Asset bubble, growth and endogenous labor supply*

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Abstract

Bubbles are often associated with an increase in growth and labor, and their collapse is accompanied by a reduction in working hours and a recession of economic growth. Taking into account this empirical evidence, I examine the interaction between asset bubbles, labor supply and economic growth. I consider an overlapping generations model with elastic labor supply, in which I introduce tax burden on capital and labor incomes that finance productive public spending and lump-sum transfers. The latter two ensure long-run growth and are involved in the appearance of the bubble and the determination of its nature. First, the existence of a bubble depends on fiscal parameters and government spending. Second, I show that labor supply is promoted by bubbles. The growth rate is also boosted by the bubbles when the positive effect of labor supply (crowding-in effect) dominates the negative effect of bubbles (crowding-out effect). I explore a transitional dynamics of bubble, employment and GDP growth following the bubble bursting caused by changes in fiscal policy.

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Keywords: Bubble, endogenous labor, endogenous growth, crowding-in effect, fiscal policy.

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1 Introduction

As identified by Brunnermeier and Oehmke (2013), economic bubble refers to periods in which the price of an asset deviates from its fundamental value. This is because investors believe that they can sell the asset at an even higher price in the future. It is characterized by a rapid escalation of its value followed by a contraction, causing economic depression. Many countries have witnessed this phenomenon, the U.S. in 1929 and 2007 and Japan in 1990. After the collapse of U.S. housing prices in 2007, two observations can be drawn. Firs, on the labor market, the average annual labor hours recorded a decline of 1.94% between 2007 and 2009 (OECD database). Second, on the goods and services market, over the same period, the average growth rate dropped from 1.87% to -2.53% (World Bank). This empirical evidence suggests that there is a link between bubble, growth and labor. Additional evidence highlights the connection between these three elements. Bahloul Zekkari and Seegmuller (2020), Shi and Suen (2014) argue that bubbly episodes are associated with high labor supply. Similarly, empirical studies show that labor supply is accompanied by a booming of the growth rate of GDP (Duranton (2001), Eriksson (1996), Smith-Morris (1990)).[1] This interplay between bubble, growth and labor supply has not been investigated in macroeconomics. The purpose of this paper is to examine this relationship.

Motivated by the evidences, I develop a growth model with bubbles and endoge-

[1]Duranton (2001) shows that depending on the amount of labor supplied, the growth rate can be either positive, negative or nil. A high labor supply implies strong economic growth. In contrast, a low labor supply implies zero or negative growth. Eriksson (1996) argues that the effect of the change in attitude to work on the endogenous productivity growth depends on their complementarity in the technology sector. Smith-Morris (1990) provides some comparative data that suggest a positive correlation between manufacturing weekly working hours and the growth rate during the 1980s.

nous labor supply. I find that the presence of this type of labor may explain that employment and economic growth are higher when there is a positive bubble. The intuition is as follows: When the bubble exists, the interest rate increases. Due to the positive correlation between the interest rate and labor, the latter always increases with the bubble. A high level of labor makes the marginal productivity of capital stock high, which favors the growth rate.

Seminal growth models dealing with asset bubbles suggest that the growth rate of the economy is reduced by the presence of bubbles (Futagami and Shibata (2000),Grossman and Yanagawa (1993), or King and Ferguson (1993)). This happens because households save in the form of non-productive assets instead of productive capital. This is the so-called crowding-out effect of bubble. However, this theory is hard to reconcile with the observations. Thus, recent works exhibit different mechanisms explaining that bubbly episodes are characterized by a boom in the growth rate, that is the existence of a crowding-in effect of the bubble. For instance, Hirano and Yanagawa (2016), Kunieda and Shibata (2016), Martin and Ventura (2012), focus on financial market imperfections and show that depending on the restrictiveness of the collateral constraint, asset bubble can promote or impair growth. Hashimoto and Im (2016), Hashimoto and Im (2019) use labor market frictions and show that asset bubbles can exist when the employment rate is high, which leads to higher economic growth. Hence, in all these papers, the positive link between the bubble and growth is explained through financial and labor market frictions.

My paper contributes to the previous literature on the positive effects of bubbles on economic growth and provides a new mechanism explaining that asset bubbles may enhance economic growth. My approach is different from theirs for two reasons: The first reason is based on the endogeneity of labor supply. Indeed, to analyze the connection between asset bubbles and endogenous growth, I introduce an elastic labor supply that is not taken into account in existing studies. The second reason consists of the fundamental drivers of growth. Indeed, to ensure long-run economic growth, my study introduces productive public spending and transfers that are financed by the tax burden on capital and labor income, while Hashimoto and Im (2016) use the AK model through a learning by doing technological capital externality, Raurich and Seegmuller (2019) benefits from an externality that depends on the average human capital–labor ratio and Hashimoto and Im (2019) introduce technological progress via research and development (R&D) innovation.

My approach builds on the recent papers of Bahloul Zekkari and Seegmuller (2020) and Shi and Suen (2014) who develop a theory of rational bubbles with endogenous labor supply. They argue that endogenous labor is a channel which may explain the crowding-in effect of bubbles, i.e. higher levels of capital and labor when there is a bubble. Shi and Suen (2014) consider a separable utility function over consumptions with an elastic labor supply. They highlight analytically that the crowding-in effect of the bubble requires a positive and strong response of the labor supply to the interest rate. This holds when the elasticity of intertemporal substitution in consumption is high. A higher interest rate at the bubbly steady state may be in accordance with higher capital when labor is higher too. The main mechanism goes through the labor supply. It should strongly increase with respect to the interest rate. Bahloul Zekkari and Seegmuller (2020) extend Shi and Suen (2014) model by considering a non-separable utility function over consumptions. Bahloul Zekkari and Seegmuller (2020) explicitly and theoretically derive the conditions to have a crowding-in effect of the bubble. The utility function they consider shows that this result does not require an arbitrarily high elasticity of intertemporal substitution in consumption. When there is a bubble, the interest rate is higher because the bubble compensates the shortage of asset and reduces over- savings. In that case, capital can increase only if labor increases too. This happens when the labor supply is positively correlated with the interest rate. The two papers show that endogenous labor is a relevant channel that may contribute to explain that bubble episodes are characterized by higher levels of capital and employment. However, these studies do not consider the possibility of economic growth and do not interested in the link between asset bubbles and growth.

This paper fills these gaps. I consider an overlapping generations (OLG) model with elastic labor supply, where each household lives two periods. At the first period, each agent works, consumes and saves, and at the second period, she only consumes. There is portfolio choice between investment in productive capital and a purely speculative asset. To ensure long-run growth, I also consider fiscal policy. Taxes on capital and labor income are used to finance public transfers, shared between young and old households. They are used also to finance productive public spending in the spirit of Barro (1990). Capital income tax and productive public spending affect the level of the interest rate, while the transfers and/or labor income tax modify the revenue at young age. Both the interest rate and the income when young are important for the existence of bubbles. The former determines the growth rate of the bubble, while the latter is a key determinant of savings, which are used in part to buy asset bubbles.

My main concern is to determine under which conditions an asset bubble can exist, and have a crowding-in effect. First, the existence of bubbles requires that the growth rate in the non-bubble equilibrium exceeds the after-tax interest rate in the non-bubble equilibrium. This condition is more likely to occur for i) a high level of capital income tax and lump-sum public transfers to young households, or ii) a low level of labor income tax, lump-sum public transfers to old households and productive public spending. High lump-sum transfers to young households or low labor income tax rate provide young households with enough income to purchase bubble. When tax rate on capital income is high, agents no longer invest in the productive capital and this enhance the emergence of economic bubbles. High transfers to old households provide them with a large income, which discourages them from investing in the bubble when they were young. Finally, investment in productive spending increases the return of the capital stock, which impedes the emergence of the bubble.

Second, asset bubble has always a crowding-in effect on labor. When there is a bubble, the interest rate rises because the bubble compensates the shortage of asset and reduces over-savings. Such an increase will push up employment because of its positive correlation with the interest rate. Growth rate is also promoted by bubble if the crowding-in effect dominates the crowding-out effect of the bubble. The main mechanism that explains this result is the following: Bubble has two opposite effects on growth; it has a direct and negative effect, i.e. crowding-out effect. It diverts savings from capital accumulation and lowers growth. This effect is a common finding in the literature (Grossman and Yanagawa (1993), King and Ferguson (1993)). The second effect is indirect and positive, i.e crowding-in effect. This type of effect occurs because of the positive connection between the labor supply and the growth rate. Indeed, a higher labor supply in the case of a positive bubble makes the marginal productivity of capital higher, which favors economic growth. The second effect is a novelty in the bubbles literature. The dominance of the crowding-in effect is possible to occur for a high a level of capital and labor income taxes, transfers to old households and productive public spending, or a low level of transfers to young households. Under these fiscal parameter conditions, the bubble value is limited and this allows growth rate to increase.

