The energy transition and fossil energy use

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Abstract

Achieving the energy transition is among global priorities of the 21^{st} century, one key element to success being the development of affordable renewable technology to compete with fossil energy. While technological progress seems already biased in favor of the renewable sector since the 70's, we had to wait until 2005 to observe a sharp increase of its share in the energy mix. In this paper I develop a theoretical model of energy transition able to explain this delay through a lasting capital effect in favor of fossil energy. The existence of a trade-off between efficient pollutant capital and less efficient carbon-free alternative, in a context of embodied technical change and long living power plants, slows down the capacity to close polluting units. This mechanism postpones the effect of biased technical change. I also show that the divergence between the EU and the US on the timing of the energy transition can be explained by subjective beliefs about future damages from pollution. Simulation of both an optimistic view for the US and a pessimistic one for the EU matches with trajectories observed in the data.

Keywords: *Energy transition, investment specific, exogenous growth, climate economics*

JEL Classification Numbers: C61, O41, Q43.

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1 Introduction

Technological and Institutional co-evolution in our use of fossil fuel during the 20^{th} century has led to today's policy inertia towards mitigation of climate change (Unruh (2000)). Escaping this "carbon lock-in" situation and achieving the energy transition is among global priorities of the 21^{st} century and requires to decrease our use of carbonized energy. Figure 1 highlights that although both the EU and the US are increasing their share of renewable energy (left panel), only the EU started to decrease its use of fossil energy after 2005 (right panel). The absence of a sharp drop in our use of carbonized energy might become problematic in a context of climate urgency,[2] pollution emissions depending on the level of fossil energy use and not on its share. In this context, 2 interesting questions might be raised: i) is the increase of the renewable energy share sufficient to limit risks of climate change ? ii) Why does the US exhibit an increase of both its share of renewable energy and its use of fossil energy ? I examine how these questions can be answered in a structural change model with embodied technological progress.

Directed technical change literature (Acemoglu et al. (2012), Lennox and Witajewski-Baltvilks (2017), Hassler et al. (2019) or Hötte (2020)) states that the use of both a carbon tax and a research subsidy helps to redirect R&D towards carbon free energy, increasing investment in the renewable sector. Figure 2 shows that research in the US has already been biased in favor of the clean sector since the 80's. However, as observed in figure 1 directed technical change mechanism seems to have had delayed impact on energy investment. The share of renewable energy has only grown more

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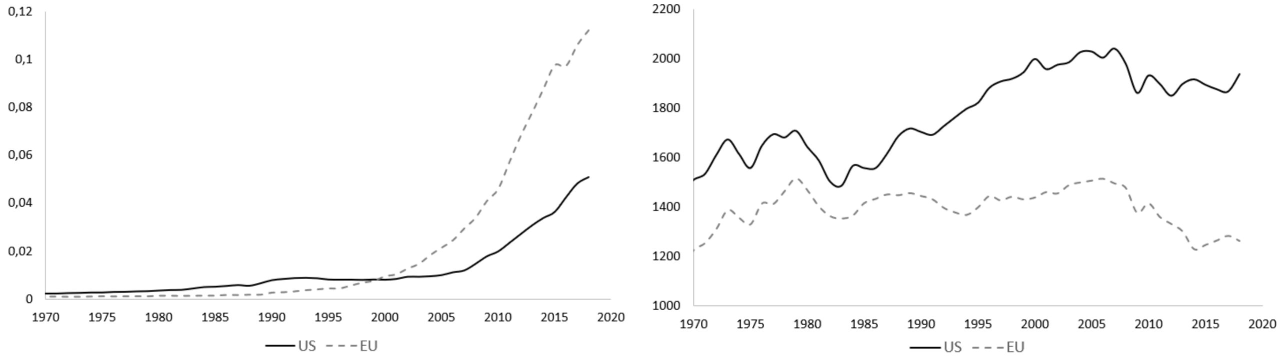
[2] 6^{th} IPCC report, Tsur and Zemel (2008), van den Bijgaart et al. (2016), Rezai and van der Ploeg (2017), Tol (2018) or Botzen et al. (2019) show how the actual path of energy use can lead us to a disastrous situation



(a) Share of renewable energy

(b) Level of fossil energy use (Mtoe)

source: BP statistical review of world energy

Figure 1: Comparison between the US and the EU

rapidly after 2005, pointing out a delay of approximately 25 years between technology differential and its direct impact on the energy mix. In this context I argue there exists an underlying mechanism able to explain why we observe such a delay and why it is compatible with an increase of both the renewable energy share and the level of fossil energy use. When technological change is embodied, and power plants long living, the trade-off between efficient fossil energy[3] and less-efficient carbon-free alternative creates a lasting capital effect in favor of carbonized sources of energy. At the beginning of the period, the economy is investing massively in long-living fossil energy plants to sustain growth, before carbon-free alternatives are able to catch-up. Power plants being meant to operate for at least 40 years (see appendix A), the closing of previously built units will be delayed through time and a quick transition become impossible. In my paper I argue that this mechanism is at the origin of the directed technical change delay observe in the energy sector. Simultaneously, I look at a possible explanation for the EU/US divergence observed in figure 1. The energy transition is required to limit the risks of climate change, but governments may have different views about risks

