Emissions pricing instruments with intermittent renewables: second-best policy☆

Preliminary Version

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Abstract

I analyze emissions pricing to support the integration of a renewable resource into an electricity mix composed of an emissions-intensive technology. I consider the intermittent nature of the resource such as wind energy and incremental externalities that become severe for high emissions levels. I show that an emissions tax is inefficient when consumers are on flat-rate electricity tariffs and do not adapt their consumption to varying production. The tax is inefficient even with flexibility in the markets when consumers are on varying tariffs. The renewable resource induces variability in fossil-fueled electricity production and associated marginal damage that does not match a predetermined tax. I study an Emissions Trading Scheme that provides flexibility at the policy level. Emissions permits are traded at market prices. Since the emissions cap must still be predetermined, I show that it leads to inefficient permits prices that do not match the marginal damages. I also find that the two emissions pricing instruments are not implemented equivalently since the tax differs from the prices of permits.

Keywords: electricity, renewables, intermittency, emissions tax, Emissions Trading Scheme

JEL classification: D24, D61, D62, Q41, Q42, Q48

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1. Introduction

Renewable energy sources play an essential role in reducing fossil fuel-based electricity production. They provide both greenhouse gas and air emissions benefits.[1] Subsequently, emissions pricing measures that have been launched to meet regional to subnational emissions reduction goals[2] are also expected to support an electricity mix in favor of renewables. Wind and solar energy are key to increasing renewables-based production (IEA 2021) but depend on variable and uncontrollable conditions resulting in intermittent electricity.[3] This intermittence raises concerns over balancing electricity production and consumption which calls for flexibility in the markets.[4] This paper studies to what extent an emissions pricing policy is efficient in response to intermittency.

Emissions pricing puts a price on each unit of emissions to capture negative externalities related to fossil-fueled electricity. It increases the variable production cost of fossil-fueled electricity and incentivizes a shift towards renewable technologies. Emissions pricing can take the form of a price-based instrument such as an emissions tax, usually referred to as the Pigouvian tax (Pigou 1960). An alternative to emissions taxation is a quantity-based instrument such as an Emissions Trading Scheme, ETS for short (Dales 1968, Montgomery 1972). Under conditions of certainty, the economic theory envisions the efficient tax or permit price under the ETS, as one that is equal to the marginal social damage. In addition, the two instruments have identical economic outcomes (Weitzman 1974).

However, ensuring an optimal regulation may be challenging in the presence of intermittent renewables. They induce variability in the electricity markets. Consequently, the regulator must decide on the policy while anticipating future variable levels of electricity consumption, production, and associated social damage. Hence, this paper studies how intermittency affects the efficiency of emissions pricing instruments. A question that also arises is whether the tax can still be equivalently implemented by an ETS. Therefore, I assess if the level of the tax differs from that of the permit price when an ETS is administered.

To investigate the implication of intermittent renewables for an optimal emissions pricing policy, I propose a stylized theoretical model of the electricity sector. Electric-

[1]Greenhouse gas emissions, mainly carbon dioxide, are global public bad that impact people through climate change. Air emissions (sulfur dioxide, nitrogen dioxide, and particulate matter) are local public bad that affect people through health risks and disamenities (Bielecki et al. 2020). See West et al. (2013) who have examined the economic impacts of climate change mitigation policy from global and regional perspectives.

[2]A summary map of regional, national and subnational carbon pricing initiatives can be found on the *World Bank Carbon Pricing Dashboard* (World Bank 2022).

[3]For example, electricity production from wind turbines fluctuates with wind speed and direction and that from solar photovoltaics with radiation intensity (see, for instance, Crawley 2013).

[4]Intermittency challenges the imperative of the electricity industry to constantly balance electricity production and consumption. Disruptions in this balance have both technical and economic impacts. See, for instance, Cochran et al. (2014), EURELECTRIC (2014), IEA (2011), and IEA-ISGAN (2019) on the flexibility in electricity markets to manage renewables intermittency.

ity production is ensured by a mix of an existing fossil-fired power plant (e.g. coal) and investment in a renewable-based technology (e.g. wind turbine). The fossil-fueled technology produces uninterrupted electricity but causes emissions.[5] The renewable technology is emissions-free but produces electricity that depends on variable conditions, hereafter also referred to as states of nature (e.g. wind (un)availability). In line with the current situation, I assume no electricity storage capacity where the two coexisting technologies supply electricity to match demand reliably.[6] Intermittency is captured through competitive wholesale markets that are state-dependent: electricity production is state-dependent and is traded at state-dependent prices.

Electricity demand comes from consumers described according to their retail electricity tariffs. Most commonly, consumers have a flat-rate tariff. They are billed at a fixed price that is independent of the state of nature that drives production. This tariff does not convey information on varying electricity production to which consumers do not necessarily adapt their consumption. Flexibility is introduced on the demand side of the electricity markets when consumers move to state-dependent tariffs. They can adapt their electricity usage to varying production.[7]

The regulator is concerned with implementing the first-best emissions pricing policy whose purpose is two-fold. It must be able to internalize the externalities that become severe for high levels of emissions from fossil-fueled electricity. In addition, the policy must implement the electricity production plan and consumption allocation that ensure social welfare. In principle, an emissions pricing policy as an incentive-based measure is an *ex-ante* regulation. The regulator announces the policy in anticipation of future consumption, production, and damage levels.

My model shows that an *ex-ante* emissions tax remains a second-best regulation when consumers move from the flat-rate to state-dependent tariffs. When consumption is constrained due to consumers on the flat-rate tariff, I find that an emissions tax does not implement the constrained social welfare allocation. The integration of the intermittent renewable results in different levels of fossil-fueled electricity production and associated marginal damage that do not match the tax. For a similar explanation, even when consumers are flexible and can adapt their consumption to changing electricity production, I find that the social welfare allocation is unreachable with a tax.

Secondly, I find that in addition to flexibility on the demand side, introducing flexibility at the policy level through an Emissions Trading Scheme is second-best as well. I study the ETS as a flexible market-based regulation. While the emissions cap is set

[5] I assume there is no capacity constraint for the fossil energy technology as in Rouillon (2015) and Twoney and Neuhoff (2010). The existing capacity provides for electricity demand reliably.

[6] I abstract from storage technologies that are presently costly but will play an important role in the future in ensuring reliable electricity supply.

[7] Flexible consumers have, for example, programmed equipment coupled with smart meters informing them on wholesale electricity prices. For parsimony, the model does not consider costs associated with smart metering. See, for instance, Ambec and Crampes (2021) and Chiba and Rouillon (2020) who study the implication of smart metering on the optimal electricity mix.

ex-ante, the emissions permits are traded *ex-post* on state-dependent markets. My model suggests that the economic agents anticipate that the regulator has the possibility to implement different levels of emissions cap to regulate emissions from variable fossil-fueled electricity production. Ultimately, no matter which cap is set, I show that it leads to *ex-post* inefficient permits prices as they do not match the marginal damage.

Finally, the results of my model indicate that the economic outcomes of administering the ETS are not the same as those when the emissions tax is implemented. The cap set by the regulator leads to prices of permits that differ in each state of nature while the emissions tax is uniform across the states. It implies that the two emissions pricing instruments are not implemented equivalently. While ranking the instruments is out of the scope of this work, I can only conclude that in the presence of intermittent renewables, the best that can be achieved with emissions pricing instruments is a second-best policy.

My work contributes principally to three strands of theoretical literature. Firstly, it adds to a growing body of literature on the electricity transition with intermittent renewables. A seminal paper is that of Ambec and Crampes (2012) who analyze the optimal electricity mix with reliable and intermittent technologies and its decentralization through competitive markets. Rouillon (2015) carries a similar analysis by considering imperfect competition from producers owning reliable technologies. This literature has been complemented by studies on how policy instruments impact the optimal electricity mix in the presence of intermittent renewables. These include Abrell et al. (2019), Ambec and Crampes (2019) and Helm and Mier (2019) who find that the optimal electricity mix can be decentralized by a Pigouvian tax that matches the constant marginal damage. This paper adds to the literature by investigating the efficiency of an incentive-based tax under the general hypothesis of increasing marginal damage due to emissions. This allows the model to circumscribe the issues when incremental damage becomes severe for high levels of emissions.

Second, this paper contributes to the scarce theoretical literature that examines the Emissions Trading Scheme to regulate the electricity transition in the presence of intermittent renewables. The closest paper is that of Abrell et al. (2019) who analyze a carbon pricing instrument that can be implemented equivalently either through a carbon tax or an ETS. However, they do not provide a formal model underlying the interaction of intermittent renewables and the emissions cap and trade mechanism. In contrast, I propose a framework for studying the decentralized electricity markets with an ETS and the efficiency of the regulation.

Finally, this paper relates to the extensive literature on the breakdown of the equivalence between the tax and ETS under uncertainty. Weitzman (1974) was the first to establish that while taxes and permits schemes have equivalent results under certainty, they perform differently under conditions of uncertainty that affect the slope of the damage function. Adar and Griffin (1976) consider uncertain marginal control cost function and risk aversion. These seminal papers laid the foundation for all subsequent works that address the difference between the two emissions pricing instruments under uncertainty.[8] This paper fits into this literature by addressing the case when intermittent renewables induce uncertainty in the level of emissions and associated damage.

The rest of the paper is organized as follows. Section 2 sets out the theoretical framework. I study the competitive equilibrium and efficiency of the emissions tax with consumers on the flat-rate tariff in section 3 and with flexible consumers in section 4. Section 5 focuses on the competitive equilibrium with flexible consumers and the Emissions Trading Scheme. I analyze the efficiency of the ETS and compare it with a taxation instrument. Section 6 concludes. All proofs are relegated to the appendices.

2. The framework

2.1. Basic assumptions

My objective is to analyze the efficiency of emissions pricing instruments that account for emissions externalities from fossil-fueled electricity production and that concomitantly foster investment in intermittent renewable-based technologies. For this purpose, I propose a stylized framework where a regulator is concerned with alleviating damage due to emissions from burning fossil fuels to produce electricity. Emissions reduction guides investment in the renewable technology. The latter depends on an intermittent source of energy. For simplicity, I assume that intermittency is depicted by two states of nature given by the set $s \in \{0, 1\}$, where 0 represents the state when the renewable energy is not available and 1 when it is.[9] They occur with probabilities $\pi_s \in]0, 1[$. Since electricity production is state-dependent and the wholesale market is organized in each state, the price per unit of electricity in state s is given by p_s. Moreover, the model is characterized by the following features.

Electricity production is described by two types of technology. The *fossil energy technology* is an existing and fully established one, for example, a coal power plant. It allows reliable electricity provision when the renewable energy is unavailable or insufficient. Electricity production from this technology is therefore state-dependent and is denoted by q_s in state s. Also, it is assumed that there is no capacity constraint where total capacity is able to provide for electricity demand reliably.[10] The production process from this type of technology being controllable, its cost is assumed to be additively separable state by state and is given by $c(q_s)$. This cost is increasing and convex ($c'(q_s) > 0$ and $c''(q_s) > 0$), inactivity is allowed ($c(0) = c'(0) = 0$) and $\lim_{q_s \to +\infty} c(q_s) = +\infty$. Using this technology creates emissions that depend on the level of production and is given by $\mathcal{E}(q_s)$ in each s . This function is increasing and convex ($\mathcal{E}'(q_s) > 0$ and $\mathcal{E}''(q_s) > 0$) with $\mathcal{E}(0) = \mathcal{E}'(0) = 0$.

The *renewable energy technology* is emissions-free, non-controllable and intermittent, for instance, a wind turbine. The capacity choice is given by κ and the intermittent

[8] This literature is extensive and it is out of the scope of this paper to review it exhaustively.

[9] Intermittency is modeled similarly as in Ambec and Crampes (2012) and (2019). State 1 may represent the state when the wind blows and 0 when it does not.

[10] I use the same assumption as in Rouillon (2015) and Twoney and Neuhoff (2010).

electricity productivity is depicted by the random variable $g_s \in [0,1]$. It describes the state-dependent production per unit capacity in each state. It is assumed that $g_0 = 0$ since there is no electricity production when the renewable energy is unavailable and $g_1 = 1$ since the renewable capacity is costly and it is inefficient to install unused capacities. Hence, renewable production occurs in state 1 only and is equal to κ. The cost of investing in capacity κ is given by $\mathcal{K}(\kappa)$ with usual assumptions: $\mathcal{K}'(\kappa) > 0$, $\mathcal{K}''(\kappa) > 0$, $\mathcal{K}(0) = \mathcal{K}'(0) = 0$ and $\lim_{\kappa \to +\infty} \mathcal{K}'(\kappa) = +\infty$.[11] Without loss of generality, short-run marginal costs of production are normalized to zero.[12]

In a competitive setting with convex technologies, a representative agent can be used to describe electricity supply.[13] *Ex-ante* of the realization of the state of nature, the optimal fossil-fuel production strategy and the optimal investment in the renewable capacity are derived from a competitive profit-maximizing behavior.

Electricity demand is characterized by consumers who derive welfare from consuming Q_s units of electricity in each s. Their *inverse demand function* is $P(Q_s)$ which is decreasing ($P'(Q_s) < 0$) and verifies that $\lim_{Q_s \to 0} P(Q_s) = +\infty$ and $\lim_{Q_s \to +\infty} P(Q_s) = 0$. Consumers' welfare, $S(Q_s)$, is given by $\int_0^{Q_s} P(v)\, dv$. In addition, electricity demand depends on the type of contracts consumers buy from retailers. The latter act as intermediaries between the consumers and the wholesale markets. I assume that retailers bear no other costs than the procurement of electricity on the wholesale markets and are risk-neutral. Most commonly, there are traditional consumers on flat-rate retail contacts. They pay a price of p_f per unit of electricity. With the no-arbitrage condition, the tariff of 1 unit of electricity is equal to its expected price on the wholesale markets: $p_f = \pi_0 p_0 + \pi_1 p_1$. Traditional consumers do not adapt their electricity consumption to state-dependent production. Irrespective of the state of nature realized, they consume the same units of electricity: $Q_s = Q, \forall s$. When consumers are billed at state-dependent tariffs p_s, they are able to adjust their consumption Q_s in each state s. I refer to them as flexible consumers. Their inverse demand function is the same as traditional consumers except that the retail prices are the state-dependent prices p_s.

Following a welfare-maximizing behavior, consumers exchange retail contracts with retailers *ex-ante* of the realization of the state of nature for *ex-post* electricity provision.

To close the model, the regulator is concerned with passing through damage due to emissions and promoting investment in the renewable capacity using an *emissions pricing policy*. It consists of implementing either a tax τ per unit of emissions or an emissions cap E with tradable permits. As an incentive-based regulation to reduce emissions, he *ex-ante* decides on the tax or emissions cap by anticipating future levels of electricity production, consumption and damage. The damage is measured by the function $\mathcal{D}(\mathcal{E}(q_s))$

[11] The strictly convex capacity investment cost function can be viewed as investment starting at the most productive site, e.g. in terms of weather conditions. See, for instance, Rouillon (2015).

[12] The resource is "free" and variable costs such as operation and maintenance costs for wind technologies tend to be typically lower than those of fossil-fueled technologies (IRENA 2018, Lazard 2018).