I numerically illustrate the effects of fiscal policy parameters (taxes on capital and labor income, transfers to young and old households, productive spending) on the existence of bubble and on the determination of its nature (productive or non-productive). I calibrate the model using the values of fiscal parameters in a sample of countries. I first show that the economies of Luxembourg, Poland, UK and U.S. may exhibit a bubble. Conversely, bubbles do not appear in Austria, Belgium, Czeck Republic, Estonia, France, Germany, Greece, Italy, Netherlands, Portugal, Spain, Finland and Turkey. Second, only in the U.S. economy the value of the fiscal parameters is consistent with the crowding-in effect of a bubble. In addition, my model allows me to examine the effects of the fiscal policy changes on bubbles bursting, economic growth, and employment. For example, I find that an increase in lump-sum transfers to young households may cause bubble burst, followed by the jump in employment and growth rate to the balanced growth path without bubble.

My paper is organized as follows. Next section presents the model and define an intertemporal equilibrium. Sections 3 describes the equilibrium with and without bubble and determines the existence condition of bubbles. In section 4 I pin down the conditions under which the asset bubble has a crowding-in effect. Section 5 describes the dynamics of the bubble bursting. The final section summarizes my findings and concludes the paper.

2 Model

I consider an economy with three types of agents, households, firms and governments. Time is discrete, $t = 0, 1, 2...+\infty$. I describe the behavior of this three types of agents and, finally, define an intertemporal equilibrium.

2.1 Households

I consider overlapping generations living two periods: young and old ages. At each period of time $t \geq 0$, a new generation of identical consumers is born with a constant population size that is normalized to one. Let c_{1t} and c_{2t+1} denote the consumption of each consumer when young and old, respectively, and let l_t his labor supply when young. Assuming separable preferences in consumption and labor supply, the utility is given by:

$$U(c_{1t}, c_{2t+1}, l_t) = \log(c_{1t}) - V\frac{l_t^{1+\mu}}{1+\mu} + \gamma \log(c_{2t+1}) \tag{1}$$

where $V > 0$ denotes the constant parameter of disutility of work, $\mu \geqslant 0$ the inverse of the Frisch elasticity of labor supply and $\gamma \in (0, 1)$ the subjective discount factor.

When young, a household earns a wage income $(l_t w_t)$ taxed at a constant rate

$\tau_L > 0$ and can benefit from a lump-sum transfer T_{1t}. This income is shared between consumption and savings through two types of assets. Indeed, each saver makes a portfolio choice between the investment in productive capital s_t and the holding of m_t units of an intrinsically useless paper asset, for instance "money", which has a positive value $P_t \geqslant 0$. When old, the household consumes all her remunerated savings, which is the sum of the after-tax return of productive investment $(R_{t+1}(1-\tau_K)s_t)$, the return from speculative asset $(P_{t+1}m_t)$, where R_{t+1}, $\tau_K > 0$ and P_{t+1} denote the gross return from productive capital, the tax rate on capital income and the price of the intrinsically worthless asset at time t + 1, respectively. She may also receive a lump-sum transfer T_{2t+1}. Taking $(w_t, P_t, P_{t+1}, R_{t+1})$ as given, the consumer's problem is to choose an allocation $(c_{1t}, c_{2t+1}, s_t, m_t, l_t)$ so as to maximize his lifetime utility (1) subject to the budget constraints:

$$c_{1t} + s_t + P_t m_t = (1-\tau_L) l_t w_t + T_{1t} \tag{2}$$

$$c_{2t+1} = (1-\tau_K) R_{t+1} s_t + P_{t+1} m_t + T_{2t+1} \tag{3}$$

The first-order conditions for this problem are given by:

$$c_{1t} = \frac{c_{2t+1}}{\gamma(1-\tau_K)R_{t+1}} \quad \text{and} \quad V l_t^{\mu} = \frac{(1-\tau_L)w_t}{c_{1t}} \tag{4}$$

According to equation (4), I deduce that labor supply is positively correlated with the interest rate. Indeed, an increase in R_{t+1} lowers the relative price of future consumption. This creates an intertemporal substitution effect, which discourages consumption when young, c_{1t}. Since consumption c_{1t} and labor supply l_t are inversely related, the intertemporal substitution effect induces consumers to work more.

Using these equations, I obtain :

$$c_{1t} = \frac{1}{1+\gamma}\left[(1-\tau_L)l_t w_t + T_{1t} + \frac{T_{2t+1}}{(1-\tau_K)R_{t+1}})\right] \tag{5}$$

$$V l_t^{\mu} = \frac{(1+\gamma)(1-\tau_L)w_t}{(1-\tau_L)l_t w_t + T_{1t} + \frac{T_{2t+1}}{(1-\tau_K)R_{t+1}}} \tag{6}$$

and

$$s_t + P_t m_t \;\; = \;\; \frac{\gamma}{1+\gamma}\left[(1-\tau_L)l_t w_t + T_{1t}\right] - \frac{1}{1+\gamma}\frac{T_{2t+1}}{(1-\tau_K)R_{t+1}} \tag{7}$$

The first-order conditions is given also by the no arbitrage condition between capital and bubble:

$$P_t(1-\tau_K)R_{t+1} = P_{t+1} \tag{8}$$

When there is a bubble, the after-tax returns of productive capital is equal to the returns of holding the speculative bubble. In other words, the bubble must grow at the after-tax interest rate for all $t \geq 0$.

2.2 Firms

The economy comprises a large number of identical and competitive firms of unit size. They rent capital K_t and hire effective labor L_t at rental price R_t and wage w_t, respectively. They use the following Cobb–Douglas technology

$$Y_t = EK_t^{\beta}(A_tL_t)^{1-\beta} \tag{9}$$

with $E > 0$ denotes productivity parameter, $A_t > 0$ and $0 < \beta < 1$. Profit maximization yields:

$$w_t = E(1-\beta)A_t^{1-\beta}K_t^{\beta}L_t^{-\beta} \quad \text{and} \quad R_t = E\beta K_t^{\beta-1}(A_tL_t)^{1-\beta} \tag{10}$$

2.3 Government

The government revenue (G_t) is financed by tax burden on capital and labor income.[2] It is used to cover productive spending (A_t) in the spirit of Barro (1990) and lump-sum transfers to young and old households. The budget is balanced at each period, so that:

$$G_t = \tau_K R_t K_t + \tau_L L_t w_t = A_t + T_{1t} + T_{2t} \tag{11}$$

Let θ_1, θ_2 and $\theta_3 \in]0,1[$, such that $\theta_1 + \theta_2 + \theta_3 = 1$. Where θ_1 denotes the share of government revenue used to finance productive spending, θ_2 (θ_3) the share of government revenue distributed to young (old) households. This means that $A_t = \theta_1 G_t$, $T_{1t} = \theta_2 G_t$ and $T_{2t} = \theta_3 G_t$. The interest of introducing productive public expenditure and lump-sum transfers in my framework is twofold. First, they ensure endogenous growth in the long run. Second, I highlight their role and interplay with labor supply in the existence and features of the bubble.

[2] I do not introduce bubble taxes in my model, because this does not guarantee long-term balanced growth.

2.4 Intertemporal equilibrium

Labor marker clears in each period, so that, $L_t = l_t$. Using equations (10), (11) and the definition of A_t, i.e. $A_t = \theta_1 G_t$, I get the government revenue expression:

$$G_t = [E\mathcal{T}(\theta_1 l_t)^{1-\beta}]^{\frac{1}{\beta}} K_t \tag{12}$$

where $\mathcal{T} \equiv \beta\tau_K + (1-\beta)\tau_L$. Therefore, the equilibrium wage and interest rate can be re-expressed as:

$$w_t = E^{\frac{1}{\beta}}(1-\beta)\frac{K_t}{l_t}[\mathcal{T}\theta_1 l_t]^{\frac{1-\beta}{\beta}} \quad \text{and} \quad R_t = E^{\frac{1}{\beta}}\beta[\mathcal{T}\theta_1 l_t]^{\frac{1-\beta}{\beta}} \tag{13}$$

Equilibrium in capital market requires that aggregate savings of young households is used to buy the asset bubble and the future capital stock.

$$K_{t+1} + P_t m_t = S_t \tag{14}$$

The total supply of the intrinsically worthless asset is constant over time and is denoted by $M > 0$, while each young household buy m_t units of asset bubbles, at price P_t.[3] At equilibrium:

$$m_t = M \tag{15}$$

Let $B_t = P_t M$ be the aggregate value of the bubble at time t. By the condition of

[3] At time 0, all asset bubbles are owned by a group of “initial-old” consumers.

