimeo, 2015.

F. Bloch and M. O. Jackson. Definitions of equilibrium in network formation games. *International Journal of Game Theory*, 34(3):305–318, Oct. 2006. ISSN 0020-7276, 1432-1270. doi: 10.1007/s00182-006-0022-9. URL http://link.springer.com/10.1007/s00182-006-0022-9.

F. Bloch, G. Genicot, and D. Ray. Informal insurance in social networks. *Journal of Economic Theory*, 143(1):36–58, Nov. 2008. ISSN 00220531. doi: 10.1016/j.jet.2008.01.008. URL https://linkinghub.elsevier.com/retrieve/pii/S0022053108000367.

S. Boucher, M. Delpierre, and B. Verheyden. The impact of index-based insurance on informal risk-sharing arrangements. *Luxembourg Institute of Socio-Economic Research (LISER) Working Paper*, 2016.

R. Bourlès, Y. Bramoullé, and E. Perez-Richet. Altruism in networks. *Econometrica*, 85(2): 675–689, 2017.

R. Bourlès, Y. Bramoullé, and E. Perez-Richet. Altruism and Risk Sharing in Networks. *Journal of the European Economic Association*, 19(3):1488–1521, June 2021. ISSN 1542-4766, 1542-4774. doi: 10.1093/jeea/jvaa031. URL https://academic.oup.com/jeea/article/19/3/1488/5878875.

Y. Bramoullé and R. Kranton. Risk-sharing networks. *Journal of Economic Behavior & Organization*, 64(3-4):275–294, Nov. 2007. ISSN 01672681. doi: 10.1016/j.jebo.2006.10.004. URL https://linkinghub.elsevier.com/retrieve/pii/S016726810700128X.

P.-A. Chiappori, K. Samphantharak, S. Schulhofer-Wohl, and R. M. Townsend. Heterogeneity and risk sharing in village economies. *Quantitative economics*, 5(1):1–27, 2014.

J. H. Cochrane. A simple test of consumption insurance. *Journal of political economy*, 99 (5):957–976, 1991.

A. De Janvry, V. Dequiedt, and E. Sadoulet. The demand for insurance against common shocks. *Journal of Development Economics*, 106:227–238, 2014.

L. Einav, A. Finkelstein, S. P. Ryan, P. Schrimpf, and M. R. Cullen. Selection on Moral Hazard in Health Insurance. *The American Economic Review*, 103(1):178–219, 2013. ISSN 0002-8282. URL http://www.jstor.org/stable/23469640. Publisher: American Economic Association.

M. Fafchamps. Risk Sharing Between Households. In *Handbook of Social Economics*, volume 1, pages 1255–1279. Elsevier, 2011. ISBN 978-0-444-53713-3. doi: 10.1016/B978-0-444-53707-2.00007-4. URL https://linkinghub.elsevier.com/retrieve/pii/B9780444537072000074.

M. Fafchamps and F. Gubert. The formation of risk sharing networks. *Journal of Development Economics*, 83(2):326–350, July 2007. ISSN 03043878. doi: 10.1016/j.jdeveco.2006.05.005. URL https://linkinghub.elsevier.com/retrieve/pii/S0304387806000782.

M. Fafchamps and S. Lund. Risk-sharing networks in rural philippines. *Journal of development Economics*, 71(2):261–287, 2003.

V. Farinha Luz, P. Gottardi, and H. Moreira. Risk Classification in Insurance Markets with Risk and Preference Heterogeneity. *The Review of Economic Studies*, page rdad017, Feb. 2023. ISSN 0034-6527. doi: 10.1093/restud/rdad017. URL https://doi.org/10.1093/restud/rdad017.

J. Gagnon and S. Goyal. Networks, markets, and inequality. *American Economic Review*, 107(1):1–30, 2017.

E. Ligon and L. Schechter. Motives for sharing in social networks. *Journal of Development Economics*, 99(1):13–26, 2012.

LLoyd's. *Insurance in Developing Countries. Exploring Opportunities in Microinsurance.* MicroInsurance Centre, 2009.

M. Mazzocco and S. Saini. Testing efficient risk sharing with heterogeneous risk preferences. *American Economic Review*, 102(1):428–468, 2012.

A. Merry and J. S. Rozo Calderon. *The Landscape of Microinsurance 2022*. Microinsurance Network, 2022.

M. R. Rosenzweig. Risk, implicit contracts and the family in rural areas of low-income countries. *The Economic Journal*, 98(393):1148–1170, 1988.

T. Sargent. *Macroeconomic Theory*. Academic Press, Orlando, 2nd edition, 1987.

K. Takahashi, C. B. Barrett, and M. Ikegami. Does index insurance crowd in or crowd out informal risk sharing? evidence from rural ethiopia. *American Journal of Agricultural Economics*, 101(3):672–691, 2019.

R. M. Townsend. Risk and insurance in village india. *Econometrica: journal of the Econometric Society*, pages 539–591, 1994.

C. Udry. Risk and insurance in a rural credit market: An empirical investigation in northern nigeria. *The Review of Economic Studies*, 61(3):495–526, 1994.

A. Veiga and Y. Levy. Optimal Contract Regulation in Selection Markets. *SSRN Electronic Journal*, 2022. ISSN 1556-5068. doi: 10.2139/ssrn.4029945. URL https://www.ssrn.com/abstract=4029945.

E. G. Weyl and A. Veiga. Pricing Institutions and the Welfare Cost of Adverse Selection. *American Economic Journal: Microeconomics*, 9(2):139–148, 2017. ISSN 1945-7669. URL http://www.jstor.org/stable/26156900. Publisher: American Economic Association.