[13] I assume no strategic behavior among producers. See, for instance, Rouillon (2015) for non-competitive strategies among electricity producers.

which is increasing and convex with emissions ($\mathcal{D}'(\mathcal{E}(q_s)) > 0$ and $\mathcal{D}''(\mathcal{E}(q_s)) > 0$) and $\mathcal{D}(0) = \mathcal{D}'(0) = 0$.

2.2. (constrained) social welfare allocation

In a social optimum where electricity demand comes from traditional consumers, the regulator is concerned with choosing the production and consumption allocation that maximize the *constrained social welfare*. The latter is given by:

$$CSW = \int_0^Q P(v)dv - \sum_0^{s=1} \pi_s \left[c(q_s) + \mathcal{D}(\mathcal{E}(q_s))\right] - \mathcal{K}(\kappa) \quad s.t. \begin{cases} Q_s = q_s + g_s\kappa, \forall s \\ Q_s = Q, \forall s \end{cases} \tag{1}$$

Social welfare is explained by an *ex-ante* consumers' welfare net of (i) expected production costs of fossil-fueled electricity with the expected damage induced by this activity, and (ii) an *ex-ante* investment in the renewable capacity. The production and consumption levels are as depicted by the first condition which is rather conventional: consumption matches production in each state of nature. What is less conventional is the second condition. It says that the potential level of state-dependent consumption is restricted to satisfy the same set of constraints on the consumption of traditional consumers at the competitive equilibrium. Traditional consumers consume $Q_s = Q$ units of electricity in each s since there are not enough retail tariffs for them to respond to intermittent electricity production (here 1 retail tariff for electricity production that is dependent on 2 states of nature). The flat-rate tariff, in some sense, depicts missing retail markets. Consequently, the consumption allocation is not necessarily Pareto optimal and I introduce the notion of constrained social welfare.

The constraint efficient allocation is given by:

$$P\left(Q^{csw}\right) = \pi_0[c'\left(q_0^{csw}\right) + \mathcal{D}'(\mathcal{E}(q_0^{csw}))\mathcal{E}'\left(q_0^{csw}\right)] + \pi_1[c'\left(q_1^{csw}\right) + \mathcal{D}'(\mathcal{E}(q_1^{csw}))\mathcal{E}'\left(q_1^{csw}\right)] \tag{2}$$

$$\mathcal{K}'\left(\kappa^{csw}\right) = \pi_1\left[c'\left(q_1^{sw}\right) + \mathcal{D}'(\mathcal{E}(q_1^{sw}))\mathcal{E}'\left(q_1^{sw}\right)\right] \tag{3}$$

$$Q = q_0 = q_1 + \kappa \tag{4}$$

If one remembers, electricity consumption is state-independent while fossil-fueled electricity production is flexible in each state. Hence, Eq.(2) says that the marginal surplus, or the willingness to pay for electricity, is equal to the expected marginal social cost of fossil-fueled electricity. This one is, in each state s, the sum of the marginal private cost, $c'\left(q_s^{csw}\right)$, and the marginal emissions damage, $\mathcal{D}'(\mathcal{E}(q_s^{csw}))\mathcal{E}'\left(q_s^{csw}\right)$. Next, Eq.(3) says the marginal investment cost in renewable capacity is equal to the expected social cost of generating 1 unit of electricity with the flexible fossil-fueled technology in state 1. Finally, Eq.(4) says that the electricity consumption is constant in both states of nature.

An interesting question that arises is how a Pareto optimal allocation can be reached. The first step is to drop the second condition of Eq.(1) so that the potential level of state-dependent consumption is no more constrained. It implies a setting of flexible consumers facing state-dependent tariffs and allowing them to adjust their potential state-dependent consumption to state-dependent production. This is a situation of complete markets with as many tariffs as states of nature allowing an efficient consumption allocation. Consumers now have an expected welfare derived from electricity consumption given by $\sum_0^{s=1} \pi_s \int_0^{Q_s} P(v)\ dv$. To sum up, *social welfare* is defined by first dropping condition 2 of Eq.(1). This suggests that there are only flexible consumers who must then be described as having an expected welfare from state-dependent electricity consumption. Formally, in a social optimum where electricity demand comes from flexible consumers, the regulator chooses the production and consumption allocation that maximize the *social welfare* given by:

$$SW = \mathbb{E}\left\{\int_0^{Q_s} P\left(v\right) dv - \left[c(\,q_s\,) + \mathcal{D}\left(\mathcal{E}\left(q_s\right)\right)\right]\right\} - \mathcal{K}(\kappa) \quad s.t.\ Q_s = q_s + g_s\kappa, \forall s \tag{5}$$

Substituting Q_0 by q_0 and Q_1 by $q_1+\kappa$, the efficient allocation is given by the following conditions:

$$P\left(q_0^{sw}\right) = c'\left(q_0^{sw}\right) + \mathcal{D}'(\mathcal{E}(q_0^{sw}))\mathcal{E}'\left(q_0^{sw}\right) \tag{6}$$

$$P\left(q_1^{sw} + \kappa^{sw}\right) = c'\left(q_1^{sw}\right) + \mathcal{D}'(\mathcal{E}(q_1^{sw}))\mathcal{E}'\left(q_1^{sw}\right) \tag{7}$$

$$\mathcal{K}'\left(\kappa^{sw}\right) = \pi_1\left[c'\left(q_1^{sw}\right) + \mathcal{D}'(\mathcal{E}(q_1^{sw}))\mathcal{E}'\left(q_1^{sw}\right)\right] \tag{8}$$

Eqs.(6) and (7) says that in each state of nature, the marginal surplus, or the willingness to pay for electricity, is equal to the marginal social cost of fossil-fueled electricity that comprises the marginal private cost, $c'\left(q_s^{sw}\right)$, and the marginal emissions damage, $\mathcal{D}'(\mathcal{E}(q_s^{sw}))\mathcal{E}'\left(q_s^{sw}\right)$. The interpretation of Eq.(8) is the same as previously (see Eq.(3)).

3. Competitive equilibrium and efficiency of emissions tax with traditional consumers

In this section, I study the emissions tax. It is implemented by the regulator to address damage from the use of the fossil-energy technology in competitive markets. Hence, I first derive the equilibrium conditions where electricity production and consumption are described by producers and consumers who are respectively profits and welfare maximizers. Producers bear an emissions tax, τ, per unit of emissions from fossil-fueled electricity production.[14] I consider the most common setting which is that of traditional consumers on a flat-rate tariff and the equilibrium market conditions are the following.

The fossil energy technology is flexible and the representative producer can adjust his production plan in each state. He decides on his optimal level of production from a

[14] It is not necessary to describe the competitive equilibrium without regulation since by construction producers will choose their fossil-fueled production plan that maximizes their profits without taking care of the damage. This situation is clearly inefficient.

profit-maximizing behavior by equating, state by state, the wholesale price of electricity to the marginal cost of production in which he incorporates the tax τ:

$$p_s^{t\tau} = c'\left(q_s^{t\tau}\right) + \tau\mathcal{E}'\left(q_s^{t\tau}\right), \quad \forall s \tag{9a}$$

The intermittent renewable technology is not flexible and produces only in state $s = 1$. It provides 1 unit of electricity per unit of the installed capacity ($g_1 = 1$). The producer therefore *ex-ante* chooses his optimal renewable capacity from a profit-maximizing behavior such that his expected additional return of a new unit is equal to his marginal cost of investment:

$$\pi_1 p_1^{t\tau} = \mathcal{K}'\left(\kappa^{t\tau}\right) \tag{9b}$$

The willingness to pay of traditional consumers is equal to the flat-rate tariff of electricity:

$$P\left(Q^{t\tau}\right) = p_f^{t\tau} \tag{9c}$$

where $p_f^{t\tau} = \pi_0 p_0^{t\tau} + \pi_1 p_1^{t\tau}$.

The wholesale markets clear under the condition that electricity production matches consumption in each state. This consumption is constrained to Q in each s through the flat-rate tariff. Hence, the market clearing condition is:

$$q_0^{t\tau} = q_1^{t\tau} + \kappa^{t\tau} = Q^{t\tau} \tag{9d}$$

The proposition hereafter summarizes the results of this equilibrium:

Proposition 1. *The competitive electricity markets with traditional consumers on a flat-rate tariff and with a tax τ per unit of emissions admit a unique equilibrium with the properties that:*

Table 1: Comparative statics w.r.t. the emissions tax (traditional consumers)

$dq_s^{t\tau}/d\tau$	$d\kappa^{t\tau}/d\tau$	$dQ^{t\tau}/d\tau$	$dp_f^{t\tau}/d\tau$
$-$	$+/-$	$-$	$+$

From a policy perspective, it is worth knowing how do electricity production and consumption vary with the tax at equilibrium. On the first hand, the tax is found to be effective at reducing emissions-intensive electricity production and interestingly in both states of nature. This follows directly from fossil-fueled electricity productions, $q_0^{t\tau}$ and $q_1^{t\tau}$, that are decreasing with the tax.

On the other hand, the tax may have an ambiguous effect on the intermittent renewable capacity $\kappa^{t\tau}$. Intuitively, it is expected that since the tax puts a price on each unit of emissions, it encourages a shift to emissions-free electricity production and thereby establishes an indirect support for the renewable technology. In fact, this is not always the case as shown in an example in Appendix A. The explanation is that the flat-rate tariff forces total electricity consumption and thereby total production in state 1 to match that in state 0 (see Eq.(9d)). Thus, even if the renewable technology produces only in state 1, the effect of the tax on the renewable capacity depends in what proportions fossil-fueled electricity production in both states decrease. It implies that in the presence of traditional consumers, reducing emissions does not always mean substituting fossil-fueled electricity with renewable electricity. It can also occur that both of them decrease.

Finally, the tax is able to provide an incentive for energy sobriety since the quantity of electricity consumed $Q^{t\tau}$ is decreasing with the tax. Any additional cost to producers due to a raise in the tax is passed along to consumers who ultimately pay for the emissions-intensive activity. In fact, the retail price of electricity, $p_f^{t\tau}$, is increasing with the tax so that each unit of electricity is costlier to consumers who lower their demand for it.

The issue that I now address is the efficiency of the emissions tax that the regulator decides on while taking into consideration the competitive equilibrium conditions. The standard definition of an efficient emissions tax is one that is set equal to the marginal damage to internalize the emissions externality. In addition, this tax must move the competitive equilibrium to its social welfare allocation.

With regard to these latter points, two problems can already be identified to implement an efficient tax. Firstly, there are two states of nature and the taxation policy must be announced *ex-ante* of the realization of these. It is expected that the tax rate that is uniform across the states of nature may not always match the marginal damage that differs in each state. This stems from different levels of state-dependent fossil-fueled electricity production in the competitive equilibrium and from incremental damage that becomes more important for high levels of emissions. Consequently, matching the tax with the marginal damage in both states is a difficulty.

Secondly, when consumption is constrained due to the flat-rate tariff, social welfare is constrained efficient as explained in Section 2.2. A tax enabling a social welfare production and consumption allocation is from the outset out of reach. The question that remains open is if the tax implements the constrained social welfare allocation.

To investigate this matter, I define the problem of the regulator. He is concerned with maximizing the constrained social welfare, CSW (Eq.(1)), with respect to the tax τ and by taking into account the competitive equilibrium conditions. The first-order condition writes:[15]

$$\begin{cases} \frac{dq_0}{d\tau}\left\{P\left(q_0\right)-\pi_0\left[c'\left(q_0\right)+\mathcal{D}'\left(\mathcal{E}\left(q_0\right)\right)\mathcal{E}'\left(q_0\right)\right]-\mathcal{K}'\left(q_0-q_1\right)\right\}+ \\ \\ \frac{dq_1}{d\tau}\left\{-\pi_1\left[c'\left(q_1\right)+\mathcal{D}'\left(\mathcal{E}\left(q_1\right)\right)\mathcal{E}'\left(q_1\right)\right]+\mathcal{K}'\left(q_0-q_1\right)\right\}=0 \end{cases} \tag{10}$$

Substituting the optimal solutions of the competitive markets as described by conditions (9a) to (9d) results in:

$$\frac{dq_0^{t\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{t\tau}\right)\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right)\right]+\frac{dq_1^{t\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{t\tau}\right)\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right)\right]=0 \tag{11}$$

[15]To simplify notation, I omit the argument τ in the equations.

I proceed by comparing the tax with the marginal damage in state 0. It is the state where the fossil energy technology is the only one to produce electricity.

Manipulating the terms of Eq.(11) as described in Appendix B results in:

$$\tau - \mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right) = \mathcal{A} \times \left[\mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right) - \mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right)\right] \tag{12}$$

where $\mathcal{A} = \left(\frac{dq_1{}^{t\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{t\tau}\right)\right) \Big/ \left(\frac{dq_0{}^{t\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0{}^{t\tau}\right) + \frac{dq_1^{t\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{t\tau}\right)\right) > 0$.

Quantity $q_0^{t\tau}$ being greater than $q_1^{t\tau}$ from condition (9d) of the competitive equilibrium, it follows that $\mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right) > \mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right)$. Since the right-hand side term is strictly negative, the equation holds when τ is less than $\mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right)$. It implies that the regulator sets the emissions tax below the marginal damage in state 0 where the fossil energy technology is the only one to operate.

Comparing the tax with the marginal damage in state 1:

$$\tau - \mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right) = \mathcal{B} \times \left[\mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right) - \mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right)\right] \tag{13}$$

where $\mathcal{B} = \left(\frac{dq_0{}^{t\tau}}{d\tau}\mathcal{E}'\left(q_0^{t\tau}\right)\right) \Big/ \left(\frac{dq_0{}^{t\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0{}^{t\tau}\right) + \frac{dq_1^{t\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{t\tau}\right)\right) > 0$.

The right-hand side term is strictly positive and the tax τ is set above the marginal damage in state 1 whereby both the fossil energy and intermittent renewable technologies produce electricity.

The emissions tax is not constrained efficient in two aspects. Firstly, the intermittent nature of renewable electricity results in different levels of fossil-fueled electricity production and thereby damage. These are not efficiently internalized by an *ex-ante* emissions tax that does not match the marginal damage in each state. Secondly, the tax deviate the production and consumption levels in the competitive electricity markets from the constrained social welfare allocation.

However, one can observe either from Eq.(12) or (13) that when the right-hand side of these equations is 0, then the tax matches the marginal damage in each state. It implies that the marginal damage is constant, i.e. $\mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right) = \mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right)$. In this case, the constrained efficient electricity mix is implemented with an *ex-ante* tax. Ambec and Crampes (2019) and Abrell et al. (2019) who study this case find similar results.

To summarize this discussion, it can be claimed that:

Proposition 2. *With the integration of an intermittent renewable energy technology into an electricity mix and with consumers on a flat-rate tariff, an ex-ante emissions tax is not even constrained efficient. It does not efficiently internalize emissions damage and implement the electricity mix with the constrained social welfare allocation unless the marginal is constant.*

In short, targeting an emissions tax that implements the social welfare allocation can be ruled out when there are traditional consumers. With the flat-rate tariff, the consumption allocation is constrained and even the constrained social welfare allocation is out of reach. To target an efficient tax, I suggest moving to a setting where consumption is not constrained. As described in Section 2.2, it implies that consumers are flexible. This case is studied hereinafter.

4. Competitive equilibrium and efficiency of emissions tax with flexible consumers

Assume a situation with flexible consumers. They face state-dependent electricity tariffs and are able to adjust their electricity consumption in each state of nature. The equilibrium conditions of the electricity markets with such consumers are described subsequently.