no-arbitrage between bubble and capital stock (8), I have:

$$B_{t+1} = (1-\tau_K)R_{t+1}B_t \tag{16}$$

The value of the bubble per efficiency of one unit labor (the normalized bubble) $b_t = B_t/A_t$ can be expressed as:

$$b_{t+1}l_{t+1}^{\frac{1-\beta}{\beta}} = \frac{(1-\tau_K)R_{t+1}}{(1+g_{K_t})}b_t l_t^{\frac{1-\beta}{\beta}} \tag{17}$$

where $g_{K_t} = \frac{K_{t+1}}{K_t} - 1$ denotes the growth rate of capital stock. Using equations (13) and (17), asset market rewrites:

$$b_{t+1} = \frac{E^{\frac{1}{\beta}}\beta(1-\tau_K)[\mathcal{T}\theta_1 l_t]^{\frac{1-\beta}{\beta}}}{1+g_{K_t}}b_t \tag{18}$$

Using equations (7), (12), (13) and (14), we deduce that the equilibrium in capital market is re-expressed as:

$$\begin{aligned} 1+g_{K_t} = {} & E^{\frac{1}{\beta}}\beta(1-\tau_K)\Big[\mathcal{T}\theta_1 l_t\Big]^{\frac{1-\beta}{\beta}} \\ & \left[\frac{\gamma\left[(1-\beta)(1-\tau_L)+\mathcal{T}\theta_2\right]}{\beta(1+\gamma)(1-\tau_K)+\mathcal{T}\theta_3} - \frac{(1+\gamma)\mathcal{T}\theta_1}{\beta(1+\gamma)(1-\tau_K)+\mathcal{T}\theta_3}b_t\right] \end{aligned} \tag{19}$$

while using equations (6), (12) and (13), the equilibrium in labor market takes the form:

$$1 + g_{K_t} = \left[\frac{E^{\frac{1}{\beta}}\beta(1-\tau_K)}{\mathcal{T}\theta_3}\right]\left[\mathcal{T}\theta_1 l_t\right]^{\frac{1-\beta}{\beta}} \left[V^{-1}(1+\gamma)(1-\beta)(1-\tau_L)l_t^{-(1+\mu)} - [(1-\beta)(1-\tau_L)+\mathcal{T}\theta_2]\right] \quad (20)$$

Definition 1 *Given the initial capital stock $K_0 > 0$, an intertemporal equilibrium is a sequence $(g_{K_t}, l_t, b_t) \in \mathbb{R}^3_{++}$ satisfying (18), (19) and (20).*

I have two static equations (19) and (20), and one dynamic equation (18) which disapears in case of non-bubble equilibrium. This system derives the dynamic of the bubble b_t for all $t > 0$. Two types of equilibria are possible, the bubbleless ones with $b_t = 0$ and the bubbly ones with $b_t > 0$.

3 Existence of bubbles

Before determining the condition of the existence, I first characterize bubbleless equilibrium. Using equation (19) and (20) with $b_t = 0$, I deduce that:

Proposition 1 *Without bubble, the economy immediately jumps on the bubbleless* *equilibrium* $(l_t, g_{K_t}, b_t) = (\tilde{l}, \tilde{g_K}, 0)$, *where*

$$\tilde{l} \equiv \left[\frac{(1-\beta)(1-\tau_L)[\beta(1+\gamma)(1-\tau_K)+\mathcal{T}\theta_3]}{V[\beta(1-\tau_K)+\mathcal{T}\theta_3][(1-\beta)(1-\tau_L)+\mathcal{T}\theta_2]}\right]^{\frac{1}{1+\mu}} \tag{21}$$

and

$$1+\tilde{g_K} \equiv E^{\frac{1}{\beta}}\gamma\beta(1-\tau_K)[\mathcal{T}\theta_1]^{\frac{1-\beta}{\beta}}\left[\frac{(1-\beta)(1-\tau_L)}{V[\beta(1-\tau_K)+\mathcal{T}\theta_3]}\right]^{\frac{1-\beta}{\beta(1+\mu)}} \left[\frac{\beta(1+\gamma)(1-\tau_K)+\mathcal{T}\theta_3}{(1-\beta)(1-\tau_L)+\mathcal{T}\theta_2}\right]^{\frac{1-\beta}{\beta(1+\mu)}-1} \tag{22}$$

Note that in bubbleless equilibrium there is no transitional dynamics.

I switch now to the bubbly equilibrium $(l_t, g_{K_t}, b_t > 0)$. In this case, I can study the transitional dynamics. The system equation (18)-(20) is simplified to one-dimensional dynamic system (b_t):

$$b_{t+1} = \frac{[\beta(1+\gamma)(1-\tau_K)+\mathcal{T}\theta_3]b_t}{\gamma[(1-\beta)(1-\tau_L)+\mathcal{T}\theta_2]-(1+\gamma)\mathcal{T}\theta_1 b_t} \tag{23}$$

where the condition $b_t < \overline{b} \equiv \frac{\gamma[(1-\beta)(1-\tau_L)+\mathcal{T}\theta_2]}{(1+\gamma)\mathcal{T}\theta_1}$ must be satisfied so that $b_{t+1} > 0$. The dynamic behavior of the bubble is shown in Figure 1, where the curve labelled BB illustrates the relationship between the size of the bubble in successive periods. The interaction between BB curve and the 45 line degree determines the bubble value in BGP b^*.

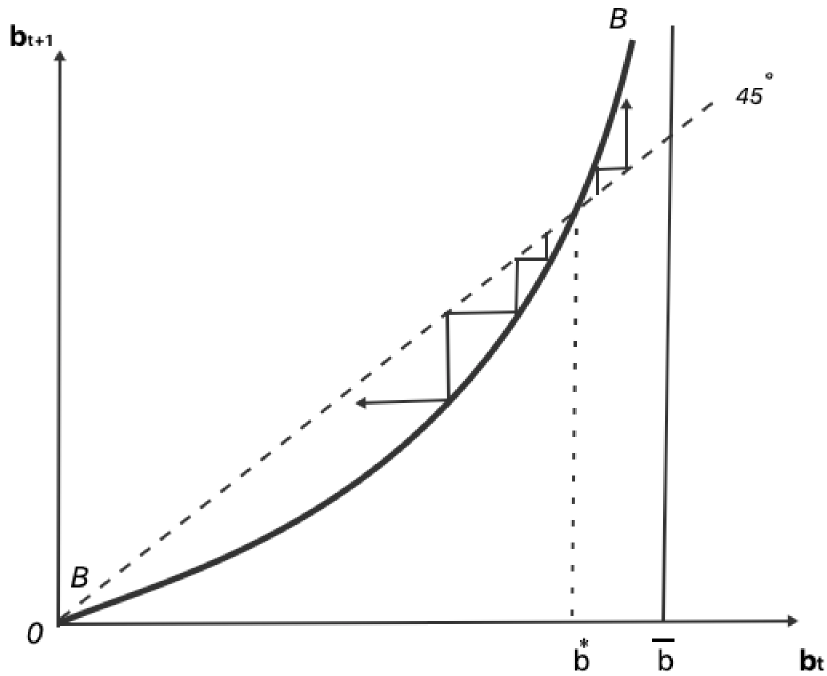


Figure 1 – Dynamic transition of bubble

Clearly, when the initial size of the bubble is such that $b_0 < b^*$. In this case, the bubble shrinks monotonically over time, and the economy converges to bubbleless equilibrium described previously. If, alternatively, the initial size of the bubble is such that $b_0 > b^*$, then the bubble grows monotonically over time until T. At some T, $b_T > \bar{b}$, the bubble cannot be sustained. This is because the aggregate savings of the young are not sufficient to allow them to acquire the bubble from the old generation. In this case, the economy converges to bubbleless BGP. The remaining possibility is that $b_0 = b^*$. In this case, the bubble and the economy remain in fixed proportion to one another and the economy immediately enters a BGP.