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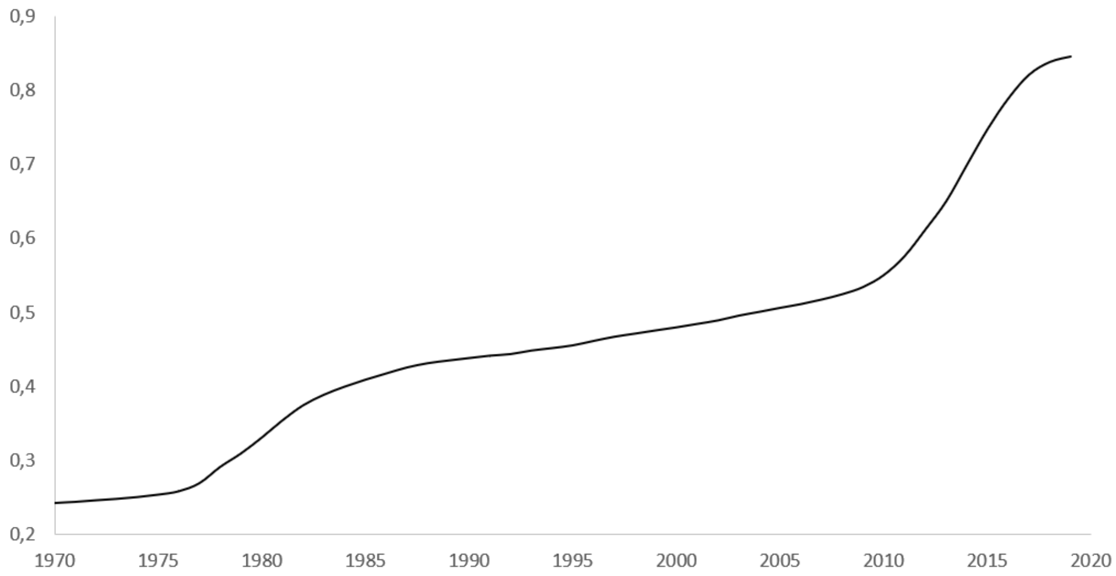
[3] In the 80's the levelized cost of energy was smaller for fossil fuels. Coal, gas and oil create more energy for a cheaper price than renewable



Author computations based on USPTO data

Figure 2: Technological delay of the renewable sector

implied by pollution. I am exploring the impact of a divergence in beliefs between the "pessimistic" European Union[4] and the "optimistic" United States[5] on damages implied by climate change. By applying different values to the damage function parameter, I show how it can shape capital accumulation in each sector.

My paper studies the existence of a lasting capital mechanism able to explain the delayed impact of biased technical change on the energy mix. For this purpose I use a multi-sector exogenous growth model with climate economics *à la* Nordhaus and Boyer (2000) and embodied technical change *à la* Krusell (1998). The two energy sectors differ in the inputs needed to produce one unit of capital ; productivity difference

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[4] "The atmosphere is warming and this is affecting citizens already now. European citizens see climate change as a serious problem and want to see increased action. Climate Change is having an increasingly severe impact on our planet's eco-systems and biodiversity" *European climate law*,03/04/2020

[5] From the Byrd-hagel resolution in 1997 to presidential declarations "It'll start getting cooler, you just watch" (president Donald Trump about global warming when California was burning)

between these sectors being exogenous. Technological progress is embedded in new capital units, each investment is then meant to stay into the economy before it fully depreciates after 40 years (Appendix A). In my analysis I follow the idea of Lennox and Witajewski-Baltvilks (2017) by adding the embodied technical change structure. Whereas their paper looks at the policies required to incentivise firms to redirect innovation toward carbon-free technology, and at the effect of embodied technical change compared to disembodied in the framework of Acemoglu et al. (2012), I depart from directed technical change literature by using a model of structural change close to Acemoglu and Guerrieri (2008) and Genna et al. (2019). I consider technology is already redirected toward the clean sector and I then add a Nordhaus' damage function, following the one used by Golosov et al. (2014). Utilization of fossil sources of energy increases the stock of pollution, which has a negative impact on the GDP. The presence of both an exogenous technology differential and a negative externality incentivize the economy to rely more intensively on the carbon free alternative but I will show it is also compatible with an increasing use of fossil energy.

This paper has 3 main contributions. Firstly, I show there exists a lasting capital effect able to explain the persistence of fossil energy accumulation in the US. In the early 80's the use of carbon-free alternatives was delayed compare to fossil energy and were noncompetitive, creating a trade-off between efficient polluting energy and less efficient carbon-free alternative. The technology differential takes time to be corrected and the economy continues to invest in fossil energy in order to sustain growth. This trade-off, paired with an extended lifetime of power plants tends to slow down closing of fossil power-plants in the economy, explaining the energy path followed by both the EU and the US.