The production side being unchanged, the optimal levels of fossil-fueled electricity production and the optimal renewable capacity are still explained by conditions similar to (9a) and (9b) respectively, that is:

The fossil-fueled production plan is profit-maximizing:

$$p_s^{s\tau} = c'\left(q_s^{s\tau}\right) + \tau\mathcal{E}'\left(q_s^{s\tau}\right), \quad \forall s \tag{14a}$$

Investment in the renewable technology maximizes profits:

$$\pi_1 p_1^{s\tau} = \mathcal{K}'\left(\kappa^{s\tau}\right) \tag{14b}$$

On the consumption side, there are now flexible consumers on state-dependent electricity tariffs, p_s. At equilibrium, for each state of nature, their willingness to pay is equal to the unit price of electricity:

$$P\left(Q_s^{s\tau}\right) = p_s^{s\tau}, \forall s \tag{14c}$$

Finally, the electricity markets clear under the condition that electricity production matches consumption in each state:

$$q_0^{s\tau} = Q_0^{s\tau}, \quad q_1^{s\tau} + \kappa^{s\tau} = Q_1^{s\tau} \tag{14d}$$

It is found that:

Proposition 3. *There exists a unique equilibrium for the competitive electricity markets with flexible consumers on state-dependent tariffs and with an emissions tax* τ*. Its properties are that* $q_1^{s\tau} < q_0^{s\tau}$ *and:*

Table 2: Comparative statics w.r.t. the emissions tax (flexible consumers)

$dq_s^{s\tau}/d\tau$	$d\kappa^{s\tau}/d\tau$	$dQ_s^{s\tau}/d\tau$	$dp_s^{s\tau}/d\tau$
−	+	−	+

Firstly, it is found that with the integration of renewables, fossil-fueled electricity production in state 1 is less than that in state 0: $q_1^{s\tau} < q_0^{s\tau}$. Also, the results of the comparative statics go in the expected directions. As in the case with traditional consumers, the tax achieves its goal of reducing emissions. For both states of nature, the level of fossil-fueled electricity $q_s^{s\tau}$ is decreasing with the tax. The explanation is that at the margin, the cost of generating electricity with the emissions-intensive technology becomes more expensive.

Secondly, the results show that the tax has a favorable impact on the intermittent renewable capacity. This is in line with the intuition that more renewable capacity should be installed when the cost of producing from the fossil energy technology increases. It now follows from condition (14d) that investment in the renewable capacity is no more related to the effect of the tax on fossil-fueled electricity production in state 0 but only to that in state 1. This is contrary to what has been observed in the previous section with traditional consumers. If one remembers, they consume the same quantity of electricity in both states of nature due to the flat-rate tariff. With state-dependent tariffs, consumers adapt their consumption to production in each state of nature. It now results that in order to meet electricity demand in state 1, renewable electricity is favored over fossil-fueled electricity production when the tax increases: $q_1^{s\tau}$ decreases while $\kappa^{s\tau}$ increases. With flexible consumers, the tax is able to support the integration of the renewable capacity. This can also be deduced from price $p_1^{s\tau}$ that increases with the tax. It implies from Eqs. (14a) & (14b) that in state 1, an increase in the tax makes investment in the renewable capacity more attractive than production from the fossil energy technology.

Finally, as in the previous section, a rise in the tax has a knock-on effect on consumption through costs of fossil-fueled electricity production that increase in both states. These lead to higher state-dependent electricity tariffs $p_s^{s\tau}$ to which consumers respond by reducing their electricity consumption: $Q_s^{s\tau}$ decreases with the tax in both states of nature.

So far, characterizing the competitive equilibrium is standard. The fundamental question that remains open concerns the efficiency of the taxation policy. If one remembers from the previous section, one of the two criteria that an efficient tax must fulfill is to be able to implement the competitive equilibrium with the social welfare allocation. *A priori*, in a setting with flexible consumers, the efficient production plan and consumption allocation can be targeted. However, the competitive equilibrium predicts different levels of damage since fossil-fueled electricity production is different in each state of nature: $q_1^{s\tau} < q_0^{s\tau}$. It is expected that reaching a tax that must match the marginal damage in each state of nature is impossible. In any case, I study this tax.

The regulator is faced by the problem of maximizing the social welfare, SW (Eq.(5)), with respect to the tax and by taking into account the set of conditions that describe the competitive equilibrium responses from the economic agents. The first-order condition is given by:

$$\begin{cases} \frac{dq_0}{d\tau}\pi_0 \left\{P\left(q_0\right) - \left[c'\left(q_0\right) + \mathcal{D}'\left(\mathcal{E}\left(q_0\right)\right)\mathcal{E}'(q_0)\right]\right\} \ + \\ \frac{dq_1}{d\tau}\pi_1 \left\{P\left(q_1+\kappa\right) - \left[c'\left(q_1\right) + \mathcal{D}'\left(\mathcal{E}\left(q_1\right)\right)\mathcal{E}'(q_1)\right]\right\} \ + \\ \frac{d\kappa}{d\tau}\left\{\pi_1 P\left(q_1+\kappa\right) - \mathcal{K}'\left(\kappa\right)\right\} = 0 \end{cases} \tag{15}$$

By substituting the optimal solutions of the competitive market into Eq.(15), I obtain:

$$\frac{dq_0^{s\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{s\tau}\right)\left[\tau - \mathcal{D}'\left(\mathcal{E}\left(q_0^{s\tau}\right)\right)\right] \ + \ \frac{dq_1^{s\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{s\tau}\right)\left[\tau - \mathcal{D}'\left(\mathcal{E}\left(q_1^{s\tau}\right)\right)\right] \ = 0 \tag{16}$$

Eventually, it is observed that this equation is similar to the one obtained previously with traditional consumers (Eq.(11)), but of course, the quantities are not necessarily the same. By carrying the same exercise of comparing the tax τ with the state-dependent marginal damage $\mathcal{D}'\left(\mathcal{E}\left(q_s^{s\tau}\right)\right)$, I find that in state 0, the tax is set below the marginal damage while in state 1, it is set above (see Appendix D). Consequently, with flexible consumers, the marginal damage is still different in each state of nature and the tax does not match any of it. With the penetration of the renewable technology, fossil-fueled electricity production is different in each state of nature so that the marginal damage is likewise. Again, the only case whereby the tax matches the marginal damage is when the latter is constant.

With these findings, it can be claimed that:

Proposition 4. *In an electricity mix with an intermittent renewable resource and consumers on state-dependent tariffs, the emissions tax is a second-best regulation. The only case when it is able to efficiently internalize the damage and move the competitive equilibrium to the social welfare allocation is when the marginal damage is constant.*

To summarize, implementing an emissions tax when consumers face state-dependent retail tariffs rather than a flat-rate tariff helps in easing the adoption of the intermittent renewable technology. Consumers are able to adapt their consumption to the availability of the renewable electricity. However, the integration of the renewable technology still results in different levels of fossil-fueled electricity production and damage. Hence, it is impossible to set a tax that is equal to the marginal damage in each state of nature and that implements the social welfare allocation in the competitive markets.

In fact, the emissions tax as an *ex-ante* regulation offers no means to be adjusted to the state-dependent marginal damage that varies. The question then arises as to whether an Emissions Trading Scheme where permits to emit are allowed to be traded *ex-post* in each state of nature produces better results than a tax in terms of efficiency. The next section looks into this.

5. Competitive equilibrium and efficiency of emissions cap with flexible consumers

This part of the paper assesses the Emissions Trading Scheme as an alternative policy to the tax for controlling emissions in a setting with flexible consumers. I analyze if *ex-post* trade of permits provides flexibility at the policy level to efficiently internalize emissions damage. The key features of the ETS are incorporated into the model as follows. The regulator sets an emissions cap E on the maximum level of emissions and equivalently sells permits to emit on competitive markets. The producer buys a permit for each unit of emissions from the regulator. The market for permits is organized in each state of nature so that the price per unit permit in state s is p_{e_s}.

With flexible consumers and the ETS, the equilibrium conditions of the electricity markets are described by similar conditions as in the previous section, except for the fossil-fueled production plan.

In each state of nature, the optimal level of fossil-fueled production that maximizes the producer's profits is reached when the wholesale price of electricity matches the full marginal cost of production. Here, the producer includes expenses incurred for the purchase of emissions permits at their price p_{e_s} in his production costs.

$$p_s^{SE} = c'\left(q_s^{SE}\right) + p_{e_s}^{SE}\mathcal{E}'\left(q_s^{SE}\right), \quad \forall s \tag{17a}$$

The remaining equilibrium conditions of the electricity markets are akin to (14b) to (14d):

The producer also maximizes his profits per unit capacity κ of the renewable technology by equating his expected revenues per unit capacity to his marginal cost of investment:

$$\pi_1 p_1^{SE} = \mathcal{K}'\left(\kappa^{SE}\right) \tag{17b}$$

Flexible consumers maximize their expected welfare by matching their willingness to pay to the price of electricity in each s:

$$P\left(Q_s^{SE}\right) = p_s^{SE}, \forall s \tag{17c}$$

The electricity markets clear in each state:

$$q_0^{SE} = Q_0^{SE}, \quad q_1^{SE} + \kappa^{SE} = Q_1^{SE} \tag{17d}$$

The state-dependent permits markets clear by the free disposal equilibrium conditions. It means that when the permits market at equilibrium clears with a strict equality of supply and demand, the price of permits is positive; otherwise, when the market clears with a non-positive excess demand at equilibrium, the price of permits is zero.

$$p_{e_s}^{SE}\left(\mathcal{E}\left(q_s^{SE}\right) - E\right) = 0 \text{ with } \mathcal{E}\left(q_s^{SE}\right) - E \leq 0, \quad \forall s \tag{17e}$$

From the latter condition, as shown in Appendix E, the economic agents anticipate that the regulator has the possibility to implement 3 different levels of emissions cap E to regulate emissions.

Firstly, the cap can be set large enough: $E \geq \max\left\{\mathcal{E}\left(q_0^{SE}\right), \mathcal{E}\left(q_1^{SE}\right)\right\}$. This results in an equilibrium that can be described as "Business-as-Usual"and is explained in the following remark:

Remark 1. *With an emissions cap set to* $E \geq \max\left\{\mathcal{E}\left(q_0^{SE}\right), \mathcal{E}\left(q_1^{SE}\right)\right\}$, *the competitive permits and electricity markets with flexible consumers have a unique solution given by* $\left(p_{e_s}^{SE}, q_s^{SE}, \kappa^{SE}, Q_s^{SE}, p_s^{SE}\right)_{|BaU}$. *It has the property that* $\forall s, {p_{e_s}^{SE}}_{|BaU} = 0$ *and* ${q_1^{SE}}_{|BaU} < {q_0^{SE}}_{|BaU}$.

When $E \geq \max \left\{ \mathcal{E}\left(q_0^{SE}\right), \mathcal{E}\left(q_1^{SE}\right) \right\}$, it follows from condition (17e) that the price of permits in each state, $p_{e_s}^{SE}$, is zero. Excess demand for permits is non-positive and drives permits prices to null. In this situation, the producer chooses his fossil-fueled production levels that maximize his profits without taking care of the externality caused by the emitting activity. Emissions cost nothing to him. This emissions cap is not effective in reducing emissions which in a sense results in a "Business-as-Usual" situation where the producer behaves as if there is no regulation. For the policy to work, the cap must be set to $E < \max\{\mathcal{E}(q_0^{SE}{}_{|BaU}), \mathcal{E}(q_1^{SE}{}_{|BaU})\}$. As shown in Appendix E, in state 1 when renewable electricity is available, quantity of fossil-fueled electricity production is less than that in state 0: $q_1^{SE}{}_{|BaU} < q_0^{SE}{}_{|BaU}$. Owing to this result, it follows that for an effective policy, E must be fixed to $E < \mathcal{E}(q_0^{SE}{}_{|BaU})$. This leads to describing the 2 other levels of the emissions cap E.

Case 1 is where the regulator sets an emissions cap E between the lowest and highest emissions levels of the "Business-as-Usual" scenario: $\mathcal{E}(q_1^{SE}{}_{|BaU}) \leq E < \mathcal{E}(q_0^{SE}{}_{|BaU})$. The equilibrium results are:

Proposition 5. *When the regulator fixes the emissions cap to $\mathcal{E}(q_1^{SE}{}_{|BaU}) \leq E < \mathcal{E}(q_0^{SE}{}_{|BaU})$, there exists a unique equilibrium for the competitive permits and electricity markets with flexible consumers. The properties of $\left(p_{e_s}^{SE}, q_s^{SE}, \kappa^{SE}, Q_s^{SE}, p_s^{SE}\right)_{|Case\ 1}$, the equilibrium solution, are as follows:*

(i) the price of permits is positive in state 0 and null in state 1: $p_{e_0}^{SE} > 0$, $p_{e_1}^{SE} = 0$

(ii) fossil-fueled electricity production in state 0 is $q_0^{SE} = \mathcal{E}^{-1}(E)$

(iii) fossil-fueled electricity production in state 1 and renewable capacity are same as the "Business-as-Usual" situation: $q_1^{SE} = q_1^{SE}{}_{|BaU}$ and $\kappa^{SE} = \kappa^{SE}{}_{|BaU}$

(iv) the effect of E on the equilibrium prices and quantities $\left(p_{e_1}^{SE}, q_1^{SE}, \kappa^{SE}, Q_1^{SE}, p_1^{SE}\right)_{|Case\ 1}$ is 0 while on the remaining, is:

Table 3: Comparative statics w.r.t. the emissions cap $\mathcal{E}(q_1^{SE}{}_{|BaU}) \leq E < \mathcal{E}(q_0^{SE}{}_{|BaU})$

dp_{e0}^{SE}/dE	dq_0^{SE}/dE	dQ_0^{SE}/dE	dp_0^{SE}/dE
$-$	$+$	$+$	$-$

When the cap is fixed to $\mathcal{E}(q_1^{SE}{}_{|BaU}) \leq E < \mathcal{E}(q_0^{SE}{}_{|BaU})$, the producer winds down his "Business-as-Usual" fossil-fueled electricity production of state 0 such that his emissions level matches the cap: $q_0^{SE} = \mathcal{E}^{-1}(E)$. In state 1, the cap covers the level of emissions of the "Business-as-Usual" situation. Hence, it follows from condition (17e) that the price of permits in state 0 is positive since the demand for permits $\mathcal{E}(q_0^{SE})$ exactly matches supply E. In state 1, excess demand is non-positive: $\mathcal{E}(q_1^{SE}) - E \leq 0$. The price of permits is zero in that state. The producer behaves as if there is no regulation in state 1 so that fossil-fueled electricity production in that state and renewable capacity are same as the "Business-as-Usual" situation.

Following these results, it is straightforward that increasing the cap E has no effect on equilibrium renewable capacity together with prices and production quantities of state 1.

Only those of state 0 are impacted. Increasing E, thereby supply of permits, pulls downs the permits price $p_{e_0}^{SE}$. This results in an increase in fossil-fueled electricity production in state 0 since, at the margin, related production costs are lower.

On the demand side, consumption in state 0 varies in the same direction as the change in the cap. A higher cap results in a lower marginal production cost in state 0 and is passed along to consumers through a lower electricity price p_0^{SE}. Flexible consumers react to this lower tariff by increasing their demand for electricity in that state.