To derive the condition under which asset bubble can appear, I study the equilibrium at the BGP. A BGP (or steady state) is a solution $g_{K_t} = g_K$, $l_t = l$ and $b_t = b$ for all $t \geq$ satisfying:

$$\begin{aligned} 1 + g_K &= \left[\frac{E^{\frac{1}{\beta}}\beta(1-\tau_K)}{\mathcal{T}\theta_3}\right]\left[\mathcal{T}\theta_1 l\right]^{\frac{1-\beta}{\beta}} \\ &\quad \left[V^{-1}(1+\gamma)(1-\beta)(1-\tau_L)l^{-(1+\mu)} - [(1-\beta)(1-\tau_L) + \mathcal{T}\theta_2]\right] \quad (24) \\ 1 + g_K &= E^{\frac{1}{\beta}}\beta(1-\tau_K)\left[\mathcal{T}\theta_1 l\right]^{\frac{1-\beta}{\beta}} \\ &\quad \left[\frac{\gamma\left[(1-\beta)(1-\tau_L) + \mathcal{T}\theta_2\right]}{\beta(1+\gamma)(1-\tau_K) + \mathcal{T}\theta_3} - \frac{(1+\gamma)\mathcal{T}\theta_1}{\beta(1+\gamma)(1-\tau_K) + \mathcal{T}\theta_3} b\right] \quad (25) \\ b &= \frac{E^{\frac{1}{\beta}}\beta(1-\tau_K)[\mathcal{T}\theta_1 l]^{\frac{1-\beta}{\beta}}}{1+g_K} b \quad (26) \end{aligned}$$

In the case of positive bubble, the relationship between labor and bubble is determined by substituting equation (24) into equation (25) with $b > 0$:

$$l^{1+\mu} = \frac{V^{-1}\xi}{\omega - \mathcal{T}^2\theta_1\theta_3 b} \quad (27)$$

where $\xi \equiv (1-\beta)(1-\tau_L)[\beta(1+\gamma)(1-\tau_K) + \mathcal{T}\theta_3]$ and $\omega \equiv [(1-\beta)(1-\tau_L) + \mathcal{T}\theta_2][\beta(1-\tau_K) + \mathcal{T}\theta_3]$. While using equations (25) and (27), the link between growth rate and bubble takes the form:

$$\begin{aligned} 1 + g_K &= E^{\frac{1}{\beta}}\beta(1-\tau_K)(\mathcal{T}\theta_1)^{\frac{1-\beta}{\beta}}\left[\frac{V^{-1}\xi}{\omega - \mathcal{T}^2\theta_1\theta_3 b}\right]^{\frac{1-\beta}{\beta(1+\mu)}} \\ &\quad \left[\frac{\gamma\left[(1-\beta)(1-\tau_L) + \mathcal{T}\theta_2\right]}{\beta(1+\gamma)(1-\tau_K) + \mathcal{T}\theta_3} - \frac{(1+\gamma)\mathcal{T}\theta_1}{\beta(1+\gamma)(1-\tau_K) + \mathcal{T}\theta_3} b\right] \quad (28) \end{aligned}$$

The two equations (27) and (28) allow us in the next section to analyze the effect of the bubble on employment and growth rate.

There are obviously two types of BGP, the bubbleless ones with $(l, g_K, b) = (\tilde{l}, \tilde{g_K}, 0)$, whose values are determined previously, and the bubbly ones with $(l, g_K, b) = (l^*, g_K^*, b^*)$. When there is a bubble, equation (26) implies that $1 + g_K^* = E^{\frac{1}{\beta}}\beta(1 - \tau_K)[\mathcal{T}\theta_1 l^*]^{\frac{1-\beta}{\beta}}$. Substituting this last expression in equation (25), yields:

$$b^* = \frac{\gamma}{\theta_1(1+\gamma)}(\theta_2 - \underline{\theta_2}) \tag{29}$$

where $\underline{\theta_2} \equiv \frac{\beta(1+\gamma)(1-\tau_K)+\mathcal{T}\theta_3-\gamma(1-\beta)(1-\tau_L)}{\gamma\mathcal{T}}$. Thus, there is a bubbly BGP with a positive bubble $b > 0$ if and only if

$$\underline{\theta_2} < \theta_2 \tag{30}$$

By inspection, I see that this inequality is more likely to occur for high θ_2 and τ_K; low θ_3 and τ_L.

The condition (30) is equivalent to $(1 + \tilde{g_K}) > (1 - \tau_K)\tilde{R}$ because, using equations (13) and (25) at the bubbleless BGP, I have:

$$\frac{1 + \tilde{g_K}}{(1-\tau_K)\tilde{R}} = \frac{\gamma\left[(1-\beta)(1-\tau_L) + \mathcal{T}\theta_2\right]}{\beta(1+\gamma)(1-\tau_K) + \mathcal{T}\theta_3} > 1 \tag{31}$$

where $\tilde{R}$ represents the interest rate at the bubleless BGP.

The following Proposition summarized these results:

Proposition 2 *There exists a unique bubbly BGP (l^*, g_K^*, b^*) if $(1 + \tilde{g_K}) > (1 - \tau_K)\tilde{R}$*

or equivalently: $\underline{\theta_2} < \theta_2$.

where, the bubbly BGP value is determined by using equations (24)-(26):

$$l^* = \left[\frac{(1+\gamma)(1-\beta)(1-\tau_L)}{V[\mathcal{T}(\theta_2+\theta_3)+(1-\beta)(1-\tau_L)]}\right]^{\frac{1}{1+\mu}} \tag{32}$$

$$1+g_K^* = E^{\frac{1}{\beta}}\beta(1-\tau_K)[\theta_1\mathcal{T}]^{\frac{1-\beta}{\beta}}\left[\frac{(1+\gamma)(1-\beta)(1-\tau_L)}{V[\mathcal{T}(\theta_2+\theta_3)+(1-\beta)(1-\tau_L)]}\right]^{\frac{1-\beta}{\beta(1+\mu)}} \tag{33}$$

$$b^* = \frac{\gamma[(1-\beta)(1-\tau_L)+\theta_2\mathcal{T}]-\beta(1+\gamma)(1-\tau_K)-\mathcal{T}\theta_3}{(1+\gamma)\mathcal{T}\theta_1} \tag{34}$$

A bubbly BGP exists if the growth rate at the bubblelss BGP exceeds the after-tax interest rate at the bubblelss BGP. This condition is consistent with the condition stated by Farhi and Tirole (2012) (2012), Grossman and Yanagawa (1993), King and Ferguson (1993), Olivier (2000)) and Tirole (1985). However, both Tirole's model and Fahri and Tirole's model are based on an exogenous growth model, while mine is based on an endogenous growth model. The interest rate in the growth models of Grossman and Yanagawa (1993), that of King and Ferguson (1993) and Olivier (2000)) is constant, contrary to mine where it depends on endogenous labor. In any case, the above condition is semilar to $\underline{\theta_2} < \theta_2$. The effects of fiscal policy τ_K, θ_2, θ_3 and τ_L on the existence of bubbles are obtained from a simple comparative static analysis on the inequality (30) or (31). The two inequalities are more likely to occur if τ_K and θ_2 are high, or θ_3 and τ_L are low. Each young household has two types of investments; capital stock and asset bubble. When the tax burden on capital income increases, the return on productive investment becomes less attractive, which promotes the appearance of bubble. The after-tax income is improved with a low level of taxation on labor income, which favors saving and thus the existence of a bubble. High transfer towards the young households allows them to have sufficient income to sustain the existence of the bubble through the purchase of the speculative asset. High transfer to old households allows them to have a significant income, which discouraged them from investing in the bubble when they were young. Using the formula $\theta_1 + \theta_2 + \theta_3 = 1$, productive public spending θ_1 has a negative effect on the existence of bubbles. Because high productive public spending increases the return on the capital stock, households prefer to invest in productive assets instead of bubbles.

4 Crowding-out versus crowding-in effect

In this section, I examine whether the existence of asset bubble raises employment and growth. The equation (27) shows the effect of bubble on employment. It is clear that this effect is always positive, it means that $\tilde{l} < l^*$. This relationship between employment with and without bubble is described in Figure 2.

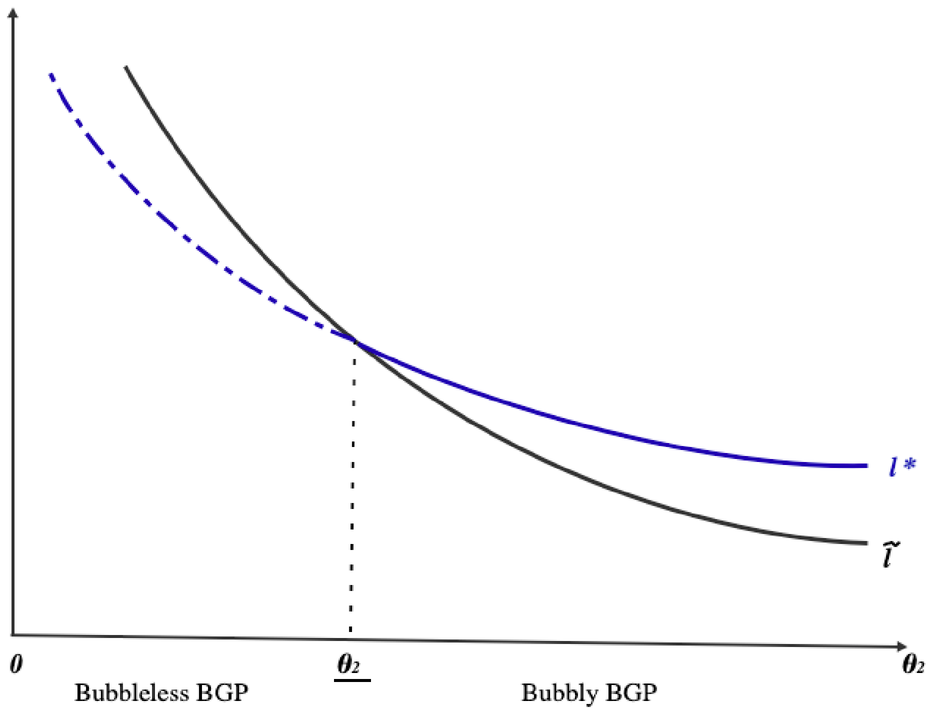


Figure 2 – Employment with versus without bubble

I now turn to the investigation of the relationship between the bubble and growth. To infer under which condition the growth rate is enhanced by the bubble, I assess the elasticity of the growth rate with respect to the bubble. According to equation (28), the elasticity of g_K with respect to b, $\xi_{g_K,b}$, is given by:

$$\xi_{g_K,b} = \frac{E^{\frac{1}{\beta}}\beta(1-\tau_K)\left(\mathcal{T}\theta_1 l\right)^{\frac{1-\beta}{\beta}}}{\beta(1+\gamma)(1-\tau_K)+\mathcal{T}\theta_3}\left[\frac{1-\beta}{\beta(1+\mu)}\frac{\mathcal{T}^2\theta_1\theta_3}{(\omega-\mathcal{T}^2\theta_1\theta_3 b)}\left(\gamma\left[(1-\beta)(1-\tau_L)+\mathcal{T}\theta_2\right] -(1+\gamma)\mathcal{T}\theta_1 b\right)-\frac{(1+\gamma)\mathcal{T}\theta_1}{\beta(1+\gamma)(1-\tau_K)+\mathcal{T}\theta_3}\right] \quad (35)$$

Then, $\xi_{g_K,b} > 0$ if

$$(1-\beta) > \beta(1+\mu) \tag{36}$$

and

$$b \;<\; \bar{b}\Big[1 - \frac{\beta^2(1+\mu)(1+\gamma)(1-\tau_K) + \beta(1+\mu)\mathcal{T}\theta_3}{[1-\beta-\beta(1+\mu)]\mathcal{T}\theta_3\gamma}\Big] \equiv \bar{\bar{b}} \tag{37}$$

Inequality (36) ensures a necessary condition for having $\tilde{g_K} < g_K^*$, which is satisfied for a high elasticity of labor supply $1/\mu$, and a low capital share β.

I will have $\tilde{g_K} < g_K^*$ if $b^* < \bar{\bar{b}}$. Using (34), the inequality (37) becomes:

$$\underline{\theta_2} < \theta_2 < \overline{\theta_2} \tag{38}$$

where $\overline{\theta_2} \equiv \frac{[1-\beta-\beta(1+\mu)]\mathcal{T}\theta_3 - \beta(1+\mu)(1-\beta)(1-\tau_L)}{\mathcal{T}\beta(1+\mu)}$. By inspection of this condition, it is nesscessary that $\underline{\theta_2} < \overline{\theta_2}$; i.e.

$$\theta_3 \;>\; \frac{\beta^2(1+\gamma)(1-\tau_K)(1+\mu)}{\mathcal{T}[\gamma(1-\beta) - \beta(1+\gamma)(1+\mu)]} \tag{39}$$

which is satisfied for a low inverse elasticity of labor supply μ and capital share β; or a high tax burden on capital and labor incomes τ_K, τ_L, and lump-sum transfers to old agents θ_3. The relationship between growth with and without bubble is illustrated in Figure 3.

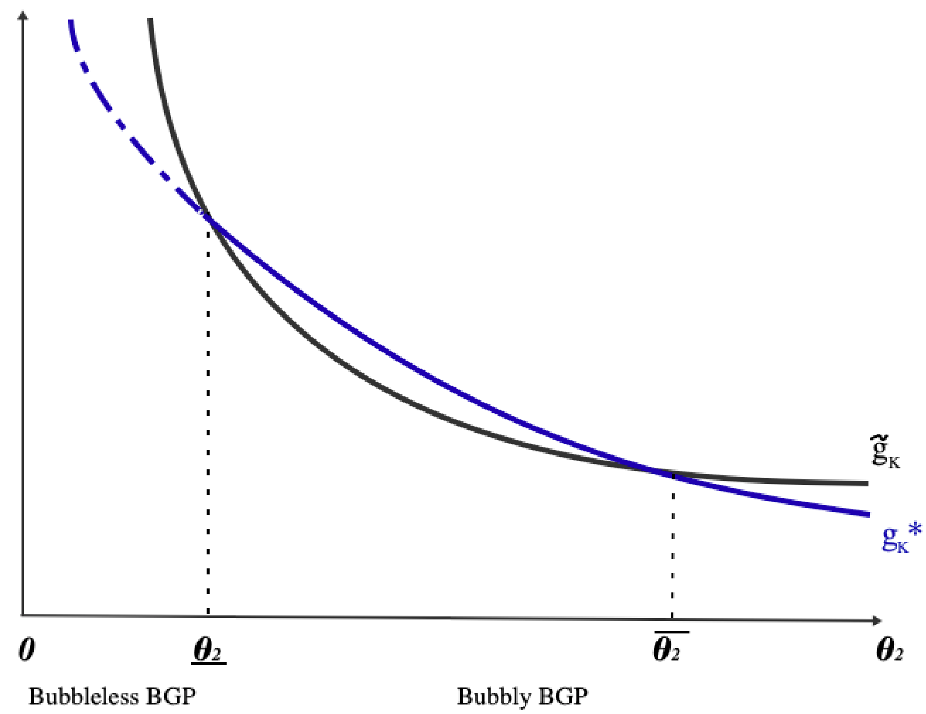


Figure 3 – Growth with versus without bubble

Note that inequality (38) ensures a sufficient condition to have $\tilde{g_K} < g_K^*$. It implies that g_K is an increasing function of b for all $b < \bar{\bar{b}}$. This does not mean that this result could not also hold if g_K is a non monotonic function of b.

My result is summarized in the following proposition:

Proposition 3 *1. Labor is higher at the bubbly BGP; i.e.* $\tilde{l} < l^*$

2. Growth rate is higher at the bubbly BGP; i.e. $\tilde{g_K} < g_K^*$ *if the inequalities (36) and (38) are satisfied, which require* τ_K, τ_L *and* θ_3 *high; or* θ_2, μ, β *low.*

Asset bubble absorbs the capital over-accumulation, resulting an increase in the interest rate. This implies a rise in employment from $\tilde{l}$ to l^* due to its positive correlation with the bubble. This result extends the view of Shi and Suen (2014), Bahloul Zekkari and Seegmuller (2020) about the positive effect of bubbles on labor to a model with endogenous growth.

Bubble has two opposite effects on growth. On the one hand, it has a direct and negative effect; i.e. crowding-out effect. It diverts savings from capital accumulation and reduces economic growth. This type of effect is a common finding in the literature (Grossman and Yanagawa (1993), King and Ferguson (1993)). On the other hand, it has an indirect and positive effect; i.e. crowding-in effect. This effect occurs through the labor supply. Indeed, the growth rate is positively related with labor: higher level of labor in bubbly BGP makes the marginal productivity of capital higher, which encourages firms to invest in the capital stock. The second effect represents a novelty in the bubble literature. So, the effect of bubble on capital growth depends on the dominance of one effect over the other. If the crowding-in (crowding-out) effect dominates the crowding-out(crowding-in) effect, the bubble promotes (hinders) growth. In such a case, the bubble is productive (unproductive). The domination of the crowding-in effect is posible when the inequality (38) is satisfied, which requires τ_K, τ_L and θ_3 to be high; or θ_2, μ, β to be low. These fical parameters values guarantee a moderate value of the bubble. The investment into productive capital stock becomes less attractive with high tax burden on capital income, which entails an increase in the bubble value. However, this rise is held back by τ_L, θ_2 and θ_3. Household income declines with labor income tax and/or low transfers to young investors, which limits the purchase of bubbles and thus reduces their values. High lump-sum transfers to ageing people increase their income, which discourages them from investing in a bubble during the first period of their youth and therefore reduces bubble values. In such a case, the the economy is characterized by the presence of bubble that is not high enough, with higher employment and economic growth compared to bubbleless BGP. Conversely, when the fiscal conditions stated in Proposition 3 are not satisfied, the value of the bubble becomes very high, leading to a large increase in the employment and a decrease of growth rate. In this case, I recover the main conclusion by Grossman and Yanagawa (1993).