Secondly, I show that the divergence between the EU and the US renewable energy path can be explained by divergent beliefs on the risks of climate change. I consider a pessimistic view for the EU and an optimistic one for the US, based on Kyoto protocol ratification decisions. This assumption allows to replicate the EU and the US trajectory differential in accumulation of fossil capital and exhibits very different timings of energy transition. In appendix C I show that if the EU overestimates future damages and the US underestimates them, the EU will be richer than the US after 2050. From a GDP point of view, it is less problematic to overestimate damages in the long run.

Lastly, I provide a simple, adaptive and somehow original theoretical framework of growth to study energy transition. The final good being produced through a Cobb-Douglas-CES production function, properties differ from a more standard case. I focus on the transitional process but I also characterize the long-run equilibrium of the model, providing the set of Non-balanced growth rate of my dynamic model. It is a flexible framework which can be enhanced with new assumptions like damage uncertainty of pollution, endogenous scrapping of capital or any other relevant hypothesis in the context of the energy transition.

For the theoretical analysis I use an exogenous growth model in a formulation close to Acemoglu and Guerrieri (2008), with two capital sectors. The final good, which can be either consumed or invested in the intermediary sectors, is produced using labor and capital. In my paper, capital can be viewed as total energy production and it is an aggregate of 2 intermediate inputs, “clean” (carbon-free energy) and “dirty” (fossil energy) capital. These two are produced using an investment specific accumulation equation *à la* Krusell (1998), with a technology differential variable set on “clean” investment. Technology differential is characterized by the relative performance of “clean” investments compare to “dirty”, investing one unit of the final good in the “dirty” sector creates one unit of capital, while it depends on the state of the technology in the “clean” sector. By construction I insure "clean" capital to be less efficient at the starting period. Technological progress is exogenous and is embodied in new units of capital as in Greenwood et al. (1997). Use of "dirty" capital emits pollutants in the atmosphere, added to the pollution stock of the economy. Finally, pollution stock has a direct impact on GDP though a damage function equivalent to Golosov et al. (2014), which is an exponential version of Nordhaus' mapping from pollution to damages. Solving the social planner allows me to simulate both the "dirty" capital stock and the share of "clean" energy in the energy mix. By doing so I am able to reproduce dynamics of the US and the EU, explaining the delayed impact of directed technical change.

Energy questions with embodied technical change have been somewhat treated by vintage capital literature, with papers by Hritonenko and Yatsenko (2012) or Díaz and Puch (2019). These papers look at the impact of energy price shocks on macroeconomic aggregates when energy efficiency is dependent of the capital vintage. Lennox and Witajewski-Baltvilks (2017) also use embodied technical change, they study optimal policy to redirect innovation toward carbon-free technology instead of polluting one. They show that with embodied technical change there is a difference for optimal tax, subsidies. Pollution damages are also greater in their model compare to a disembodied framework. My approach uses embodied technical change to study its impact when technological progress is already biased. My methodology is then closer to Greenwood et al. (1997) but with an emphasis on the energy transition while the latter is developed to account for post-war growth differential in the US. Additionally, my paper shows embodied technical change may have a negative impact on transition speed, while in the existing literature it is use to account for growth processes.

A second branch of the literature tries to compute the optimal tax rate of the economy and estimate the social cost of carbon. Golosov et al. (2014) uses a DSGE model to show that the optimal tax rate is proportional to GDP when some plausible assumptions hold, making taxation dynamic. Li et al. (2016) enhances this framework by adding an uncertainty measure on future damages from pollution to GDP, using robust control theory. The robust path slows down significantly the use of coal in the economy but the carbon tax is still dynamic. Acemoglu et al. (2016) builds a tractable microeconomic model of endogenous growth, estimated with microdata, to study optimal environmental policy required to accompany the energy transition, they find that relying only on a carbon tax or delaying the intervention has significant welfare costs on the economy. Acemoglu and Rafey (2018) assess the risks implied by geoengineering alternatives to reduce climate change damages as a way to only postpone the problem instead of solving it. In my paper I am also able to develop a dynamic taxation scheme but I differ in the theoretical framework use, my theory is based on a structural change model instead of directed technical change, the social cost of carbon is then implicit and characterized by the damage function of the economy.

The paper is organized as follow, section 2 details the economic model of structural change while section 3 characterizes optimal growth path and theoretical results. Section 4 presents calibration and numerical results of the paper.