Case 2 is where the emissions cap E is set below the lowest level of emissions that occurs at equilibrium in the "Business-as-Usual"scenario: $E < \mathcal{E}(q_1^{SE}{}_{|BaU})$. Solving this case gives the following results:

Proposition 6. *When the regulator sets the emissions cap to $E < \mathcal{E}(q_1^{SE}{}_{|BaU})$, the competitive permits and electricity markets admit a unique solution. It is given by $\left(p_{e_s}^{SE}, q_s^{SE}, \kappa^{SE}, Q_s^{SE}, p_s^{SE}\right)_{|Cas}$ with the properties that:*

(i) the prices of permits are positive: $\forall s, p_{e_s}^{SE} > 0$ with $p_{e_0}^{SE} > p_{e_1}^{SE}$

(ii) the state-dependent fossil-fueled electricity productions are equal: $\forall s, q_s^{SE} = \mathcal{E}^{-1}(E)$

(iii) the renewable capacity is greater than the "Business-as-Usual"situation: $\kappa^{SE} > \kappa^{SE}{}_{|BaU}$

(iv) the effect of E on the equilibrium is:

Table 4: Comparative statics w.r.t. the emissions cap $E < \mathcal{E}(q_1^{SE}{}_{|BaU})$

$dp_{e_s}^{SE}/dE$	dq_s^{SE}/dE	$d\kappa^{SE}/dE$	dQ_s^{SE}/dE	dp_s^{SE}/dE
$-$	$+$	$-$	$+$	$-$

The results first show that the cap $E < \mathcal{E}(q_1^{SE}{}_{|BaU})$ has an immediate effect on the level of fossil-fueled electricity production in both states of nature. To be in conformity with the cap, the producer is led to coordinate his production activity to drive down emissions cost-effectively. Since the cap E is the same for both states of nature, he reduces his state-dependent "Business-as-Usual"fossil-fueled electricity productions such that corresponding emissions match the cap. Consequently, quantity of emissions-intensive electricity in state 0 is equal to that in state 1: $q_0^{SE} = q_1^{SE} = \mathcal{E}^{-1}(E)$

In addition, the demand for emissions permits in each state, $\mathcal{E}\left(q_s^{SE}\right)$, is exactly covered by the quantity of permits supplied E. It follows from condition (17e) that the prices of permits are positive in each state: $p_{e_0}^{SE}$, $p_{e_1}^{SE} > 0$. Also, as shown in Appendix E for Case 2, $p_{e_0}^{SE} > p_{e_1}^{SE}$. This difference can be explained by the fact that the permits markets is organized in each state thereby resulting in prices of permits that are different.

Moreover, setting the cap to $E < \mathcal{E}(q_1^{SE}{}_{|BaU})$ prompts investment in the intermittent renewable technology to a capacity that is higher when compared to the situation without regulation (see Appendix E).

In light of these results, the outcomes of the comparative statics are self-explanatory. Increasing E, thereby supply of permits, pulls downs permits prices, $p_{e_s}^{SE}$. This results in an increase in state-dependent fossil-fueled electricity productions since production costs are lower at the margin. Also, a higher cap is at the expense of investment in the renewable capacity. Price p_1^{SE} decreases with E. It implies, from Eqs. (17a) & (17b), that a more important cap E is prejudicial to investment in the renewable technology. It lowers way more the marginal cost of fossil-fueled electricity production in state 1 than the marginal cost of investment in the renewable capacity.

On the demand side, the results of the comparative statics are analogous to that of Case 1, except that it is now extended to both states of nature 0 and 1. Following a higher cap, marginal production costs in both states of nature are reduced resulting in lower electricity prices p_s^{SE}. Benefitting from lower state-dependent tariffs, flexible consumers increase their demand for electricity in each state.

As usual, having described the competitive equilibrium solutions, I can investigate the efficiency of the policy. Until now, the paper has shown that the regulator can only achieve a second-best policy with an emissions tax. The constrained social welfare allocation is unreachable with an emissions tax when consumers are on the flat-rate tariff. Even with flexible consumers, the social welfare allocation cannot be implemented in decentralized markets. This mainly follows from the renewable technology that induces variability in fossil-fueled electricity production on the competitive markets. This results in different levels of damage and it is impossible to match the tax with the marginal damage in each state of nature. The question that I address is if an Emissions Trading Scheme, with permits markets being state-dependent, provides flexibility in reaching an efficient policy. *A priori*, the permits prices at the equilibrium conditions are different in each state both for Case 1 and 2. It now remains to verify if the cap is efficient. It must be able to equalize the permits price and marginal damage in each state of nature so as to account for the externality efficiently. In addition, it must move the competitive equilibrium to the social welfare allocation.

The regulator's problem is to choose the emissions cap E that maximizes social welfare, SW (Eq. (5)), and to consider the equilibrium solutions of the electricity and permits markets. The first-order condition is given by[16]:

$$\left\{\begin{array}{c} \frac{dq_0}{dE}\pi_0\{P\left(q_0\right)-\left[c'\left(q_0\right)+\mathcal{D}'\left(\mathcal{E}\left(q_0\right)\right)\mathcal{E}'(q_0)\right]\} \quad + \\ \frac{dq_1}{dE}\pi_1\{P\left(q_1+\kappa\right)-\left[c'\left(q_1\right)+\mathcal{D}'\left(\mathcal{E}\left(q_1\right)\right)\mathcal{E}'(q_1)\right]\ \} \quad + \\ \frac{d\kappa}{dE}\{\pi_1 P\left(q_1+\kappa\right)-\mathcal{K}'\left(\kappa\right)\} \ = 0 \end{array}\right. \tag{18}$$

As shown in the concluding part of Appendix E, the solutions of the competitive equilibrium are continuous in E, but are not necessarily differentiable. I therefore study case by case if the emissions cap is efficient as per the definition given earlier.

I start with Case 1 where for a cap $\mathcal{E}(q_1^{SE}{}_{|BaU}) \leq E < \mathcal{E}(q_0^{SE}{}_{|BaU})$, the optimal solutions of the competitive equilibrium are as summarized in Proposition 5. Substituting them

[16]For simplicity of notation, I omit the argument E in the equations.

into the first-order condition gives:

$$\mathcal{D}'(E) = p_{e_0}^{SE}\,_{|Case\ 1} \tag{19}$$

where $E = \mathcal{E}(q_0^{SE}\,_{|Case\ 1})$.

If the regulator considers the competitive solutions of Case 1, then the cap E that he chooses is such that only the marginal damage in state 0 is equalized with the price of permits in that state. As for state 1, the marginal damage is matched with a price of zero. By setting a cap E that regulates emissions from the fossil energy technology only in state 0, the social welfare allocation cannot be decentralized in the competitive markets since the damage in state 1 is not taken care of.

Similarly, I analyze Case 2 with a cap $E < \mathcal{E}(q_1^{SE}\,_{|BaU})$ and optimal solutions as given in Proposition 6. Substituting the latter into the first-order condition of the regulator's problem results in:

$$\mathcal{D}'(E) = \pi_0 p_{e_0}^{SE}\,_{|Case\ 2} + \pi_1 p_{e_1}^{SE}\,_{|Case\ 2}, \tag{20}$$

where $E = \mathcal{E}(q_0^{SE}\,_{|Case\ 2}) = \mathcal{E}(q_1^{SE}\,_{|Case\ 2})$.

As shown in Proposition 6, the price of permits in state 0 is greater than that in state 1: $p_{e_0}^{SE} > p_{e_1}^{SE}$. If the regulator considers the optimal solutions of Case 2, then the cap that he implements is such that in each s the marginal damage is not matched with the price of permits $p_{e_s}^{SE}$. Instead, the marginal damage in each s is equalized with the expected prices of permits. This cap is clearly inefficient: it does not internalize the emissions damage efficiently and does not decentralize the efficient electricity mix through the competitive markets.

These findings show that an emissions cap is not efficient in regulating a "Business-as-Usual" electricity mix where fossil-fueled electricity production differs in each state due to the integration of an intermittent renewable. This impossibility to reach an optimal E arises from the fact that the regulator decides on the cap *ex-ante* of the realization of the states to which the *ex-post* emitting production must conform to state by state. When a uniform emissions cap E is implemented for both states of nature, it leads to *ex-post* inefficient permits prices that do not match the marginal social damage.

It can therefore be set forth that:

Proposition 7. *An Emissions Trading Scheme with an ex-ante emissions cap is a second-best policy in an electricity mix with intermittent renewables and flexible consumers. It is not able to internalize emissions damage in each state of nature efficiently and implement the social welfare allocation in the competitive markets.*

Also, if one remembers from Section 4, in the case when the marginal damage is constant, an efficient emissions tax is reached. When this case is studied with an Emissions

Trading Scheme, it means to reach the social welfare allocation, the regulator must set a cap such that the marginal damage is equal to the price of permits in each state of nature.

Denoting the marginal damage by δ, it implies $p_{e_0}^{SE} = p_{e_1}^{SE} = \delta$. This is impossible since the permits prices are not the same in each state of nature as shown in the equilibrium solutions of Cases 1 & 2 (see Propositions 5 and 6).

Corollary 1. *In a setting with flexible consumers, an Emissions Trading Scheme is still a second-best policy when the marginal damage is constant.*

To conclude, switching from one emissions pricing instrument to another, here from an emissions tax to an Emissions Trading Scheme, does not allow to reach an efficient policy. Even if an ETS can provide flexibility in the regulation by allowing for *ex-post* trade of permits, the emissions cap as an incentive-based policy is still announced *ex-ante* by the regulator and is uniform across the states of nature. Consequently, in the presence of an intermittent renewable, the *ex-ante* cap results in permit prices that inefficiently internalize the damage. They also do not implement the social welfare allocation in the competitive markets.

Finally, by comparing the results of Sections 4 and 5, it is observed that the tax and an ETS are not implemented equivalently even in the case where the incremental damage due to emissions is constant. The cap set by the regulator leads to permits prices that differ in each state of nature while the emissions tax is uniform across the states. It follows that the instruments do not have identical economic outcomes.

6. Concluding Remarks

Emissions damage associated with fossil-fueled electricity production is a notable driver to implement policies and incentivize a shift from emissions-intensive to emissions-free technologies. In this respect, renewable generators such as wind turbines and solar photovoltaics are key technologies on which the sector can rely. However, their adoption is challenging due to their intermittent nature. Focusing on emissions pricing, this paper analyzes the efficiency of the policy in the presence of intermittency.

I first study an emissions tax in a common setting of the electricity markets with traditional consumers who do not adapt their consumption to varying production. I show that the constrained efficient electricity mix is out of reach with an *ex-ante* tax. The integration of intermittent renewables results in different levels of fossil-fueled electricity production and associated damage that the tax is unable to internalize efficiently. For similar reasons, even with flexible consumers who adapt their consumption to varying production, the tax does not decentralize the efficient mix through competitive markets. As an alternative to emissions taxation, I analyze an Emissions Trading Scheme that provides flexibility at the policy level: the permits are traded on state-dependent markets. The regulation is still second-best. Due to variability induced by renewables on the electricity markets, the emissions cap leads to inefficient permits prices that do not match the marginal damage. In addition, the model indicates that the tax and ETS are not equivalently implemented since the level of the tax differs from the prices of permits.

The results show that accounting for intermittency has non-trivial implications for incentive-based emissions pricing instruments when incremental damage due to emissions is increasing. The key to achieving efficiency points to a state-dependent policy. It implies the emissions tax rate or cap must be tailored to the availability of the renewable resource. In this model, intermittency is captured through 2 states of nature which are sufficient to show distortions in the markets but in the real world, there are certainly more than 2 states. In this respect, it is hard to imagine implementing a state-dependent policy, for instance, for institutional reasons. It implies managing a regulation with a level of granularity matching that of the states of nature. Moreover, the policy may lose its incentive character to trigger long-term investment in renewables if it is administered *ex-post* rather than *ex-ante*.

Several extensions of this paper can be expected. An immediate one is to study the efficiency of direct subsidies to support renewables. Abrell et al. (2019) and Ambec and Crampes (2019) examine how the presence of subsidies affects the constrained efficient electricity mix with intermittent renewables. This literature can be complemented through the present framework to study the coexistence of emissions pricing and renewable support schemes, advocated as key for an accelerated energy transition (Hepburn et al. 2020 and Rosenbloom et al. 2020).

A second extension can be to consider flexibility on the production side through the storage of electricity that will be part of the electricity transition. Since storage is intrinsically a dynamic process, it implies extending the present framework to a dynamic one. This is in line with a recent literature such as Helm and Mier (2018). The authors propose a dynamic model of an optimal mix of fossil-fueled, renewables-based and storage technologies. They account for the intermittent effect of renewables and study the implementation of capacity subsidies for renewable and storage technologies. This work can be broadened by studying other widely-implemented subsidy schemes for renewable electricity production such as the feed-in tariffs and market premiums.

References

Abrell, J., Rausch, S., Streitberger, C., 2019. The economics of renewable energy support. Journal of Public Economics 176, 94–117.

Adar, Z., Griffin, J. M., 1976. Uncertainty and the choice of pollution control instruments. Journal of Environmental Economics and Management 3 (3), 178–188.

Ambec, S., Crampes, C., 2012. Electricity provision with intermittent sources of energy. Resource and Energy Economics 34 (3), 319 – 336.

Ambec, S., Crampes, C., 2019. Decarbonizing Electricity Generation with Intermittent Sources of Energy. Journal of the Association of Environmental and Resource Economists 6 (6), 1105–1134.

Ambec, S., Crampes, C., 2021. Real-time electricity pricing to balance green energy intermittency. Energy economics 94, 105074.

Bielecki, A., Ernst, S., Skrodzka, W., Wojnicki, I., 2020. The externalities of energy production in the context of development of clean energy generation. Environmental Science and Pollution Research International 27 (11), 11506–11530.

Chiba, F., Rouillon, S., 2020. Intermittent Electric Generation Technologies and Smart Meters: Substitutes or Complements. Revue d'économie politique 130 (4), 573–613.

Cochran, J., Miller, M., Zinaman, O., Milligan, M., Arent, D., Palmintier, B., O'Malley, M., Mueller, S., Lannoye, E., Tuohy, A., Kujala, B., Sommer, M., Holttinen, H., Kiviluoma, J., Soonee, S. K., 2014. Flexibility in 21st Century Power Systems.

Crawley, G. M., 2013. The World Scientific Handbook of Energy. World Scientific.

Dales, J. H., 1968. Pollution, Property and Prices. University Press, Toronto.

EURELECTRIC, 2014. Flexibility and Aggregation. Requirements for their interaction in the market.

Helm, C., Mier, M., 2018. Subsidising renewables but taxing storage? second-best policies with imperfect carbon pricing. Oldenburg Discussion Papers in Economics V-413-18, Oldenburg.

Helm, C., Mier, M., 2019. On the efficient market diffusion of intermittent renewable energies. Energy Economics 80, 812–830.

Hepburn, C., Stern, N., Stiglitz, J. E., 2020. "Carbon pricing" special issue in the European economic review. European economic review 127, 103440.

IEA, 2011. Harnessing Variable Renewables: A Guide to the Balancing Challenge. OECD Publishing, Paris.

IEA, 2021. Renewables 2021. Report, International Energy Agency, Paris.

IEA-ISGAN, 2019. Power Transmission & Distribution Systems. Flexibility needs in the future power system.

IRENA, 2018. Renewable power generation costs in 2017.

Lazard, 2018. Lazard's levelized cost of energy analysis: Version 12.0.