4.1 Numerical illustration

I now provide a specific numerical example in which asset bubbles will lead to an expansion in labor and growth. I set the parameters values as follows: disutility of work V is normalized to one, productivity parameter $E = 10$, the share of capital stock $\beta = 0.2$ (Senhadji (2000)). I follow Shi and Suen (2014) and set the annual subjective discount rate to 0.9950, which matches the average annual growth rate of U.S. employment over the period 1953–2008. So, I have $\gamma = (0.9950)^3 0 = 0.8604$. I follow King and Rebelo (1999) and set also the inverse of the Frisch elasticity of labor supply $\mu = 0.25$. Fiscal parameters τ_K, τ_L, θ_1 θ_2 and θ_3 are obtained from the OECD data set in 2006. The transfers to young households (θ_2) correspond to income maintenance, public expenditure on family (family allowances, maternity and parental leave) and other cash benefits. Lump-sum transfers to old households (θ_3) and productive public spending (θ_1) represent pensions and infrastructure spending, respectively. The calibration parameters are displayed in Table 1. The results of this calibration are displayed in Table 2.

Table 1 – ***Fiscal parameters***

Countries	θ_1	θ_2	θ_3	τ_K	τ_L
Austria	0.08	0.22	0.7	0.25	0.49
Belgium	0.05	0.23	0.72	0.36	0.56
Czeck Republic	0.18	0.18	0.64	0.24	0.43
Estonia	0.16	0.23	0.61	0.23	0.39
France	0.07	0.17	0.76	0.35	0.5
Germany	0.06	0.13	0.81	0.39	0.53
Greece	0.08	0.07	0.85	0.29	0.43
Italy	0.14	0.06	0.8	0.38	0.46
Luxembourg	0.1	0.4	0.5	0.3	0.34
Netherlands	0.08	0.24	0.68	0.3	0.39
Poland	0.125	0.115	0.76	0.19	0.1
Portugal	0.13	0.09	0.78	0.19	0.38
Spain	0.20	0.08	0.72	0.35	0.4
Finland	0.05	0.2	0.75	0.26	0.44
Turkey	0.1	0.05	0.85	0.2	0.43
UK	0.11	0.29	0.6	0.3	0.34
U.S.	0.07	0.13	0.8	0.39	0.31

Source. OECD data base (2006)

Table 2 shows the existence of bubble, with a negative sign implying that the economy does not exhibit a bubble and a positive sign meaning that the economy has a bubble. The same table shows also the value of interest rate, employment and growth rate in both economies (bubble and no bubble). As it is clear from this table, the economies of Luxembourg, Poland, UK and U.S. may exhibit a bubble. Conversely, bubbles do not appear in Austria, Belgium, Czeck Republic, Estonia, France, Germany, Greece, Italy, Netherlands, Portugal, Spain, Finland and Turkey. The non-existence of bubble can be explained either by the high τ_L (Austria, Belgium, Czeck Republic, France, Germany, Greece, Italy), or a low τ_K (Austria, Czeck

Table 2 – ***Numerical results***

Countries	Existence	$\tilde{R}$	R^*	$\tilde{l}$	l^*	$(1+\tilde{g_K})$	$1+g_K^*$
Austria	—	0.034	- - -	1.027	- - -	0.0192	- - -
Belgium	—	0.0067	- - -	0.9273	- - -	0.002865	- - -
Czeck Republic	—	0.7525	- - -	1.11	- - -	0.4855	- - -
Estonia	—	0.3387	- - -	1.12	- - -	0.2535	- - -
France	—	0.02534	- - -	1.0198	- - -	0.011	- - -
Germany	—	0.01760	- - -	1.0170	- - -	0.006436	- - -
Greece	—	0.036	- - -	1.14	- - -	0.017	- - -
Italy	—	0.4990	- - -	1.137	- - -	0.2083	- - -
Luxembourg	+	0.034	0.041	1.089	1.14	0.031	0.028
Netherlands	—	0.021	- - -	1.082	- - -	0.01456	- - -
Poland	+	0.0036	0.0044	1.40	1.47	0.0047	0.0035
Portugal	—	0.15	- - -	1.19	- - -	0.1024	- - -
Spain	—	1.30	- - -	1.1528	- - -	0.7151	- - -
Finland	—	0.0043	- - -	1.068	- - -	0.002	- - -
Turkey	—	0.084	- - -	1.1810	- - -	0.0443	- - -
UK	+	0.055	0.062	1.1152	1.15	0.045	0.043
U.S.	+	0.0091	0.01	1.14	1.16	0.0058	0.006

Republic , Estonia, Portugal, Turkey), or the low θ_2 (France, Germany, Greece, Italy, Portugal, Spain, Turkey), or the high θ_3 (Germany, Greece, Turkey). These results confirm my finding regarding the effects of fiscal parameters on the bubble existence.

Based on the comparison between bubbly and bubbleless BGP at the level of empoyment and growth rate, I find that the bubble is productive and has a crowding-in effect only in the U.S. . A higher interest rate in the bubbly BGP leads to an increase in employment from $\tilde{l}$ to l^* and growth from $\tilde{g_K}$ to g_K^*. The U.S. fiscal parameters are consistent with the crowding-in effect of a bubble.

4.2 Robustness

By inspection of the condition of existence of the bubble (30), productive public spending (θ_1) has no direct effect on the existence and features of the bubble. However, my objective in introducing this type of policy is twofold. First, to ensure a long run balanced growth path and second, to show that bubbles have a crowding-in effect on labor and growth. To reinforce my argument, let me choose another engine of growth such as the AK model through a learning by doing capital externality (Romer et al. (1988)). In such a case, I will show that the bubble has a crowding-out effect.

I keep the previous model as it is, the only change that will be made is located at the level of A_t. I incorporate capital externality, making it linear in the aggregate capital stock, i.e. $A_t = \overline{K_t}$. In a such a case, the government revenue finances only lump-sum transfers to old and young households. The framework without productive spending is indexed by a hat $\hat{}$. Since there is a continuum of identical firms, at equilibrium $K_t = \overline{K_t}$. Therefore, the government revue becomes:

$$\hat{G}_t \quad = \quad E\mathcal{T}\hat{l_t}^{1-\beta}\hat{K}_t \tag{40}$$

and the equilibrium prices becomes:

$$\hat{w}_t = E(1-\beta)\hat{K}_t\hat{l_t}^{-\beta} \quad \text{and} \quad \hat{R}_t = E\beta\hat{l_t}^{1-\beta} \tag{41}$$

Capital market, labor market and asset market, respectively, are defined by the following equations:

$$1 + g_{\hat{K}_t} = \left[\frac{E\beta(1-\tau_K)\hat{l}_t^{1-\beta}}{\mathcal{T}\theta_3}\right] \left[V^{-1}(1+\gamma)(1-\beta)(1-\tau_L)\hat{l_t}^{-(1+\mu)} - [(1-\beta)(1-\tau_L) + \mathcal{T}\theta_2]\right] \quad (42)$$

$$1 + g_{\hat{K}_t} = E\beta(1-\tau_K)\hat{l}_t^{1-\beta} \left[\frac{\gamma\left[(1-\beta)(1-\tau_L) + \mathcal{T}\theta_2\right]}{\beta(1+\gamma)(1-\tau_K) + \mathcal{T}\theta_3} - \frac{(1+\gamma)}{\beta(1+\gamma)(1-\tau_K) + \mathcal{T}\theta_3}\hat{b_t}\right] \quad (43)$$

$$\hat{b}_{t+1} = \frac{E\beta(1-\tau_K)\hat{l}_t^{1-\beta}}{1 + g_{\hat{K}_t}}\hat{b_t} \quad (44)$$

where $\hat{b_t} \equiv \frac{B_t}{EA_t l_t^{1-\beta}}$. Without productive public spending model, I have to types of equilibria in BGP. The bubbleless ones $(\hat{l_t}, \hat{g}_{K_t}, \hat{b_t}) = (\tilde{\hat{l}}, \tilde{\hat{g}}_K, 0)$, and the bubbly ones $(\hat{l_t}, \hat{g}_{K_t}, \hat{b_t}) = (\hat{l^*}, \hat{g^*}_K, \hat{b^*})$.

I follow the same steps as before and I find that, first, there exists a unique bubbleless BGP equilibrium $(\tilde{\hat{l}}, \tilde{\hat{g}}_K)$. Where

$$\tilde{\hat{l}} = \tilde{l} \quad (45)$$

$$1 + \tilde{\hat{g}}_K = E\gamma\beta(1-\tau_K)\left[\frac{(1-\beta)(1-\tau_L)}{V[\beta(1-\tau_K) + \mathcal{T}\theta_3]}\right]^{\frac{1-\beta}{1+\mu}} \left[\frac{\beta(1+\gamma)(1-\tau_K) + \mathcal{T}\theta_3}{(1-\beta)(1-\tau_L) + \mathcal{T}\theta_2}\right]^{\frac{1-\beta}{1+\mu}-1} \quad (46)$$

Second, the condition for the existence of bubble is similar to that of the model with productive spending; i.e. the growth rate at the bubblelss BGP $(1+\tilde{\hat{g}}_K)$ exceeds the after-tax interest rate at the bubblelss BGP $\tilde{\hat{R}}$, or $\underline{\theta_2} < \theta_2$.