2 The model

To study the drivers of the energy transition I develop a multi-sector growth model of structural change with both exogenous growth and climate economics. The baseline model is a mix of Lennox and Witajewski-Baltvilks (2017) and Acemoglu and Guerrieri (2008). Time is continuous and intermediate energy capital is accumulated through an investment specific accumulation equation, which is a continuous version of Krusell

(1998). Structural change comes from the supply side and is then induced by a price effect from a productivity differential. This mechanism is therefore close to Ngai and Pissarides (2007) but is also shaped by the presence of a negative externality from pollution. As in the directed technical change literature, the presence of a negative externality incites the economy to limit its investment toward the polluting sector. The intermediary "clean" and "dirty" sectors are aggregated through a CES function to produce the capital good of the model, associated to labor within a Cobb-Douglas function to produce the final good. Intermediary sectors are imperfect substitutes and here I am focusing on the case of substitutable inputs, based on the work by Papageorgiou et al. (2017).

2.1 Household

The representative household maximizes an instantaneous separable logarithmic utility function by choosing her consumption and labor participation, discounted through time.

$$U = \max \quad \int_0^{\infty} (ln(c_t) - \chi ln(L_t))e^{-\rho t}dt \tag{1}$$

Where ρ is the discount factor, c_t the instantaneous consumption, L_t labor participation and χ is a scale parameter for disutility of work. Preferences are homogeneous and compatible with both exogenous and endogenous growth. I exclude the possibility to add environment quality in the utility function, it is let for future research.

The representative household also owns firms and decide on the amount to invest on new machines, therefore she maximizes her lifetime utility subject to the following budget constraint,

$$c(t) = Y(t) - I(t) \tag{2}$$

Where $Y(t)$ is the production of the economy, and $I(t)$ is the total amount invested in intermediate goods. Therefore, the revenue can be either consumed or invested by the representative household. $I(t)$ will be used to invest in two intermediate goods, either “clean” or “dirty” capital and also serve as savings.

2.2 Production sector

The final good is produced through a standard Cobb-Douglas function, without aggregate technological progress. Labor, $L(t)$, and capital, $K(t)$, are used with constant return to scale. In the model the final good is also used as the numéraire.

$$\tilde{Y}(t) = L(t)^{1-\alpha} K(t)^{\alpha}$$

Where $0 < \alpha < 1$ is the capital intensity of final good production, $L(t)$ is labor and $K(t)$ is an aggregate of “dirty" and “clean" capital. These two intermediary goods represent energy capital units and are aggregated through a CES function to produce the final capital good.

$$K(t) = (K_c(t)^{\sigma} + K_d(t)^{\sigma})^{\frac{1}{\sigma}} \tag{3}$$

$K_c(t)$ and $K_d(t)$ are respectively clean and dirty capital, and are considered as imperfect substitutes. They are produced using an investment specific accumulation equation *à la* Krusell (1998). $-1 < \sigma < 1$ is a transformation of the elasticity of substitution between clean and dirty inputs, such as $\sigma = \frac{\varepsilon-1}{\varepsilon}$ where ε is the elasticity of substitution.

Assumption 1 *“Clean” and “dirty” capital are substitutable inputs,* $0 < \sigma < 1$

Assumption 1 is based on the paper by Papageorgiou et al. (2017) in which they show that in the framework of Acemoglu et al. (2012) “clean” and “dirty” capital units are substitutable. Based on this finding I will only consider the substitutable case between the two intermediate inputs, which make them imperfect substitutes. This framework will create a trade-off mechanism on capital use that will be derive from investments decisions.

Accumulation of both type of capital is done using an investment specific accumulation equation, which is a continuous time version of Greenwood et al. (1997) or Krusell (1998), as mentioned in Greenwood and Jovanovic (2001). Technological progress is embodied, new technologies are incorporated in new capital units and are unable to spread over already existing capital. I then have the following accumulation equations:

$$\dot{K}_d = i_{dt} - \delta K_{dt} \tag{4}$$

$$\dot{K}_c = q_t i_{ct} - \delta K_{ct} \tag{5}$$

Where δ is the depreciation rate of capital and is the same for both type and i_j is the amount invested in new machines for sector $j = c, d$. As observed in 2, the representative household decides how much she wants to invest in the acquisition of new capital units, this amount is then allocated to "clean" and "dirty" capital accumulation through the following equality,

$$I(t) = i_c(t) + i_d(t)$$

The amount invested is optimally allocated between the two different type of capital, such that the budget constraint can be rewritten as:

$$c(t) = Y(t) - i_c(t) - i_d(t)$$

The variable q in equation 5 is the relative efficiency of clean sector, it determines the amount of "clean" capital produced with one unit of the final good. Here I do not assume there is no technological progress within the "dirty" sector, this $q(t)$ is a variable of relative performance of the "clean" sector compare to the "dirty" one. The first paper using this double accumulation equation is the quasi-accountability paper by Greenwood et al. (1997) in which they use embodied technical change to account for post-war growth in the US. In their paper they consider 2 types of capital, structure and equipment, the latter being the one concerned by embodied technical change. As they argue, the relative performance variable, $q(t)$, might be interpreted in two different ways: i) $1/q$ could be interpreted as the relative cost of producing one unit of "clean" capital in terms of final output. ii) q represents the relative productivity of a new unit of "clean" capital, and, because I consider technological progress is biased in favor of the clean sector, it is increasing over time. However, the following assumption ensures a productivity gap in favor of the "dirty" sector at the starting point.