Montgomery, W. D., 1972. Markets in licenses and efficient pollution control programs. Journal of Economic Theory 5 (3), 395–418.

Pigou, A. C., 1960. The Economics of Welfare, 4th Edition. Macmillan, London.

Rosenbloom, D., Markard, J., Geels, F. W., Fuenfschilling, L., 2020. Why carbon pricing is not sufficient to mitigate climate change-and how "sustainability transition policy" can help. Proceedings of the National Academy of Sciences 117 (16), 8664–8668.

Rouillon, S., 2015. Optimal and Equilibrium Investment in the Intermittent Generation Technologies. Revue d'économie politique 125 (3), 415–452.

Twoney, P., Neuhoff, K., 2010. Wind power and market power in competitive markets : Large-scale wind power in electricity markets. Energy Policy 38 (7), 3198–3210.

Weitzman, M., 1974. Prices vs. quantities. Review of Economic Studies 41 (4), 477–491.

West, J. J., Smith, S. J., Silva, R. A., Naik, V., Zhang, Y., Adelman, Z., Fry, M. M., Anenberg, S., Horowitz, L. W., Lamarque, J.-F., 2013. Co-benefits of mitigating global greenhouse gas emissions for future air quality and human health. Nature Climate Change 3 (10), 885–889.

World Bank, 2022. Carbon pricing dashboard, ETS and carbon taxes.

Appendix

Appendix A. Proof of proposition 1

(i) Competitive equilibrium with an emissions tax and traditional consumers

The producer's fossil-fuel production plan and investment in the intermittent renewable technology are profit-maximizing:

$$q_s^{t\tau}, \kappa^{t\tau} \in \arg\max \sum_{0}^{s=1} \pi_s \left(p_s q_s - c\left(q_s\right) - \tau \mathcal{E}\left(q_s\right) + p_s g_s \kappa\right) - \mathcal{K}(\kappa)$$

$$\therefore \quad p_s^{t\tau} = c'\left(q_s^{t\tau}\right) + \tau \mathcal{E}'\left(q_s^{t\tau}\right), \quad \forall s$$

$$\text{and} \quad \pi_1 p_1^{t\tau} = \mathcal{K}'\left(\kappa^{t\tau}\right), \text{ since } g_0 = 0 \text{ and } g_1 = 1$$

Traditional consumers maximize their welfare:

$$\therefore \quad P\left(Q^{t\tau}\right) = p_f$$

The electricity markets clear in each state:

$$q_s^{t\tau} + g_s \kappa^{t\tau} = Q_s^{t\tau}, \forall s$$

$$\therefore \quad q_0^{t\tau} = q_1^{t\tau} + \kappa^{t\tau} = Q^{t\tau} \text{ since } \forall s, Q_s = Q \text{ and } g_0 = 0 \;\&\; g_1 = 1$$

(ii) Existence and uniqueness of a competitive equilibrium

Substituting $Q^{t\tau}$ and $q_0^{t\tau}$ by $q_1^{t\tau} + \kappa^{t\tau}$, the equilibrium is characterized by the following first-order conditions:

$$\begin{cases} f_1\left(q_1^{t\tau}, \kappa^{t\tau}\right) = P\left(q_1^{t\tau} + \kappa^{t\tau}\right) - \pi_0\left[c'\left(q_1^{t\tau} + \kappa^{t\tau}\right) + \tau \mathcal{E}'\left(q_1^{t\tau} + \kappa^{t\tau}\right)\right] - \pi_1\left[c'\left(q_1^{t\tau}\right) + \tau \mathcal{E}'\left(q_1^{t\tau}\right)\right] = 0 \\ f_2\left(q_1^{t\tau}, \kappa^{t\tau}\right) = \mathcal{K}'\left(\kappa^{t\tau}\right) - \pi_1\left[c'\left(q_1^{t\tau}\right) + \tau \mathcal{E}'\left(q_1^{t\tau}\right)\right] = 0 \end{cases} \tag{A.1}$$

For simplicity, I omit noting the superscript in the proofs.

Step 1: Construction of $q_1(\kappa)$ satisfying $f_2(q_1(\kappa), \kappa) = 0$ with $0 < q_1 < \infty$.

Following the model's assumptions, it can be observed that for $\forall\ \kappa > 0$, $\lim_{q_1 \to 0} f_2(q_1, \kappa) > 0$ and $\lim_{q_1 \to q_0} f_2(q_1, \kappa) = -\infty$. Also,

$$\frac{\partial f_2\left(q_1, \kappa\right)}{\partial q_1} = -\pi_1\left[c''\left(q_1\right) + \tau \mathcal{E}''\left(q_1\right)\right] < 0$$

By the Implicit Function Theorem, it follows that $\exists\ q_1 :]0, +\infty[\ \to]0, +\infty[$ with the property that $\forall\ \kappa > 0$, (i) $f_2(q_1, \kappa) = 0$ and (ii)

$$\frac{dq_1}{d\kappa} = \frac{\mathcal{K}''\left(\kappa\right)}{\pi_1\left[c''\left(q_1\right) + \tau \mathcal{E}''\left(q_1\right)\right]} > 0 \tag{A.2}$$

In addition, under the model's assumptions, $f_2(q_1, \kappa) = 0$ verifies that:

- $\lim_{\kappa \to 0} q_1 = 0$
- $\lim_{\kappa \to +\infty} q_1 = +\infty$

Step 2: Existence of a solution

From previous observations, it can be found that (i) $\lim_{\kappa\to 0} f_1(q_1,\kappa) = +\infty$ and $\lim_{\kappa\to+\infty} f_1(q_1,\kappa) = -\infty$. Moreover, using Eq.(A.2):

$$\frac{df_2}{d\kappa} = P' - \pi_0\left(c'' + \tau\mathcal{E}''\right)_{|q_1+\kappa} + \left[P' - \pi_0\left(c'' + \tau\mathcal{E}''\right)_{|q_1+\kappa} - \pi_1\left(c'' + \tau\mathcal{E}''\right)_{|q_1}\right]\frac{dq_1}{d\kappa} < 0$$

It therefore follows that there exists a unique $\kappa^{t\tau}$ which solves $f_2\left(q_1^{t\tau},\kappa^{t\tau}\right)$ and consequently a unique $\left(q_1^{t\tau},\kappa^{t\tau}\right)$ satisfying the F.O.C.

(iii) Comparative statics

Applying the Implicit Function Theorem to the F.O.C given by Eq.(A.1) results in:

$$\underbrace{\begin{bmatrix} P' - \pi_0\left(c'' + \tau\mathcal{E}''\right)_{|q_1+\kappa} - \pi_1\left(c'' + \tau\mathcal{E}''\right)_{|q_1} & P' - \pi_0\left(c'' + \tau\mathcal{E}''\right)_{|q_1+\kappa} \\ -\pi_1\left(c'' + \tau\mathcal{E}''\right)_{|q_1} & \mathcal{K}'' \end{bmatrix}}_{A}\begin{bmatrix} dq_1 \\ d\kappa \end{bmatrix} - \begin{bmatrix} \pi_0\mathcal{E}'(q_1+\kappa) + \pi_1\mathcal{E}'(q_1) \\ \pi_1\mathcal{E}'(q_1) \end{bmatrix} d\tau = 0 \tag{A.3}$$

The determinant of A is $\left(P' - \pi_0\left(c'' + \tau\mathcal{E}''\right)_{|q_1+\kappa}\right)\left(\mathcal{K}'' + \pi_1\left(c'' + \tau\mathcal{E}''\right)_{|q_1}\right) - \pi_1\mathcal{K}''\left(c'' + \tau\mathcal{E}''\right)_{|q_1}$ which is negative. It therefore follows that:

$$\frac{dq_1^{t\tau}}{d\tau} = \frac{1}{det(A)}\left[\left(\pi_0\mathcal{E}'(q_1+\kappa) + \pi_1\mathcal{E}'(q_1)\right)\mathcal{K}'' + \pi_1\mathcal{E}'(q_1)\left(\pi_0\left(c'' + \tau\mathcal{E}''\right)_{|q_1+\kappa} - P'\right)\right] < 0 \tag{A.4}$$

$$\frac{d\kappa^{t\tau}}{d\tau} = \frac{\pi_1}{det(A)}\left[\mathcal{E}'(q_1)P' + \pi_0\mathcal{E}'(q_1+\kappa)\left(c'' + \tau\mathcal{E}''\right)_{|q_1} - \pi_0\mathcal{E}'(q_1)\left(c'' + \tau\mathcal{E}''\right)_{|q_1+\kappa}\right] = +/- \tag{A.5}$$

From condition (9d) of the competitive equilibrium:

$$\frac{dq_0^{t\tau}}{d\tau} = \frac{dq_1}{d\tau} + \frac{d\kappa}{d\tau} = \frac{1}{det(A)}\left[\left(\pi_0\mathcal{E}'(q_1+\kappa) + \pi_1\mathcal{E}'(q_1)\right)\mathcal{K}'' + \pi_0\pi_1\mathcal{E}'(q_1+\kappa)\left(c'' + \tau\mathcal{E}''\right)_{|q_1}\right] < 0 \tag{A.6}$$

$$\frac{dQ^{t\tau}}{d\tau} = \frac{dq_0}{d\tau} < 0 \tag{A.7}$$

Moreover, from condition (9a) of the competitive equilibrium:

$$\frac{p_s^{t\tau}}{d\tau} = \left[c''\left(q_s^{t\tau}\right) + \tau\mathcal{E}''\left(q_s^{t\tau}\right)\right]\frac{dq_s}{d\tau} + \mathcal{E}'(q_s), \quad \forall s$$

Using previous results, it can be found that:

$$\frac{dp_0^{t\tau}}{d\tau} = \frac{\pi_1\mathcal{E}'(q_1)\mathcal{K}''\left(c'' + \tau\mathcal{E}''\right)_{|q_1+\kappa} - \pi_1\mathcal{E}'(q_1+\kappa)\mathcal{K}''\left(c'' + \tau\mathcal{E}''\right)_{|q_1} + \mathcal{E}'(q_1+\kappa)P'\left(\mathcal{K}'' + \pi_1\left(c'' + \tau\mathcal{E}''\right)_{|q_1}\right)}{det(A)} \tag{A.8}$$

$$\frac{dp_1^{t\tau}}{d\tau} = \frac{\pi_0\mathcal{E}'(q_1+\kappa)\mathcal{K}''\left(c'' + \tau\mathcal{E}''\right)_{|q_1} - \pi_0\mathcal{E}'(q_1)\mathcal{K}''\left(c'' + \tau\mathcal{E}''\right)_{|q_1+\kappa} + \mathcal{E}'(q_1)\mathcal{K}''P'}{det(A)} \tag{A.9}$$

Finally,

$$\frac{dp_f^{t\tau}}{d\tau} = \pi_0\frac{dp_0^{t\tau}}{d\tau} + \pi_1\frac{dp_1^{t\tau}}{d\tau} = \frac{\pi_0\mathcal{E}'(q_1+\kappa)P'\left(\mathcal{K}'' + \pi_1\left(c'' + \tau\mathcal{E}''\right)_{|q_1}\right) + \pi_1\mathcal{E}'(q_1)\mathcal{K}''P'}{det(A)} > 0 \tag{A.10}$$

(iv)Example: $d\kappa^{t\tau}/d\tau$

Consider an electricity market where the cost of producing from the fossil-fuel technology is given by $c(q_s) = 0.5q_s^2$ and emissions by $\mathcal{E}(q_s) = 0.5eq_s^2$ where e is the emission factor of the fossil-fuel energy. Moreover the cost of investing in the intermittent renewable capacity κ is given by $\mathcal{K}(\kappa) = 0.5\kappa^2$ and the inverse demand function by $P(v) = 1 - v$ with $0 < v < 1$. The states of nature $s \in \{0, 1\}$ occur each with probability 0.5.

The F.O.C. of the competitive equilibrium are then given by:

$$(3 + e\tau)\, q_0^{t\tau} + (1 + e\tau)\, q_1^{t\tau} = 2$$

$$2q_0^{t\tau} - (3 + e\tau)\, q_1^{t\tau} = 0$$

The interior solutions are:

$$q_0^{t\tau} = Q^{t\tau} = \frac{6 + 2e\tau}{11 + 8e\tau + e^2\tau^2} \quad , \quad q_1^{t\tau} = \frac{4}{11 + 8e\tau + e^2\tau^2} \quad , \quad \kappa^{t\tau} = \frac{2 + 2e\tau}{11 + 8e\tau + e^2\tau^2}$$

The comparative statics show that:

$$\frac{dq_0^{t\tau}}{d\tau} = \frac{dQ^{t\tau}}{d\tau} = -\frac{26e + 12e^2\tau + 2e^3\tau^2}{(11 + 8e\tau + e^2\tau^2)^2} < 0 \quad , \quad \frac{dq_1^{t\tau}}{d\tau} = -\frac{32e + 8e^2\tau}{(11 + 8e\tau + e^2\tau^2)^2} < 0$$

$$\frac{d\kappa^{t\tau}}{d\tau} = \frac{2e\,(3 + e\tau)\,(1 - e\tau)}{(11 + 8e\tau + e^2\tau^2)^2}$$

It is found that $d\kappa^{t\tau}/d\tau \geq 0$ when $e \leq 1/\tau$.