Third, labor is always higher when there is a positive bubble; i.e. $\tilde{\hat{l}} < \hat{l^*}$. Because using equations (42) and (43) in BGP, I have:

$$\hat{l}^{(1+\mu)} = \frac{V^{-1}\xi}{\omega - \mathcal{T}\theta_3\hat{b}} \tag{47}$$

In addition, bubbles promote growth rate if $\xi_{\hat{g}_K,\hat{b}} > 0$. Where

$$\begin{aligned}\xi_{\hat{g}_K,\hat{b}} &= \beta(1-\tau_K)\hat{l}^{1-\beta}\Bigg[\frac{1-\beta}{(1+\mu)}\frac{\mathcal{T}\theta_1\theta_3}{(\omega-\mathcal{T}\theta_3\hat{b})}\Bigg(\frac{\gamma\left[(1-\beta)(1-\tau_L)+\mathcal{T}\theta_2\right]}{\beta(1+\gamma)(1-\tau_K)+\mathcal{T}\theta_3} \\ &\quad -\frac{(1+\gamma)\mathcal{T}}{\beta(1+\gamma)(1-\tau_K)+\mathcal{T}\theta_3}\hat{b}\Bigg) - \frac{\beta(1+\gamma)(1-\tau_K)}{\beta(1+\gamma)(1-_K)+\mathcal{T}\theta_3}\Bigg]\end{aligned} \tag{48}$$

So, I will have $\tilde{\hat{g}}_K < \hat{g^*}_K$ if

$$(1-\beta) > (1+\mu) \tag{49}$$

and

$$\hat{b} < \bar{\bar{b}} \tag{50}$$

In the macroeconomic calibrations, the Frisch elasticity needs to be set somewhere in the range of 2-4 (King and Rebelo (1999), Cho and Cooley (1994)). In contrast to the large calibration values, the seminal microeconometric estimates of the Frisch elasticity are in the range of 0-0.54 (MaCurdy (1981), Altonji (1986)). Whatever the nature of the study, the calibrated value of the inverse elasticity of labour supply is $\mu \geq 0$. Under the parameters values $\mu \geq 0$ and $0 < \beta < 1$, the inequality (49) cannot be satisfied. Therefore, in this case the bubble has a crowding-out effect on growth, and I recover the main conclusion by Grossman and Yanagawa (1993).

5 Dynamics of bubble burst

In this section, I investigate under what conditions the bubble can burst. As pointed by Weil (1987), Miao et al. (2016), Miao and Wang (2018), Clain-Chamosset-Yvrard et al. (2020), the bubble burst stems from the realization of a sunspot (pessimistic beliefs). According to Hashimoto and Im (2019), the bubbles burst is the results of the parameters changes of employment rate such as, an increase in search cost, R& D cost, unemployment benefit and bargaining power of worker. Whatever the reason of the crash, they are unanimous on its consequences. It is accompanied by a decline in employment and/or in growth rate. In my framework, the condition for the existence of a bubble depends on the labor supply which, in turn, depends on fiscal parameters. Therefore, my framework focuses on the bursting of an asset bubble caused by changes in fiscal parameters, such as the tax burden on capital and labor incomes, lump-sum transfers to young and old households, productive public spending.

The dynamic behavior of the bubble is represented in equation (23) and illustrated in Figure 4. The curve labelled BB depicts the transition of bubble over time, its interaction with the 45 line determines the BGP value b^*. By using comparative dynamics in response to parameter changes, I can analyze the dynamic properties of bubble burst in the phase diagram. I consider a rise in productive public spending θ_1, lump-sum transfers to old households θ_3 and labor income tax τ_L; or a decline in lump-sum transfers to young households θ_2 and capital income tax τ_K. Note that changes of these parameters still allow the bubble to exist and satifies the condition (30). In this case, whatever the value of the bubble $b^* < \bar{b}$, the latter decreases and the BB curve moves upwards. Therefore, the bubble may burst when the BB curve is completely above the 45 line, followed by a jump of employment and growth on the bubbleless equilibrium (bubbleless BGP). The dynamic behavior of a bubble burst is shown in Figure 4.

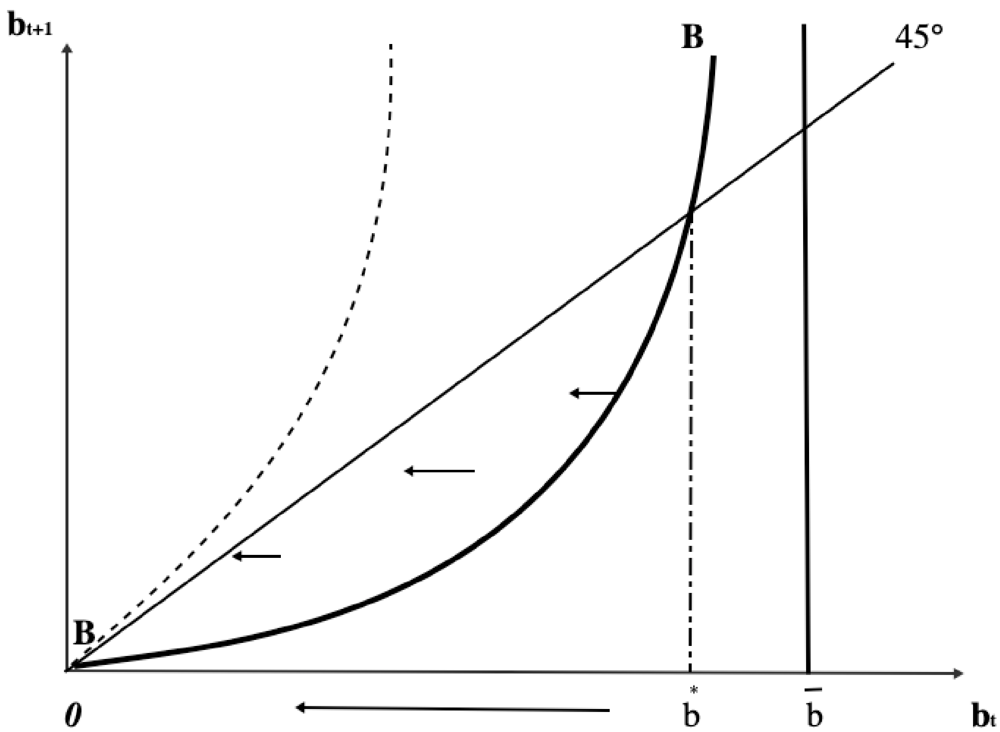
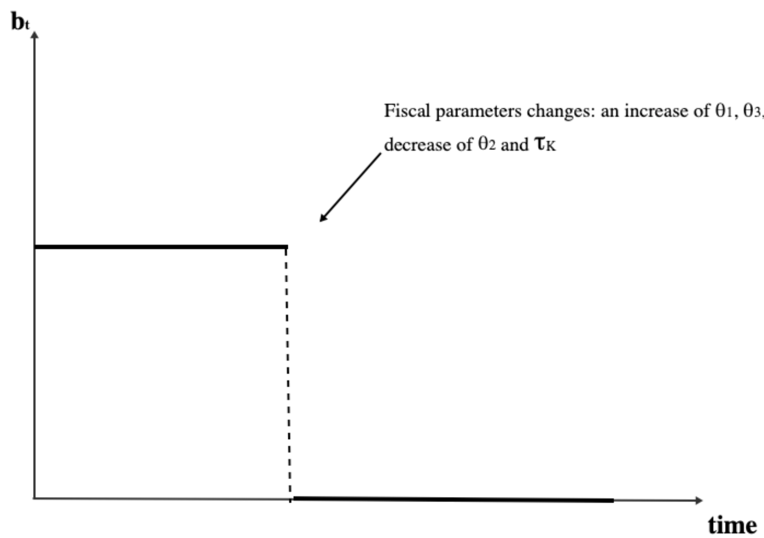


Figure 4 – The pattern of bubble burst

These results are formally stated in the following proposition:

Proposition 4 *Changes to the following fiscal parameters;* θ_1*,* θ_2*,* θ_3*,* τ_K *and* τ_L *satisfy the condition (30) and can cause a bubble to burst. This leads to the jump of employment and growth on the bubbleless equilibrium (bubbleless BGP).*

Figure 5 summarizes the dynamic paths of bubbles, employment and growth rate after bubble burst.