Assumption 2 *The initial condition of relative efficiency in the clean sector is such that:* $q(0) < 1$

Assumption 2 creates inertia in the dirty sector, it ensures "dirty" capital to be more efficient in a first time creating the actual trade-off in energy investment: fossil sources are cheaper but more polluting on the long term while carbon-free alternative are more expensive. Renewable technology catch-up through an exogenous process, the relative efficiency evolves at a constant rate γ such that,

$$\dot{q}_t = \gamma q_t \tag{6}$$

Imposing a restriction on q_0 allows existence of both capital at the same time but only the “dirty” type will have an impact on the pollution level of the economy, creating a negative externality. As it will be detailed in the next section, the production suffers from the level of pollution in the economy, this mechanism is introduced through a Nordhaus’ damage function.

I can then incorporate equation 3 in the final good production function to obtain a Cobb-Douglas-CES form in the final good sector.

$$\tilde{Y}_t = L_t^{1-\alpha} \left(K_{ct}^{\sigma} + K_{dt}^{\sigma}\right)^{\alpha/\sigma} \tag{7}$$

The final good is produced using both labor and a capital good aggregated from both type of intermediate inputs. It ends up with a “CES-Cobb-Douglas” formulation which is not standard in macroeconomic literature. Capital dynamics implied by this formulation will be at the core of the structural change mechanism. $1/q$ being the price of investment in the “clean” sector and q being increasing it will results into a structural change mechanism in favor of the “clean” sector due to assumption 1, but it will be shaped by the imperfect substitution imposed by the CES part of the function, additionally to the capital share of the model. As mentioned, the use of “dirty” capital emits pollutant in the atmosphere that are added to the pollution stock. The latter being a negative externality it will reduced the level of GDP through a damage function introduced in the following section.

2.3 Pollution stock and damage function

Following the literature on energy transition and climate change, see Acemoglu et al. (2012), Golosov et al. (2014), Li et al. (2016), Nordhaus (2014b) or Lennox and Witajewski-Baltvilks (2017) among other, a pollution stock equation is introduced. Carbon accumulation, through use of “dirty” capital has a negative impact on the economy. Justification of this effect can be found in Graff Zivin and Neidell (2012), Chang et al. (2019), Pindyck (2019) or Nordhaus and Boyer (2000). The baseline model assumes “dirty” capital is the only source of new pollution, accumulated in the global carbon stock. The environment is regenerating itself at a constant rate through photosynthesis and other carbon absorption mechanism, $S(t)$ represents the carbon stock of the economy and is described by,

$$\dot{S}(t) = -\varphi_1 S(t) + \varphi_2 K_d(t) \tag{8}$$

Where φ_1 is the natural rate of absorption and φ_2 the linear transformation rate of dirty capital into carbon. If “dirty" capital stock falls under a sufficiently low level, the environment starts to decarbonize. This assumption about pollution stock might seems too simple but allows to reproduce short term behavior of the economy, which is the objective here compare to asymptotic properties. Nevertheless, section 5 introduces a multi-level pollution stock equation, with permanent and transitory carbon emissions like it is done in Li et al. (2016) or Adao et al. (2017).

In this model, pollution stock has a negative impact on GDP, the damage function is an exponential version of Nordhaus mapping and is the same than Golosov et al. (2014), such that GDP is given by $Y(t) = (1 - d(S(t)))\tilde{Y}(t)$ where $d(S(t))$ is the fraction of GDP lost because of pollution. The damage function is then characterized by $1 - d(S(t)) = exp(\theta(S(t) - \bar{S}))$ with $\bar{S}$ the pre-industrial level of pollution and θ the scale parameter for the mapping from pollution to damages to GDP. In the calibration section I will dissociate 2 different values for this parameter according to the region considered. As evoked in the introduction, I will consider a pessimistic European Union toward the risks implied by climate change and pollution accumulation, based on the European climate law, and based on Trump's declaration about global warming I will consider the US is more optimistic toward damages from pollution to GDP. Everything being considered, I can rewrite the final expression for GDP as:

$$Y_t = L^{1-\alpha} \left(K_c^\sigma + K_d^\sigma\right)^{\alpha/\sigma} \exp(-\theta(S_t - \bar{S})) \tag{9}$$

This model aims at computing the timing of the energy transition according to 3 phenomena. First, the decreasing price of investment in the "clean" sector favors the use of "clean" capital, which will be observed as an increasing share for this type of input. Second, the dirty capital sector exhibits an higher relative efficiency in a first time, characterizing the advancement of fossil technologies, at the starting point the economy is still relying massively on fossil energy, which have a lasting effect due to capital lifetime. Third, the accumulation of dirty capital increases the damages to GDP, there will be a trade-off between growth and environment preservation. These three effects are characterizing the optimal growth of the economy and evolution of the energy transition, the dynamic implied by this model is consistent with the data about fossil energy use and renewable energy share. The embodied technical change added to the CES-Cobb-Douglas production function creates a lasting effect in favor of the "dirty" capital and postpone the energy transition implied by biased technical change. Each effect will be then described in the simulation section of the paper.