Appendix B. Proof of Proposition 2

Effienciency of emissions tax

$$\max_{\tau} \quad CSW\,(Q, q_s, \kappa) \tag{B.1}$$

Substituting Q by q_0 and κ by $q_0 - q_1$ as per the 2 constraints on social welfare, Eq.(B.1) can be simplified to:

$$\max_{\tau} \int_0^{q_0} P(v)dv - \pi_0\left[c(q_0) + \mathcal{D}(\mathcal{E}(q_0))\right] - \pi_1\left[c(q_1) + \mathcal{D}(\mathcal{E}(q_1))\right] - \mathcal{K}(q_0 - q_1) \tag{B.2}$$

The first-order condition is:

$$\tfrac{dq_0}{d\tau}\left\{P(q_0) - \pi_0\left[c'(q_0) + \mathcal{D}'(\mathcal{E}(q_0))\,\mathcal{E}'(q_0)\right] - \mathcal{K}'(q_0 - q_1)\right\} \quad +$$
$$\tfrac{dq_1}{d\tau}\left\{ - \pi_1\left[c'(q_1) + \mathcal{D}'(\mathcal{E}(q_1))\,\mathcal{E}'(q_1)\right] + \mathcal{K}'(q_0 - q_1)\right\} \quad = 0$$

Substituting the optimal solutions of the competitive market as described by Eq. (A.1) results in:

$$\tfrac{dq_0^{t\tau}}{d\tau}\pi_0\mathcal{E}'(q_0^{t\tau})\left[\tau - \mathcal{D}'(\mathcal{E}(q_0^{t\tau}))\right] \quad + \quad \tfrac{dq_1^{t\tau}}{d\tau}\pi_1\mathcal{E}'(q_1^{t\tau})\left[\tau - \mathcal{D}'(\mathcal{E}(q_1^{t\tau}))\right] \quad = 0 \tag{B.3}$$

To compare the tax τ with the marginal damage in state 0, Eq.(B.3) can be rewritten as:

$$\tfrac{dq_0^{t\tau}}{d\tau}\pi_0\mathcal{E}'(q_0^{t\tau})\left[\tau - \mathcal{D}'(\mathcal{E}(q_0^{t\tau}))\right] \quad + \quad \tfrac{dq_1^{t\tau}}{d\tau}\pi_1\mathcal{E}'(q_1^{t\tau})\left[\tau - \mathcal{D}'(\mathcal{E}(q_1^{t\tau})) + \mathcal{D}'(\mathcal{E}(q_0^{t\tau})) - \mathcal{D}'(\mathcal{E}(q_0^{t\tau}))\right] \quad = 0$$

Factorizing and rearranging the terms results in:

$$\left[\tau - \mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right)\right]\left[\frac{dq_0^{t\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{t\tau}\right) + \frac{dq_1^{t\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{t\tau}\right)\right] = \frac{dq_1^{t\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{t\tau}\right)\left[\mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right) - \mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right)\right]$$

Now comparing the tax τ with the marginal damage in state 1, Eq.(B.3) is rewritten as:

$$\frac{dq_0^{t\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{t\tau}\right)\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right)+\mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right)-\mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right)\right] \quad + \quad \frac{dq_1^{t\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{t\tau}\right)\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right)\right] \; = 0$$

Factorizing and rearranging the terms gives:

$$\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right)\right]\left[\frac{dq_0^{t\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{t\tau}\right)+\frac{dq_1^{t\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{t\tau}\right)\right]=\frac{dq_0^{t\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{t\tau}\right)\left[\mathcal{D}'\left(\mathcal{E}\left(q_0^{t\tau}\right)\right)-\mathcal{D}'\left(\mathcal{E}\left(q_1^{t\tau}\right)\right)\right]$$

Assuming that the marginal damage is constant and is denoted by δ, then $\forall s, \mathcal{D}'\left(\mathcal{E}\left(q_s^{t\tau}\right)\right)=\delta$. Eq.(B.3) is then simplified to:

$$(\tau-\delta)\underbrace{\left[\frac{dq_0^{t\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{t\tau}\right)+\frac{dq_1^{t\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{t\tau}\right)\right]}_{\mathcal{A}}=0 \tag{B.4}$$

From the results of the comparative statics as in (A.4) and the assumptions of the framework, it is straightforward to find that $\mathcal{A}$ is strictly negative. Then, for the equation to hold, the tax τ must be equal to the constant marginal damage δ.

Appendix C. Proof of proposition 3

(ii) Competitive equilibrium with an emissions tax and flexible consumers

The producer's fossil-fuel production plan and investment in the intermittent renewable technology is profit maximizing:

$$q_s^{s\tau}, \kappa^{s\tau} \in \arg\max \quad \sum_0^{s=1}\pi_s\left[p_s q_s - c\left(q_s\right)-\tau\mathcal{E}\left(q_s\right)+p_s g_s\kappa\right]-\mathcal{K}\left(\kappa\right)$$

$$\therefore \quad p_s^{s\tau}=c'\left(q_s^{s\tau}\right)+\tau\mathcal{E}'\left(q_s^{s\tau}\right), \quad \forall s$$

$$\text{and} \quad \pi_1 p_1^{s\tau}=\mathcal{K}'\left(\kappa^{t\tau}\right), \text{ since } g_0=0 \text{ and } g_1=1$$

Flexible consumers maximize their expected welfare:

$$\therefore \quad P\left(Q_s^{s\tau}\right)=p_s^{s\tau}, \forall s$$

The electricity markets clear in each state:

$$q_s^{s\tau}+g_s\kappa^{s\tau}=Q_s^{s\tau}, \forall s$$

$$\therefore \quad q_0^{s\tau}=Q_0^{s\tau} \text{ and } q_1^{s\tau}+\kappa^{s\tau}=Q_1^{s\tau} \text{ since } g_0=0 \text{ and } g_1=1$$

(ii) Existence and uniqueness of a competitive equilibrium

Substituting $Q_0^{s\tau}$ by $q_0^{s\tau}$ and $Q_1^{s\tau}$ by $(q_0^{s\tau}+\kappa^{s\tau})$, the equilibrium is characterized by the following first-order conditions:

$$f_1(q_0^{s\tau})=P\left(q_0^{s\tau}\right)-c'\left(q_0^{s\tau}\right)-\tau\mathcal{E}'\left(q_0^{s\tau}\right)=0 \tag{C.1}$$

$$\begin{cases} f_2(q_1^{s\tau},\kappa^{s\tau})=P\left(q_1^{s\tau}+\kappa^{s\tau}\right)-c'\left(q_1^{s\tau}\right)-\tau\mathcal{E}'\left(q_1^{s\tau}\right)=0 \\ f_3(q_1^{s\tau},\kappa^{s\tau})=\mathcal{K}'\left(\kappa^{s\tau}\right)-\pi_1\left[c'\left(q_1^{s\tau}\right)+\tau\mathcal{E}'\left(q_1^{s\tau}\right)\right]=0 \end{cases} \tag{C.2}$$

It follows from the assumptions of the model that for $0 < q_0 < \infty$, (i) $\lim_{q_0 \to 0} f_1(q_0) = +\infty$ and (ii) $\lim_{q_0 \to +\infty} f_1(q_0) = -\infty$. Also,

$$\frac{df_1(q_0)}{dq_0} = P'(q_0) - c''(q_0) - \tau\mathcal{E}''(q_0) < 0$$

It is now a matter of fact to conclude that there exists a unique $q_0^{s\tau}$ which satisfies the F.O.C. as given by Eq.(C.1).

To verify existence and uniqueness of a solution for f_2 and f_3, the steps are as follows:

Step 1: Construction of $q_1(\kappa)$ satisfying $f_3(q_1(\kappa), \kappa) = 0$ with $0 < q_1 < \infty$.

Following the assumptions of the model, it can be observed that for $\forall\, \kappa > 0$, $\lim_{q_1 \to 0} f_3(q_1, \kappa) > 0$ and $\lim_{q_1 \to +\infty} f_3(q_1, \kappa) = -\infty$. Also,

$$\frac{\partial f_3(q_1, \kappa)}{\partial q_1} = -\pi_1 \left[c''(q_1) + \tau\mathcal{E}''(q_1)\right] < 0$$

By the Implicit Function Theorem, it follows that $\exists\, q_1 :]0, +\infty[\to]0, +\infty[$ with the property that $\forall\, \kappa > 0$, (i) $f_3(q_1, \kappa) = 0$ and (ii)

$$\frac{dq_1}{d\kappa} = \frac{\mathcal{K}''(\kappa)}{\pi_1 \left[c''(q_1) + \tau\mathcal{E}''(q_1)\right]} > 0 \tag{C.3}$$

Also, $f_3(q_1, \kappa) = 0$ verifies that:

- $\lim_{\kappa \to 0} q_1 = 0$
- $\lim_{\kappa \to +\infty} q_1 = +\infty$

Step 2: Existence of a solution

From previous observations, it can be found that (i) $\lim_{\kappa \to 0} f_2(q_1, \kappa) = +\infty$ and $\lim_{\kappa \to +\infty} f_2(q_1, \kappa) = -\infty$. Moreover from Eq.(C.3)

$$\frac{df_2}{d\kappa} = P'(q_1 + \kappa) + \left[P'(q_1 + \kappa) - c''(q_1) - \tau\mathcal{E}''(q_1)\right] \frac{dq_1}{d\kappa} < 0$$

It follows that there exists a unique $\kappa^{s\tau}$ which solves $f_2(q_1^{s\tau}, \kappa^{s\tau})$ and consequently a unique $(q_1^{s\tau}, \kappa^{s\tau})$ that satisfies the F.O.C. as given by Eq.(C.2).

Also, it can be found that $q_1^{s\tau} < q_0^{s\tau}$. In view of Eq.(C.1), if it is supposed that $q_1^{s\tau} \geq q_0^{s\tau}$, then from f_2 of Eq.(C.2), it implies that $\kappa^{s\tau} \leq 0$ which violates f_3.

(iii) Comparative statics

Applying the Implicit Function Theorem to the F.O.C given by Eq.(C.1) results in:

$$\frac{dq_0^{s\tau}}{d\tau} = \frac{\mathcal{E}'(q_0)}{P'(q_0) - c''(q_0) - \tau\mathcal{E}''(q_0)} < 0 \tag{C.4}$$

Now applying the Implicit Function Theorem to the F.O.C given by Eq.(C.2) gives:

$$\underbrace{\begin{bmatrix} P'(q_1 + \kappa) - (c'' + \tau\mathcal{E}'')_{|q_1} & P'(q_1 + \kappa) \\ -\pi_1 (c'' + \tau\mathcal{E}'')_{|q_1} & \mathcal{K}''(\kappa) \end{bmatrix}}_{B} \begin{bmatrix} dq_1 \\ d\kappa \end{bmatrix} - \begin{bmatrix} \mathcal{E}'(q_1) \\ \pi_1\mathcal{E}'(q_1) \end{bmatrix} d\tau = 0 \tag{C.5}$$

The determinant of B is $\left(P'\left(q_1+\kappa\right)-\left(c''+\tau\mathcal{E}''\right)_{|q_1}\right)\ \mathcal{K}''\left(\kappa\right)+\pi_1\left(c''+\tau\mathcal{E}''\right)_{|q_1}P'\left(q_1+\kappa\right)$ which is negative. It then follows that:

$$\frac{dq_1^{s\tau}}{d\tau}=\frac{1}{det\left(B\right)}\left\{\mathcal{E}'\left(q_1\right)\left[\mathcal{K}''\left(\kappa\right)-\pi_1 P'\left(q_1+\kappa\right)\right]\right\}<0 \tag{C.6}$$

$$\frac{d\kappa^{s\tau}}{d\tau}=\frac{1}{det\left(B\right)}\left[\pi_1\mathcal{E}'\left(q_1\right)P'\left(q_1+\kappa\right)\right]>0 \tag{C.7}$$

From condition (14d) of the competitive equilibrium:

$$\frac{dQ_0^{s\tau}}{d\tau}=\frac{dq_0}{d\tau}<0 \tag{C.8}$$

$$\frac{dQ_1^{s\tau}}{d\tau}=\frac{dq_1^{s\tau}}{d\tau}+\frac{d\kappa^{s\tau}}{d\tau}=\frac{\mathcal{E}'\left(q_1\right)\mathcal{K}''\left(\kappa\right)}{det\left(B\right)}<0 \tag{C.9}$$

Remembering that:

$$\frac{p_s^{s\tau}}{d\tau}=\left[c''\left(q_s^{s\tau}\right)+\tau\mathcal{E}''\left(q_s^{s\tau}\right)\right]\frac{dq_s^{s\tau}}{d\tau}+\mathcal{E}'\left(q_s^{s\tau}\right),\quad\forall s$$

It can be found that:

$$\frac{dp_0^{s\tau}}{d\tau}=\frac{\mathcal{E}'\left(q_0\right)P'\left(q_0\right)}{det\left(B\right)}>0 \tag{C.10}$$

$$\frac{dp_1^{s\tau}}{d\tau}=\frac{\mathcal{E}'\left(q_1\right)P'\left(q_1+\kappa\right)}{det\left(B\right)}>0 \tag{C.11}$$

Appendix D. Proof of Proposition 4

Efficiency of emissions tax

$$\max_{\tau} SW:=\sum_{0}^{s=1}\left\{\pi_s\int_0^{Q_s}P(v)dv-\left[c(q_s)+\mathcal{D}(\mathcal{E}(q_s))\right]\right\}-\mathcal{K}(\kappa)\ \text{ s.t. } Q_s=q_s+g_s\kappa,\forall s \tag{D.1}$$

Substituting Q_0 by q_0 and Q_1 by $(q_1+\kappa)$, Eq.(D.1) becomes:

$$\max_{\tau}\pi_0\int_0^{q_0}P(v)dv+\pi_1\int_0^{q_1+\kappa}P(v)dv-\pi_0\left[c\left(q_0\right)+\mathcal{D}\left(\mathcal{E}\left(q_0\right)\right)\right]-\pi_1\left[c\left(q_1\right)+\mathcal{D}\left(\mathcal{E}\left(q_1\right)\right)\right]-\mathcal{K}\left(\kappa\right)$$

The first-order condition is given by:

$$\begin{aligned}\tfrac{dq_0}{d\tau}\pi_0\{P\left(q_0\right)-\left[c'\left(q_0\right)+\mathcal{D}'\left(\mathcal{E}\left(q_0\right)\right)\mathcal{E}'(q_0)\right]\}\quad&+\\ \tfrac{dq_1}{d\tau}\pi_1\{P\left(q_1+\kappa\right)-\left[c'\left(q_1\right)+\mathcal{D}'\left(\mathcal{E}\left(q_1\right)\right)\mathcal{E}'(q_1)\right]\ \}\quad&+\\ \tfrac{d\kappa}{d\tau}\{\pi_1 P\left(q_1+\kappa\right)-\mathcal{K}'\left(\kappa\right)\}\quad&=0\end{aligned}$$

Substituting the optimal solutions of the competitive market as obtained in (9a), (9b), (14c) and (14d) results in:

$$\tfrac{dq_0^{s\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{s\tau}\right)\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_0^{s\tau}\right)\right)\right]\quad+\quad\tfrac{dq_1^{s\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{s\tau}\right)\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_1^{s\tau}\right)\right)\right]\ =0 \tag{D.2}$$

To compare the tax τ with the marginal damage in state 0, Eq.(D.2) can be rewritten as:

$$\tfrac{dq_0^{s\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{s\tau}\right)\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_0^{s\tau}\right)\right)\right]\quad+\quad\tfrac{dq_1^{s\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{s\tau}\right)\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_1^{s\tau}\right)\right)+\mathcal{D}'\left(\mathcal{E}\left(q_0^{s\tau}\right)\right)-\mathcal{D}'\left(\mathcal{E}\left(q_0^{s\tau}\right)\right)\right]\ =0$$

Factorizing and rearranging the terms results in:

$$\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_0^{s\tau}\right)\right)\right]\left[\frac{dq_0^{s\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{s\tau}\right)+\frac{dq_1^{s\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{s\tau}\right)\right]=\frac{dq_1^{s\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{s\tau}\right)\left[\mathcal{D}'\left(\mathcal{E}\left(q_1^{s\tau}\right)\right)-\mathcal{D}'\left(\mathcal{E}\left(q_0^{s\tau}\right)\right)\right]$$

Now comparing the tax τ with the marginal damage in state 1, Eq.(D.2) is rewritten as:

$$\frac{dq_0^{s\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{s\tau}\right)\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_0^{s\tau}\right)\right)+\mathcal{D}'\left(\mathcal{E}\left(q_1^{s\tau}\right)\right)-\mathcal{D}'\left(\mathcal{E}\left(q_1^{s\tau}\right)\right)\right]\quad+\quad\frac{dq_1^{s\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{s\tau}\right)\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_1^{s\tau}\right)\right)\right]\ =0$$

Factorizing and rearranging the terms gives:

$$\left[\tau-\mathcal{D}'\left(\mathcal{E}\left(q_1^{s\tau}\right)\right)\right]\left[\frac{dq_0^{s\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{s\tau}\right)+\frac{dq_1^{s\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{s\tau}\right)\right]=\frac{dq_0^{s\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{s\tau}\right)\left[\mathcal{D}'\left(\mathcal{E}\left(q_0^{s\tau}\right)\right)-\mathcal{D}'\left(\mathcal{E}\left(q_1^{s\tau}\right)\right)\right]$$

Assuming that the marginal damage is constant and is denoted by δ, then $\forall s, \mathcal{D}'\left(\mathcal{E}\left(q_s^{s\tau}\right)\right)=\delta$. Eq.(D.2) is then simplified to:

$$(\tau-\delta)\underbrace{\left[\frac{dq_0^{s\tau}}{d\tau}\pi_0\mathcal{E}'\left(q_0^{s\tau}\right)+\frac{dq_1^{s\tau}}{d\tau}\pi_1\mathcal{E}'\left(q_1^{s\tau}\right)\right]}_{\mathcal{A}}=0 \tag{D.3}$$

From the results of the comparative statics as in (C.4) & (C.6) and the assumptions of the framework, it is straightforward to find that $\mathcal{A}$ is strictly negative. Then, for the equation to hold, the tax τ must be equal to the marginal constant damage δ.