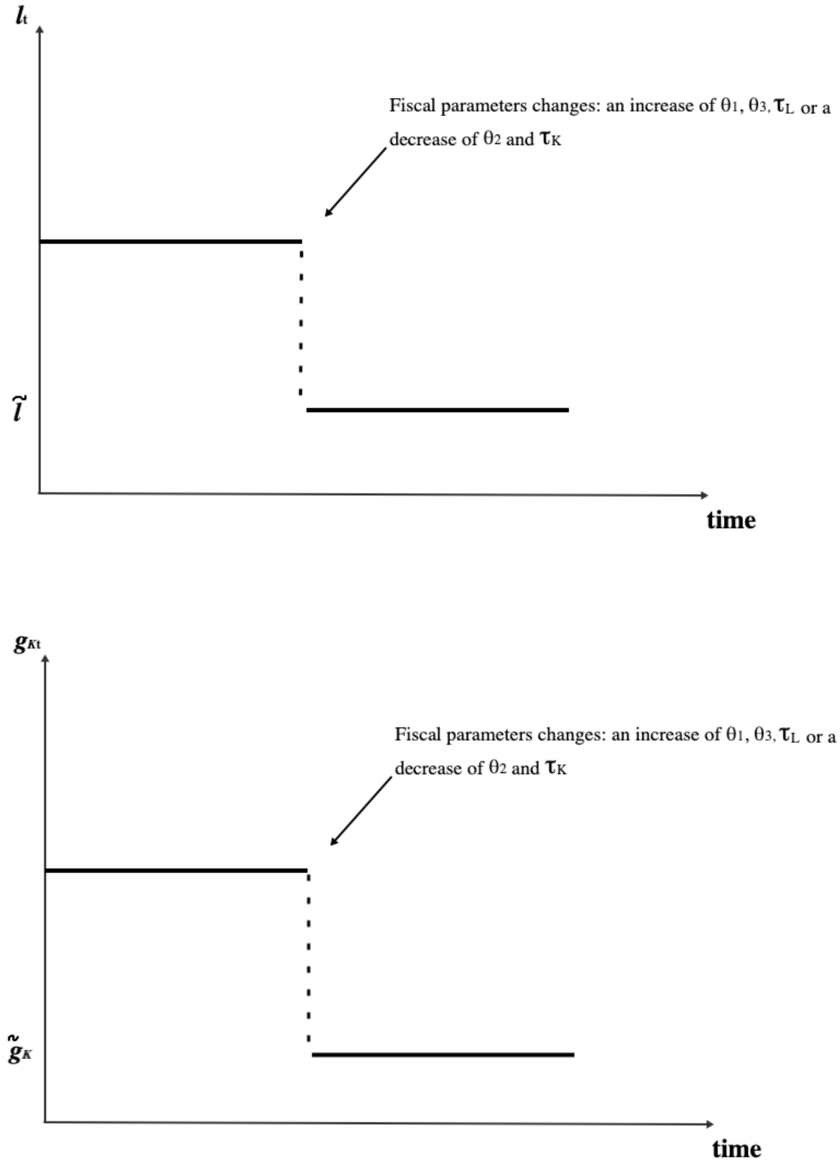


Figure 5 – Dynamic paths

6 Conclusion

This paper has highlighted the effect of asset bubble on labor and growth. I thus consider an OLG model with elastic labor supply in which I introduce tax burden on capital and labor incomes. The two taxes finance productive public spending and transfers. First, my results indicated that the existence of the bubble is subject to the conditions of fiscal parameters. High transfer to young households or low income tax rate modify the income upward, which promote the appearance of the bubble. Moreover, high level of capital income tax makes the investment into capital stock less attractive, which facilitates the existence of a speculative bubble. High transfers to old households allow them to have a significant income, which discouraged them from investing in the bubble when they were young.

Second , I have exhibited a new mechanism explaining that bubbles promote labor and economic growth. When the bubble exists, labor supply is high because of the its positive correlation with the interest rate. Moreover, the growth rate may increase with the bubble thanks to labor supply. Indeed, bubble has two opposite effects on growth. The first one is negative (crowding-out effect), because households save in the form of non-productive assets instead of productive capital. The second effect is positive (crowding-in effect), higher labor supply when there is a bubble rises the marginal product of capital, which leads to an increase in investment in the capital stock. Thus, when the crowding-in effect dominates the crowding out-effect, then the bubbles promotes growth. This type of dominance is possible for a high level of capital and labor income taxation and lump-sum transfers to old households or low level of lump-sum transfers to young households. These results challenge the conventional view supported by Grossman and Yanagawa (1993), Futagami and Shibata (2000) about a negative effect of bubbles in endogenous growth. Finally, I have found that changes to the fiscal parameters can cause a bubble to burst. This leads to the jump of employment and growth on the bubbleless equilibrium.

I note that my model with transfers and taxes on capital and labor incomes introduce some distortions that encompass some models with labor market imperfections as a particular case. For instance, it is possible to show that when there are unions which efficiently bargain with firms on employment and wages, the unions' bargaining power plays a role of capital income taxation and redistributes a share of rental cost of capital to labor income. Therefore, using my analysis, I can deduce the implications of such type of labor market imperfection on speculative bubbles.

Bibliography

Altonji, J. G. (1986). Intertemporal substitution in labor supply: Evidence from micro data. *Journal of Political Economy*, 94:176–215.

Bahloul Zekkari, K. and Seegmuller, T. (2020). Asset bubble and endogenous labor supply: a clarification. *Economics Letters*, 196:109537.

Barro, R. J. (1990). Government spending in a simple model of endogeneous growth. *Journal of political economy*, 98:103–125.

Brunnermeier, M. K. and Oehmke, M. (2013). Bubbles, financial crises, and systemic risk. *Handbook of the Economics of Finance*, 2:1221–1288.

Cho, J.-O. and Cooley, T. F. (1994). Employment and hours over the business cycle. *Journal of Economic Dynamics and Control*, 18:411–432.

Clain-Chamosset-Yvrard, L., Raurich, X., and Seegmuller, T. (2020). Are the liquidity and collateral roles of asset bubbles different? *Available at SSRN 3572024*.

Duranton, G. (2001). Endogenous labor supply, growth and overlapping generations. *Journal of economic behavior & Organization*, 44:295–314.

Eriksson, C. (1996). Economic growth with endogenous labour supply. *European Journal of Political Economy*, 12:533–544.

Farhi, E. and Tirole, J. (2012). Bubbly liquidity. *The Review of Economic Studies*, 79:678–706.

Futagami, K. and Shibata, A. (2000). Growth effects of bubbles in an endogenous growth model. *Japanese Economic Review*, 51:221–235.

Grossman, G. M. and Yanagawa, N. (1993). Asset bubbles and endogenous growth. *Journal of Monetary Economics*, 31:3–19.

Hashimoto, K.-i. and Im, R. (2016). Bubbles and unemployment in an endogenous growth model. *Oxford Economic Papers*, 68:1084–1106.

Hashimoto, K.-i. and Im, R. (2019). Asset bubbles, labour market frictions and r&d-based growth. *Canadian Journal of Economics/Revue canadienne d'économie*, 52:822–846.

Hirano, T. and Yanagawa, N. (2016). Asset bubbles, endogenous growth, and financial frictions. *The Review of Economic Studies*, 84:406–443.

King, I. and Ferguson, D. (1993). Dynamic inefficiency, endogenous growth, and ponzi games. *Journal of Monetary Economics*, 32:79–104.

King, R. G. and Rebelo, S. T. (1999). Resuscitating real business cycles. *Handbook of macroeconomics*, 1:927–1007.

Kunieda, T. and Shibata, A. (2016). Asset bubbles, economic growth, and a self-fulfilling financial crisis. *Journal of Monetary Economics*, 82:70–84.

MaCurdy, T. E. (1981). An empirical model of labor supply in a life-cycle setting. *Journal of political Economy*, 89:1059–1085.

Martin, A. and Ventura, J. (2012). Economic growth with bubbles. *American Economic Review*, 102:3033–58.

Miao, J. and Wang, P. (2018). Asset bubbles and credit constraints. *American Economic Review*, 108:2590–2628.

Miao, J., Wang, P., and Xu, L. (2016). Stock market bubbles and unemployment. *Economic Theory*, 61:273–307.

Olivier, J. (2000). Growth-enhancing bubbles. *International Economic Review*, 41:133–152.

Raurich, X. and Seegmuller, T. (2019). Growth and bubbles: Investing in human capital versus having children. *Journal of Mathematical Economics*, 82:150–159.

Romer, P. M. et al. (1988). Capital accumulation in the theory of long run growth. Technical report, University of Rochester-Center for Economic Research (RCER).

Senhadji, A. (2000). Sources of economic growth: An extensive growth accounting exercise. *IMF staff papers*, 47:129–157.

Shi, L. and Suen, R. M. (2014). Asset bubbles in an overlapping generations model with endogenous labor supply. *Economics Letters*, 123:164–167.

Smith-Morris, M. (1990). *Economist book of vital world statistics*. Times Books.

Tirole, J. (1985). Asset bubbles and overlapping generations. *Econometrica*, 53:1499–1528.

Weil, P. (1987). Confidence and the real value of money in an overlapping generations economy. *Quarterly Journal of Economics*, 102:1–22.