3 Optimal energy transition

3.1 The planner problem

We analyze the first best solution, the social planner maximizes utility of the representative household.

$$
\begin{aligned}
\max_{L_t, i_{ct}, i_{dt}} \quad & \int_0^{+\infty} \left(ln(c_t) - \chi ln(L_t)\right) e^{-\rho t} dt \\
s.t. \quad & (3) - (7) \text{ and} \\
& c_t = y_t - i_{ct} - i_{dt}
\end{aligned} \tag{10}
$$

The central planner solves the Hamiltonian in current value,

$$
\begin{aligned}
\mathcal{H} &= ln(Y - i_c - i_d) - \chi ln(L) + P[L^{1-\alpha}\left(K_c^{\sigma} + K_d^{\sigma}\right)^{\alpha/\sigma} e^{-\theta(S_t - \bar{S})} - Y] \\
&+ P_d[i_d - \delta k_d] + P_c[q i_c - \delta k_c] + Q_t[-\varphi_1 S_t + \varphi_2 K_{dt}]
\end{aligned}
$$

Giving first order conditions,

$$
\begin{aligned}
Y &= \frac{\chi}{(1-\alpha)P} \qquad (11) \\
P &= P_d = qP_c = \frac{1}{Y - i_c - i_d} \qquad (12)
\end{aligned}
$$

Equation (12) shows shadow price of production and dirty capital are the same and they equal the product $P_c q$, at t_0 $q(0) < 1$ means $P_c(0) > P_d(0)$ validating actual empirical facts of cheapest fossil energy compared to renewable. Equation (11) show the direct relationship between the shadow price of the final good and production itself in a straightforward equation.

Dynamic equations of state variables are the following,

$$\begin{aligned}
\frac{\dot{P}_c}{P_c} &= \rho + \delta - \alpha q K_c^{\sigma-1}\frac{Y}{K_c^\sigma + K_d^\sigma} \\
\frac{\dot{P}_d}{P_d} &= \rho + \delta - \alpha K_d^{\sigma-1}\frac{Y}{K_c^\sigma + K_d^\sigma} - \varphi_2\frac{Q}{P_d} \\
\frac{\dot{Q}}{Q} &= \rho + \varphi_1 + \frac{\theta Y P_d}{Q}
\end{aligned}$$

As expected evolution of both shadow prices are similar but differ in the presence of q for the clean sector and the term $-\varphi_2\frac{Q}{P_d}$, the decentralized equilibrium will show that the latter is equivalent to the carbon tax. Further in the paper we will see $Q < 0$, ensuring the dynamic equation for the shadow price of "dirty" capital to be bigger with carbon emissions than without.

In order to go further in the analysis the ratio κ is introduced, such that

$$\kappa \equiv \frac{K_c^\sigma}{K_c^\sigma + K_d^\sigma}$$

This ratio will be the proxy for energy transition, the closer from 1 κ is, the higher the share of "clean" energy. Calibration of the model will match $\kappa(0)$ with renewable share in energy mix in 2010.

This proxy allows to rewrite dynamic equation of both P_d and P_c,

$$\frac{\dot{P}_c}{P_c} = \rho + \delta - \alpha q \kappa \frac{Y}{K_c} \tag{13}$$

$$\frac{\dot{P}_d}{P_d} = \rho + \delta - \alpha(1-\kappa)\frac{Y}{K_d} - \varphi_2\frac{Q}{P_d} \tag{14}$$

Shadow price of both type of capital depends on clean energy ratio. As one might expect, the closer from 1 κ is, the higher the growth difference will be. Next section will characterize the steady growth path and transitional patterns of the model, κ will be the central element of the analysis as it drives energy transition and all other variables.

3.2 Steady growth path

This section aims at computing the asymptotic behavior of the model, to derive steady growth rate is the first step before characterizing the transitional path of the economy. Model behavior is in line with some papers of structural changes like Acemoglu and Guerrieri (2008) or Genna et al. (2019). Economic transition and structural change occur along the growth path of other variables. In this paper, the energy transition takes place along with constant growth of prices, labor and GDP. As mentioned above, κ is the proxy for energy transition, therefore the final goal of this section is to derive the asymptotic and transitional growth rate of the “clean" capital ratio. By differentiating its definition one obtains,

$$\frac{\dot{\kappa}}{\kappa} = \sigma(1-\kappa)(g_{K_c} - g_{K_d})$$

In the following the term g_x refers to growth rate of variable x. κ growth rate depends on its own value and on the difference between “clean" and “dirty" capital growth. If “clean" capital grows faster (slower) than “dirty" one, κ is increasing (decreasing), and there are no inconsistent behavior because if κ is equal to one, the growth rate is equal to 0. To derive the complete characterization of κ's growth rate one uses the following statement: growth rate of shadow prices are assumed to be constant, such that $g_{\dot{P_c}} = g_{\dot{P_d}} = 0$. Using this property on equations (13) and (14) gives,