Appendix E. Proof of propositions 5 and 6

(ii) Competitive equilibrium with Emissions Trading Scheme and flexible consumers

The producer's fossil-fuel production plan and investment in the intermittent renewable technology are profit maximizing:

$$q_s^{SE}, \kappa^{SE} \in \arg\max \quad \sum_0^{s=1}\pi_s\left[p_s q_s - c\left(q_s\right) - p_{e_s}\mathcal{E}\left(q_s\right)+p_s g_s\kappa\right]-\mathcal{K}\left(\kappa\right)$$

$$p_s^{SE}=c'\left(q_s^{SE}\right)+p_{e_s}^{SE}\mathcal{E}'\left(q_s^{SE}\right), \quad \forall s$$

$$\pi_1 p_1^{SE}=\mathcal{K}'\left(\kappa^{SE}\right), \text{ since } g_0=0 \text{ and } g_1=1$$

Flexible consumers maximize their expected welfare:

$$P\left(Q_s^{SE}\right)=p_s^{SE}, \forall s$$

The electricity markets clear in each state:

$$q_s^{SE}+g_s\kappa^{SE}=Q_s^{SE}, \forall s$$

$$q_0^{SE}=Q_0^{SE} \text{ and } q_1^{SE}+\kappa^{SE}=Q_1^{SE} \text{ since } g_0=0 \text{ and } g_1=1$$

The state-dependent markets for emissions permits clear:

$$p_{e_s}^{SE}\left(\mathcal{E}\left(q_s^{SE}\right)-E\right)=0 \text{ with } \mathcal{E}\left(q_s^{SE}\right)-E\leq 0, \quad \forall s$$

(ii) Existence and uniqueness of a competitive equilibrium

Substituting Q_0^{SE} by q_0^{SE} and Q_1^{SE} by $(q_0^{SE}+\kappa^{SE})$, the equilibrium is characterized by the following first-order conditions:

$$\begin{cases} f_1(q_0^{SE})=P\left(q_0^{SE}\right)-c'\left(q_0^{SE}\right)-p_{e_0}^{SE}\mathcal{E}'\left(q_0^{SE}\right)=0 & \text{(E.1)}\\ f_2(q_1^{SE},\kappa^{SE})=P\left(q_1^{SE}+\kappa^{SE}\right)-c'\left(q_1^{SE}\right)-p_{e_1}^{SE}\mathcal{E}'\left(q_1^{SE}\right)=0 & \text{(E.2)}\\ f_3(q_1^{SE},\kappa^{SE})=\mathcal{K}'\left(\kappa^{SE}\right)-\pi_1\left[c'\left(q_1^{SE}\right)+p_{e_1}^{SE}\mathcal{E}'\left(q_1^{SE}\right)\right]=0 & \text{(E.3)}\end{cases}$$

$$\text{s.t. } p_{e_s}^{SE}\left(\mathcal{E}_s\left(q_s^{SE}\right)-E\right)=0 \text{ with } \mathcal{E}_s\left(q_s^{SE}\right)-E\leq 0, \quad \forall s$$

Following the emissions markets clearing conditions, 4 different cases for equilibrium can be identified.

Case 0 : $\forall s, \mathcal{E}_s\left(q_s^{SE}\right)-E\leq 0 \iff \forall s, p_{e_s}^{SE}=0$

This case implies that $E>\max\left\{\mathcal{E}\left(q_0^{SE}\right), \mathcal{E}\left(q_1^{SE}\right)\right\}$.

The F.O.C.s then write:

$$\begin{cases} f_1(q_0^{SE}) = P\left(q_0^{SE}\right)-c'\left(q_0^{SE}\right)=0 & \text{(E.4)}\\ f_2(q_1^{SE},\kappa^{SE}) = P\left(q_1^{SE}+\kappa^{SE}\right)-c'\left(q_1^{SE}\right)=0 & \text{(E.5)}\\ f_3(q_1^{SE},\kappa^{SE}) = \mathcal{K}'\left(\kappa^{SE}\right)-\pi_1 c'\left(q_1^{SE}\right)=0 & \text{(E.6)}\end{cases}$$

It follows from the assumptions of the model that for $0<q_0<\infty$, (i) $\lim_{q_0\to 0} f_1(q_0)=+\infty$ and (ii) $\lim_{q_0\to+\infty} f_1(q_0)=-\infty$. Also,

$$\frac{df_1\left(q_0\right)}{dq_0}=P'\left(q_0\right)-c''\left(q_0\right)<0$$

It can thus be concluded that there exists a unique q_0^{SE} which satisfies the F.O.C. as given by Eq.(E.4).

The existence and uniqueness of a solution for f_2 and f_3 is verified as follows:

Step 1: Construction of $q_1\left(\kappa\right)$ satisfying $f_3\left(q_1\left(\kappa\right),\kappa\right)=0$ with $0<q_1<\infty$.

Following the assumptions of the model, it can be observed that for $\forall\,\kappa>0$, $\lim_{q_1\to 0} f_3\left(q_1,\kappa\right)>0$ and $\lim_{q_1\to+\infty} f_3\left(q_1,\kappa\right)=-\infty$. Also,

$$\frac{\partial f_3\left(q_1,\kappa\right)}{\partial q_1}=-\pi_1 c''\left(q_1\right)<0$$

By the Implicit Function Theorem, it follows that $\exists\, q_1 :]0,+\infty[\,\to]0,+\infty[$ with the property that $\forall\,\kappa>0$, (i) $f_3\left(q_1,\kappa\right)=0$ and (ii)

$$\frac{dq_1}{d\kappa}=\frac{\mathcal{K}''\left(\kappa\right)}{\pi_1 c''\left(q_1\right)}>0 \tag{E.7}$$

Also, $f_3\left(q_1,\kappa\right)=0$ verifies that:

- $\lim_{\kappa\to 0} q_1=0$
- $\lim_{\kappa\to+\infty} q_1=+\infty$

Step 2: Existence of a solution

From previous observations, it can be found that (i) $\lim_{\kappa\to 0} f_2\left(q_1,\kappa\right)=+\infty$ and $\lim_{\kappa\to+\infty} f_2\left(q_1,\kappa\right)=-\infty$. Moreover from Eq.(E.7)

$$\frac{df_2}{d\kappa}=P'\left(q_1+\kappa\right)+\left[P'\left(q_1+\kappa\right)-c''\left(q_1\right)\right]\frac{dq_1}{d\kappa}<0$$

It follows that there exists a unique κ^{SE} which solves $f_2\left(q_1^{SE},\kappa^{SE}\right)$ and consequently a unique $\left(q_1^{SE},\kappa^{SE}\right)$ that satisfies the F.O.C.s as given by Eqs.(E.5) & (E.6).

Also, it is found that $q_1^{SE} < q_0^{SE}$. In view of Eq.(E.4), if it is supposed that $q_1^{SE} \geq q_0^{SE}$, then from f_2 of Eq.(E.5), it implies that $\kappa^{SE} \leq 0$ which violates f_3 as given by Eq.(E.6).

It therefore exists a unique equilibrium when the emissions cap E is set equal or above the highest emissions that can occur which is in state 0 when the fossil-fuel technology is the only one to produce: $E \geq \mathcal{E}\left(q_0^{SE}\right)$. In fact, this case corresponds to the situation when there is no emissions cap policy in force so that polluting producers operate as they usually do. This can be described as a "Business-as-Usual". The equilibrium solutions will be referred as: $q_s^{SE}{}_{|BaU}$ and $\kappa^{SE}{}_{|BaU}$.

Case 1 : $\mathcal{E}\left(q_0^{SE}\right) - E = 0 \iff p_{e_0}^{SE} > 0$ and $\mathcal{E}\left(q_1^{SE}\right) - E \leq 0 \iff p_{e_1}^{SE} = 0$

Existence and uniqueness of an equilibrium

In this case, $\mathcal{E}\left(q_0^{SE}\right) > \mathcal{E}\left(q_1^{SE}\right)$ and $q_0^{SE} = \mathcal{E}^{-1}(E)$.

The F.O.C.s are then:

$$\begin{cases} f_1(q_0^{SE}) = P\left(\mathcal{E}^{-1}(E)\right) - c'\left(\mathcal{E}^{-1}(E)\right) - p_{e_0}^{SE}\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right) = 0 & \text{(E.8)} \\ f_2(q_1^{SE}, \kappa^{SE}) = P\left(q_1^{SE} + \kappa^{SE}\right) - c'\left(q_1^{SE}\right) - = 0 & \text{(E.9)} \\ f_3(q_1^{SE}, \kappa^{SE}) = \mathcal{K}'\left(\kappa^{SE}\right) - \pi_1 c'\left(q_1^{SE}\right) = 0 & \text{(E.10)} \end{cases}$$

From the results in Case 0, it follows that there exists a unique $\left(q_1^{SE} = q_1^{SE}{}_{|BaU}, \kappa^{SE} = \kappa^{SE}{}_{|BaU}\right)$ that satisfies the F.O.C.s as given by Eqs. (E.9) & (E.10).

From Eq. (E.8),

$$p_{e_0} = \frac{P\left(\mathcal{E}^{-1}(E)\right) - c'\left(\mathcal{E}^{-1}(E)\right)}{\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)}, \tag{E.11}$$

where $P\left(\mathcal{E}^{-1}(E)\right) - c'\left(\mathcal{E}^{-1}(E)\right)$ must be > 0 for p_{e_0} to be > 0.

With the results in Case 0 for a solution for f_1, it can be deduced that in the present case q_0^{SE} must be less than $q_0^{SE}{}_{|BaU}$ for $P\left(\mathcal{E}^{-1}(E)\right) - c'\left(\mathcal{E}^{-1}(E)\right)$ to be positive. It implies that here the emissions cap E is set as: $\mathcal{E}(q_1^{SE}{}_{|BaU}) \leq E < \mathcal{E}(q_0^{SE}{}_{|BaU})$.

It therefore exists a unique solution for an equilibrium for Case 1 when $\mathcal{E}(q_1^{SE}{}_{|BaU}) \leq E < \mathcal{E}(q_0^{SE}{}_{|BaU})$.

Here, the equilibrium solutions will be denoted with a subscript "$_{|c1}$".

Comparative statics

From Eq. (E.11)

$$\frac{dp_{e_0}}{dE} = \frac{P'\left(\mathcal{E}^{-1}(E)\right) - c''\left(\mathcal{E}^{-1}(E)\right) - \left[P\left(\mathcal{E}^{-1}(E)\right) - c'\left(\mathcal{E}^{-1}(E)\right)\right]\frac{\mathcal{E}''\left(\mathcal{E}^{-1}(E)\right)}{\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)}}{\left[\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)\right]^2} < 0, \tag{E.12}$$

since where $P\left(\mathcal{E}^{-1}(E)\right) - c'\left(\mathcal{E}^{-1}(E)\right) > 0$

$$\frac{dq_0}{dE} = \frac{1}{\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)} > 0 \tag{E.13}$$

$$\frac{dq_1}{dE} = \frac{d\kappa}{dE} = 0 \tag{E.14}$$

Also, from condition (17d) of the competitive equilibrium:

$$\frac{dQ_0}{dE} = \frac{dq_0}{dE} > 0 \tag{E.15}$$

$$\frac{dQ_1}{dE} = \frac{dq_1}{dE} + \frac{d\kappa}{dE} = 0 \tag{E.16}$$

Replacing p_{e_0} of Eq. (E.11) into the fossil production equilibrium conditions (17a):

$$\frac{dp_0}{dE} = P'\left(\mathcal{E}^{-1}(E)\right) < 0 \tag{E.17}$$

$$\frac{dp_1}{dE} = 0 \tag{E.18}$$

Case 2 : $\mathcal{E}\left(q_0^{SE}\right) - E = 0 \iff p_{e_0}^{SE} > 0$ and $\mathcal{E}\left(q_1^{SE}\right) - E = 0 \iff p_{e_1}^{SE} > 0$

Existence and uniqueness of an equilibrium

Here, $q_0^{SE} = q_1^{SE} = \mathcal{E}^{-1}(E)$.

The F.O.C.s write:

$$\begin{cases} f_1(q_0^{SE}) = P\left(\mathcal{E}^{-1}(E)\right) - c'\left(\mathcal{E}^{-1}(E)\right) - p_{e_0}^{SE}\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right) = 0 & \text{(E.19)} \\ f_2(q_1^{SE}, \kappa^{SE}) = P\left(\mathcal{E}^{-1}(E) + \kappa^{SE}\right) - c'\left(\mathcal{E}^{-1}(E)\right) - p_{e_1}^{SE}\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right) = 0 & \text{(E.20)} \\ f_3(q_1^{SE}, \kappa^{SE}) = \mathcal{K}'\left(\kappa^{SE}\right) - \pi_1\left[c'\left(\mathcal{E}^{-1}(E)\right) + p_{e_1}^{SE}\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)\right] = 0 & \text{(E.21)} \end{cases}$$

From Eq. (E.19),

$$p_{e_0} = \frac{P\left(\mathcal{E}^{-1}(E)\right) - c'\left(\mathcal{E}^{-1}(E)\right)}{\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)}, \tag{E.22}$$

where $P\left(\mathcal{E}^{-1}(E)\right) - c'\left(\mathcal{E}^{-1}(E)\right) > 0$ for p_{e_0} to be > 0.

With the results in Case 0 for a solution for f_1, it can be deduced that in the present case q_0^{SE} must be less than ${q_0^{SE}}_{|BaU}$ for $P\left(\mathcal{E}^{-1}(E)\right) - c'\left(\mathcal{E}^{-1}(E)\right)$ to be positive. It implies that $E < \mathcal{E}({q_0^{SE}}_{|BaU})$.

From Eq. (E.21),

$$p_{e_1} = \frac{\mathcal{K}'(\kappa) - \pi_1 c'\left(\mathcal{E}^{-1}(E)\right)}{\pi_1\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)}, \tag{E.23}$$

For p_{e_1} to be > 0, κ must be $> \mathcal{K}'^{-1}\left[\pi_1 c'\left(\mathcal{E}^{-1}(E)\right)\right]$.

From Eq. (E.20),

$$p_{e_1} = \frac{P\left(\mathcal{E}^{-1}(E) + \kappa\right) - c'\left(\mathcal{E}^{-1}(E)\right)}{\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)} \tag{E.24}$$

For p_{e_1} to be > 0, $P\left(\mathcal{E}^{-1}(E) + \kappa\right) - c'\left(\mathcal{E}^{-1}(E)\right)$ must be > 0 implying that $E < \mathcal{E}({q_1^{SE}}_{|BaU})$. This can be observed from Case 0 for a solution for f_2.