$$\begin{aligned} g_{K_c} &= \gamma + \frac{\dot{\kappa}}{\kappa} + g_Y && (15) \\ g_{K_d} &= g_Y - \frac{\dot{\kappa}}{\kappa}\frac{\kappa}{1-\kappa} + \frac{\varphi_2 Q K_d}{\alpha(1-\kappa)\chi}(g_Q - g_{P_d}) && (16) \end{aligned}$$

g_{K_d} can be simplified by differentiating (11) and using the following proposition,

Proposition 1 *The shadow price of pollution, Q, is always at its steady-state value and is negative.*

Proof. Using (11) and (12), the shadow price of pollution growth can be rewritten as $\frac{\dot{Q}}{Q} = \rho + \varphi_1 + \frac{\theta\chi}{(1-\alpha)Q}$, it depends on the value of Q and on model's parameter. As for every variable of the model, asymptotically this growth rate should be constant, $\dot{g}_Q = 0 \Leftrightarrow g_Q = 0$. One is able to derive $Q^* = -\frac{\theta\chi}{(1-\alpha)(\rho+\varphi_1)}$, the steady state value of the shadow price of pollution, and this value is negative. In the long-run, Q must converge to its steady-state level, however it appears that if Q deviates from this value, its trend is explosive and cannot converge. The conclusion is there exist only one value for the shadow price of pollution leading to a stable steady-state, Q is a jump variable and is always at its steady-state level. □

At first, proposition 1 seems counter-intuitive, one should expects the constraint on pollution stock to variate with the level of pollution, to capture the constraint induced, by definition, by a shadow price. However, each new unit of pollutant emitted in the atmosphere has the same impact on this economy because of (11) and (12), the marginal impact of pollution is expected to co-move with the level of GDP($\frac{\partial Y(t)}{\partial S(t)} = -\theta Y(t)$), but the equivalence between $Y(t)$ and $P(t)$ is cutting this co-movement. The increasing impact of one unit of pollution is compensate by a drop in prices, such that the constraint is always the same, $Q(t)$ is then constant.

Equation (16) can be rewritten using proposition 1 and differentiating (11)

$$g_{K_d} = g_Y \left(1 - \frac{\varphi_2 \theta K_d}{\alpha(1-\kappa)(1-\alpha)(\rho+\varphi_1)}\right) - \frac{\dot{\kappa}}{\kappa}\frac{\kappa}{1-\kappa} \qquad \textbf{(15*)}$$

Output growth, dirty capital level and evolution of the clean share are the three variables defining “dirty" capital growth rate. Because of equation (11) and the assumption made on g_{P_d} the output growth rate is constant, then the only variables at play are K_d and κ. Combining this result with equation (15) in κ's growth rate and rearranging it characterizes the rhythm of energy transition,

$$\frac{\dot{\kappa}}{\kappa} = \frac{\sigma}{1-\sigma}\gamma(1-\kappa) + \frac{\sigma\varphi_2\theta K_d}{(1-\sigma)\alpha(\rho+\varphi_1)}g_Y \tag{17}$$

This expression can be divided in 2 parts, $\frac{\sigma}{1-\sigma}\gamma(1-\kappa)$ represents the transition implied by technology, it relies on γ, the efficiency differential between the two intermediates, and on the imperfect substitution reflected by the parameter σ. Without any environmental damages, growth rate of κ is only defined by this first part. $\frac{\sigma\varphi_2\theta K_d}{(1-\sigma)\alpha(\rho+\varphi_1)}g_Y$ represents the second part in which transition is implied by damages from pollution. The term $\theta\varphi_2 K_d$ represents the damages induced by “dirty" capital on GDP, the higher it is the faster transition is, Pollution has an acceleration effect on the energy transition. There are also 2 straightforward remarks, i) it appears that the bigger κ is, the slower the transition, which is due to scale effect ; ii) $\frac{\sigma}{1-\sigma}$ is present in each part, it represents the substituability effect implied by the CES function for capital aggregation.

κ growth rate depends on $(1-\kappa)$ and on K_d, it is straightforward that asymptotically κ will tend to 1. Growth rate of the clean ratio proxy is always larger than 0 and κ cannot be higher than 1 by construction. We have $\kappa^* = 1$ as asymptotic condition. When $t \to \infty$ the clean technology will be the dominant on the energy market, it does not mean that dirty technologies will totally disappear but its level will be non significant in the energy mix. Their existence will be discussed below and is of first importance when it comes to the energy sector. Structural change occurs, production becomes relatively more green but if we continue to buy new dirty inputs, it will not reduce greenhouse gas emissions.