Now substituting p_{e_1} as obtained in Eq. (E.24) into f_3:

$$f_3(q_1^{SE}, \kappa^{SE}) = P\left(\mathcal{E}^{-1}(E) + \kappa\right) - \frac{\mathcal{K}'(\kappa)}{\pi_1} = 0 \tag{E.25}$$

Taking into consideration the assumptions of the model, it is found that for $a < \kappa < \infty$ where $a = \mathcal{K}'^{-1}\left[\pi_1 c'\left(\mathcal{E}^{-1}(E)\right)\right]$, $\lim_{\kappa \to a} f_3(q_1, \kappa) = P\left(\mathcal{E}^{-1}(E) + a\right) - \pi_1 c'\left(\mathcal{E}^{-1}(E)\right) > 0$ when $E < \mathcal{E}({q_1^{SE}}_{|BaU})$. From Eq. (E.23), $p_{e_1} = 0$ when $\kappa = \mathcal{K}'^{-1}\left[\pi_1 c'\left(\mathcal{E}^{-1}(E)\right)\right]$. So with the results in Case 0 for a solution for f_2, it is deduced that $P\left(\mathcal{E}^{-1}(E) + a\right) - \pi_1 c'\left(\mathcal{E}^{-1}(E)\right)$ is positive when $q_1^{SE} < {q_1^{SE}}_{|BaU}$. In other words, when $E < \mathcal{E}({q_1^{SE}}_{|BaU})$. Also, $\lim_{\kappa \to \infty} f_3(q_1, \kappa) = -\infty$ with:

$$\frac{df_3(q_1, \kappa)}{dk} = P'\left(\mathcal{E}^{-1}(E) + \kappa\right) - \frac{\mathcal{K}''(\kappa)}{\pi_1} < 0$$

It therefore exists a unique κ^{SE} which solves $f_3\left(q_1^{SE}, \kappa^{SE}\right)$ and consequently a unique solution for an equilibrium of this Case 2 when $E < \mathcal{E}(q_1^{SE}{}_{|BaU})$. It is also deduced that $\kappa^{SE} > \kappa^{SE}{}_{|BaU}$. Replacing p_{e_1} of Eq. (E.24) into f_3 of Eq. (E.10) gives

$$f_3(q_1^{SE}, \kappa^{SE}) = \mathcal{K}'\left(\kappa^{SE}\right) - \pi_1 P'\left(\mathcal{E}^{-1}\left(E\right) + \kappa^{SE}\right) = 0$$

This equation is similar to the one hereunder obtained from Case 0:

$$f_3(q_1^{SE}, \kappa^{SE})_{|BaU} = \mathcal{K}'\left(\kappa^{SE}{}_{|BaU}\right) - \pi_1 P'\left(q_1^{SE}{}_{|BaU} + \kappa^{SE}{}_{|BaU}\right) = 0$$

So, in this case $\kappa^{SE} > \kappa^{SE}{}_{|BaU}$.
Also, it is noted that from Eqs. (E.22) and (E.24) that $p_{e_1} < p_{e_0}$ since $P\left(\mathcal{E}^{-1}\left(E\right) + \kappa\right) < P\left(\mathcal{E}^{-1}\left(E\right)\right)$.
Here, the equilibrium solutions will be denoted with a subscript "$|_{c2}$ ".

<u>*Comparative statics*</u>
From Eq. (E.22),

$$\frac{dp_{e_0}}{dE} = \frac{P'\left(\mathcal{E}^{-1}\left(E\right)\right) - c''\left(\mathcal{E}^{-1}\left(E\right)\right) - \left[P\left(\mathcal{E}^{-1}\left(E\right)\right) - c'\left(\mathcal{E}^{-1}\left(E\right)\right)\right]\frac{\mathcal{E}''\left(\mathcal{E}^{-1}(E)\right)}{\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)}}{\left[\mathcal{E}'\left(\mathcal{E}^{-1}\left(E\right)\right)\right]^2} < 0 \tag{E.26}$$

since $P\left(\mathcal{E}^{-1}\left(E\right)\right) - c'\left(\mathcal{E}^{-1}\left(E\right)\right) > 0$

From Eq. (E.24),

$$\frac{dp_{e_1}}{dE} = \frac{P'\left(\mathcal{E}^{-1}\left(E\right) + \kappa\right) - c''\left(\mathcal{E}^{-1}\left(E\right)\right) - \left[P\left(\mathcal{E}^{-1}\left(E\right) + \kappa\right) - c'\left(\mathcal{E}^{-1}\left(E\right)\right)\right]\frac{\mathcal{E}''\left(\mathcal{E}^{-1}(E)\right)}{\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)}}{\left[\mathcal{E}'\left(\mathcal{E}^{-1}\left(E\right)\right)\right]^2} < 0, \tag{E.27}$$

since $P\left(\mathcal{E}^{-1}\left(E\right) + \kappa\right) - c'\left(\mathcal{E}^{-1}\left(E\right)\right) > 0$

$$\frac{dq_0}{dE} = \frac{dq_1}{dE} = \frac{1}{\mathcal{E}'\left(\mathcal{E}^{-1}\left(E\right)\right)} > 0 \tag{E.28}$$

Applying the Implicit Function Theorem to Eq. (E.25):

$$\frac{d\kappa}{dE} = \frac{\pi_1 P'\left(\mathcal{E}^{-1}\left(E\right) + \kappa\right)}{\mathcal{E}'\left(\mathcal{E}^{-1}\left(E\right)\right)\left[\mathcal{K}''\left(\kappa\right) - \pi_1 P'\left(\mathcal{E}^{-1}\left(E\right) + \kappa\right)\right]} < 0 \tag{E.29}$$

Also, from condition (17d) of the competitive equilibrium:

$$\frac{dQ_0}{dE} = \frac{dq_0}{dE} > 0 \tag{E.30}$$

$$\frac{dQ_1}{dE} = \frac{dq_1}{dE} + \frac{d\kappa}{dE} = \frac{\mathcal{K}''\left(\kappa\right)}{\mathcal{E}'\left(\mathcal{E}^{-1}\left(E\right)\right)\left[\mathcal{K}''\left(\kappa\right) - P'\left(\mathcal{E}^{-1}\left(E\right) + \kappa\right)\right]} > 0 \tag{E.31}$$

Replacing p_{e_0} and p_{e_1} as obtained in Eqs. (E.22) and (E.24) respectively into the fossil production equilibrium conditions (17a):

$$\frac{dp_0}{dE} = P'\left(\mathcal{E}^{-1}\left(E\right)\right) < 0 \tag{E.32}$$

$$\frac{dp_1}{dE} = P'\left(\mathcal{E}^{-1}\left(E\right) + \kappa\right) < 0 \tag{E.33}$$

Case 3 : $\mathcal{E}\left(q_0^{SE}\right) - E \leq 0 \iff p_{e_0}^{SE} = 0$ and $\mathcal{E}\left(q_1^{SE}\right) - E = 0 \iff p_{e_1}^{SE} > 0$

In this case, $\mathcal{E}\left(q_0^{SE}\right) < \mathcal{E}\left(q_1^{SE}\right)$ and $q_1^{SE} = \mathcal{E}^{-1}(E)$.

The F.O.C.s are therefore given by:

$$\begin{cases} f_1(q_0^{SE}) = P\left(q_0^{SE}\right) - c'\left(q_0^{SE}\right) = 0 & \text{(E.34)} \\ f_2(q_1^{SE}, \kappa^{SE}) = P\left(\mathcal{E}^{-1}(E) + \kappa^{SE}\right) - c'\left(\mathcal{E}^{-1}(E)\right) - p_{e_1}^{SE}\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right) = 0 & \text{(E.35)} \\ f_3(q_1^{SE}, \kappa^{SE}) = \mathcal{K}'\left(\kappa^{SE}\right) - \pi_1\left[c'\left(\mathcal{E}^{-1}(E)\right) + p_{e_1}^{SE}\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)\right] = 0 & \text{(E.36)} \end{cases}$$

It follows from previous results in Case 0 that there exists a unique solution for f_1. In fact, $q_0^{SE} = q_0^{SE}{}_{|BaU}$. It now remains to verify the condition of existence of a solution for f_2 and f_3.

From Eq.(E.36),

$$p_{e_1} = \frac{\mathcal{K}'(\kappa) - \pi_1 c'\left(\mathcal{E}^{-1}(E)\right)}{\pi_1 \mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)} \tag{E.37}$$

For p_{e_1} to be > 0, it follows that κ must be $> \mathcal{K}'^{-1}\left[\pi_1 c'\left(\mathcal{E}^{-1}(E)\right)\right]$.

Substituting p_{e_1}into Eq.(E.35):

$$f_2(q_1^{SE}, \kappa^{SE}) = P\left(\mathcal{E}^{-1}(E) + \kappa\right) - \frac{\mathcal{K}'(\kappa)}{\pi_1} = 0 \tag{E.38}$$

With the assumptions of the model, it is found that for $a < \kappa < \infty$ where $a = \mathcal{K}'^{-1}\left[\pi_1 c'\left(\mathcal{E}^{-1}(E)\right)\right]$, $\lim_{\kappa \to a} f_2(q_1, \kappa) < 0$. This is because in view of the solution of Eq.(E.34) and that $q_0^{SE} < q_1^{SE}$, $P\left(\mathcal{E}^{-1}(E) + a\right) - c'\left(\mathcal{E}^{-1}(E)\right)$ must be less than 0. Also, $\lim_{\kappa \to -\infty} f_3(q_1, \kappa) = -\infty$. In addition,

$$\frac{df_3(q_1, \kappa)}{dk} = P'\left(\mathcal{E}^{-1}(E) + \kappa\right) - \frac{\mathcal{K}''(\kappa)}{\pi_1} < 0$$

Here, it does not exist a solution that satisfies f_2 and f_3. This case can be eliminated.

Now, putting together Case 1 where $\mathcal{E}(q_1^{SE}{}_{|BaU}) \leq E < \mathcal{E}(q_0^{SE}{}_{|BaU})$ and Case 2 where $E < \mathcal{E}(q_1^{SE}{}_{|BaU})$, the following observations can be made:

- $q_0^{SE}{}_{|c1}$ and $q_0^{SE}{}_{|c2}$ are both given by the function $\mathcal{E}^{-1}(E)$
- $\lim_{E \to \mathcal{E}(q_1^{SE}{}_{|BaU})} q_1^{SE}{}_{|c1} = q_1^{SE}{}_{|BaU}$
- When $E \to \mathcal{E}(q_1^{SE}{}_{|BaU})$, then from Eqs. (E.20) & (E.21), the solution for $\kappa^{SE}{}_{|c1}$ is given by:

$$P\left(q_1^{SE}{}_{|BaU} + \kappa^{SE}\right) - \frac{\mathcal{K}'\left(\kappa^{SE}\right)}{\pi_1} = 0$$

From Eqs. (E.5) & (E.6), it can be found that $\kappa^{SE}{}_{|c1} = \kappa^{SE}{}_{|BaU}$.

From these, it is concluded that the equilibrium solutions are continuous in E.

Appendix F. Proof of Proposition 7

Efficiency of emissions cap

$$\max_E SW := \sum_0^{s=1} \left\{ \pi_s \int_0^{Q_s} P(v)dv - [c(q_s) + \mathcal{D}(\mathcal{E}(q_s))] \right\} - \mathcal{K}(\kappa) \text{ s.t. } Q_s = q_s + g_s\kappa, \forall s \tag{F.1}$$

Substituting Q_0 by q_0 and Q_1 by $(q_1+\kappa)$, Eq.(F.1) becomes:

$$\max_E \pi_0 \int_0^{q_0} P(v)dv + \pi_1 \int_0^{q_1+\kappa} P(v)dv - \pi_0 \left[c(q_0) + \mathcal{D}(\mathcal{E}(q_0))\right] - \pi_1 \left[c(q_1) + \mathcal{D}(\mathcal{E}(q_1))\right] - \mathcal{K}(\kappa)$$

The first-order condition writes:

$$\begin{aligned} &\tfrac{dq_0}{dE}\pi_0\{P(q_0) - [c'(q_0) + \mathcal{D}'(\mathcal{E}(q_0))\,\mathcal{E}'(q_0)]\} \quad + \\ &\tfrac{dq_1}{dE}\pi_1\{P(q_1+\kappa) - [c'(q_1) + \mathcal{D}'(\mathcal{E}(q_1))\,\mathcal{E}'(q_1)]\ \} \quad + \\ &\tfrac{d\kappa}{dE}\{\pi_1 P(q_1+\kappa) - \mathcal{K}'(\kappa)\} \quad = 0 \end{aligned}$$

Substituting the optimal solutions of the competitive equilibrium for Case 1 where $\mathcal{E}\left(q_1^{SE}{}_{|BaU}\right) \leq E < \mathcal{E}\left(q_0^{SE}{}_{|BaU}\right)$, results in:

$$\begin{aligned} &\frac{dq_0^{SE}}{dE}\pi_0\{p_{e_0}^{SE} - \mathcal{D}'(E))\} \quad + \\ &\underbrace{\frac{dq_1^{SE}}{dE}}_{=0}\pi_1\{P(q_1^{SE}+\kappa^{SE}) - c'(q_1^{SE}) - \mathcal{D}'(\mathcal{E}(q_1^{SE}))\,\mathcal{E}'(q_1^{SE})\} \quad + \\ &\underbrace{\frac{d\kappa^{SE}}{dE}}_{=0}\{\pi_1 P(q_1^{SE}+\kappa^{SE}) - \mathcal{K}'(\kappa^{SE})\} \quad = 0 \end{aligned} \tag{F.2}$$

$$\mathcal{D}'(E) = p_{e_0}^{SE}{}_{|c1} \tag{F.3}$$

where $p_{e_0}^{SE}{}_{|c1} = \frac{P(\mathcal{E}^{-1}(E)) - c'(\mathcal{E}^{-1}(E))}{\mathcal{E}'(\mathcal{E}^{-1}(E))}$

Now, substituting the optimal solutions of the competitive equilibrium for Case 2 where $E < \mathcal{E}\left(q_1^{SE}{}_{|BaU}\right)$, results in:

$$\begin{aligned} &\tfrac{\pi_0}{\mathcal{E}'(\mathcal{E}^{-1}(E))}\{p_{e_0}^{SE}\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right) - \mathcal{D}'(E)\,\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)\} \quad + \\ &\tfrac{\pi_1}{\mathcal{E}'(\mathcal{E}^{-1}(E))}\{p_{e_0}^{SE}\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right) - \mathcal{D}'(E)\,\mathcal{E}'\left(\mathcal{E}^{-1}(E)\right)\} \quad + \\ &\tfrac{d\kappa^{SE}}{dE}\{0\} \quad = 0 \end{aligned} \tag{F.4}$$

$$\mathcal{D}'(E) = \pi_0 p_{e_0}^{SE}{}_{|c2} + \pi_1 p_{e_1}^{SE}{}_{|c2}, \tag{F.5}$$

where $p_{e_0}^{SE}{}_{|c2} = \frac{P(\mathcal{E}^{-1}(E)) - c'(\mathcal{E}^{-1}(E))}{\mathcal{E}'(\mathcal{E}^{-1}(E))}$ and $p_{e_1}^{SE}{}_{|c2} = \frac{P(\mathcal{E}^{-1}(E)+\kappa) - c'(\mathcal{E}^{-1}(E))}{\mathcal{E}'(\mathcal{E}^{-1}(E))}$