Using the clean capital ratio result, I am able to derive the other growth rates of the economy. When $t \to \infty$ economy tends toward the Non Balanced Growth Path (NBGP) detailed in following theorem.

Theorem 1 *Under assumptions 1 and 2, the set of Non Balanced Growth Rates (NBGR) of this model are as follow:*

$$\begin{aligned} g_Y &= \frac{\alpha\gamma}{1-\alpha} \quad ; \quad g_{K_c} = \gamma + g_Y \quad ; \quad g_{K_d} = g_Y - \frac{\sigma\gamma}{1-\sigma} \\ g_{P_c} &= -g_Y - \gamma \quad ; \quad g_{P_d} = -g_Y \quad ; \quad g_{i_c} = g_Y \\ g_{i_d} &= g_{K_d} \end{aligned}$$

Proof: see Appendix (in construction) □

Theorem 1 shows the set of non-balanced growth rates, when energy transition has been completed $\kappa \to 1$, some features can be derived from the asymptotic behavior of the model. In the long run, “dirty" capital might still be increasing even if its share becomes non significant, if $\alpha > \sigma$ one obtains $g_{K_d} > 0$ which has no consequences for the energy transition but has some disastrous effect on environment quality, through the carbon stock. In the illustrative calibration in section **4.4**, $\alpha = 0.4$ and $\sigma = 0.44$, it coincides with an asymptotically decreasing growth of “dirty" capital. Nevertheless, it seems difficult to imagine an infinitely increasing fossil energy due to resources limitations, however this paper omits intentionally to include a resource stock because it is not the problematic here. Jaffe et al. (2011) survey about world oil reserves lets one think resource constraint will not be the major problematic of tomorrow. The idea of a negligible Hotelling effect in the short run is also present in Hart and Spiro (2011), in which they argue that scarcity rents do not dominate prices of fossil resources . Therefore, the main limitation of the non-balanced growth path is the lack of an Hotelling rule, but in the short and middle-run this absence seems less problematic.

3.3 Stability analysis

In the previous section I have characterized the asymptotic properties of the model, in this section I will show this Non-balanced growth path (NBGP) is stable and unique. For this puprpose I will reformulate the dynamical system of my model by using the normalization of variables introduced by Caballé and Santos (1993). I obtain the stationarized NBGP by deflating my variables by their long run growth rate, I then obtain: $k_c(t) = K_c(t)e^{-g_{K_c}}, k_d(t) = K_d(t)e^{-g_{K_d}}$ and $p_c(t) = P_c(t)e^{-g_{p_c}}$, for all $t > 0$, with $k_c(t), k_d(t)$ and $p_c(t)$ the stationarized values of $K_c(t), K_d(t)$ and $P_c(t)$.

Substituting these values into (5), (6) and (13) I obtain a stationarized system of differential equations able to characterize the equilibrium path. The expression of this 3 variables is sufficient to describe the full dynamic of the model, and in this stationarized system I am still assuming an elasticity of substitution such as: $0 < \sigma < 1$.

Lemma 1 *Let assumptions 1 and 2 hold. Along a stationarized equilibrium path and for any given q_0, "clean" capital k_c, "dirty" capital k_d and price of the "clean" capital p_c*

are solutions of the following dynamical system

$$\begin{aligned}\frac{\dot{k}_c}{k_c} &= \frac{q_0 i_c(k_c,k_c,p_c)}{k_c} - \delta - g_{k_c} \\ \frac{\dot{k}_d}{k_d} &= \frac{i_d(k_c,k_d,p_c)}{k_d} - \delta - g_{k_d} \\ \frac{\dot{p}_c}{p_c} &= \rho + \delta - \alpha\kappa(k_c,k_d)\frac{\chi}{(1-\alpha)p_c k_c} - g_{p_c}\end{aligned} \tag{18}$$

Proof: See appendix (in construction).

I now have a stationarized dynamical system characterized in Lemma 1, I can prove the existence of a unique steady-state that will corresponds to the set of non balanced growth rate given by theorem 1.

Theorem 2 *Let q_0 be given and suppose assumptions 1 and 2 hold. There exists a unique steady-state (k_c^*, k_d^*, p_c^*) solution of the dynamical system (18).*

Proof: See appendix (in construction).

Theorem 2 proves there exists a unique and stable steady state for the “clean” capital stock k_c, the “dirty” capital stock k_d and the price of “clean” capital p_c.

3.4 Decentralized equilibrium

Previous section aimed at solving the social planner problem, correcting for pollution damages from use of a “dirty" technology. To solve the decentralized equilibrium will help at characterizing the optimal tax rate required for this economy to reach the optimal growth path. In this model there is only one externality, pollution from use of dirty capital, which needs one instrument to be corrected, the tax rate. The final good is used as a numéraire, its price is normalized to 1.

3.4.1 Household

The representative household owns the intermediate firms and lend his work to the final good producer, he exhibits the same utility function than for the social planner but his budget constraint is: $c(t) = L(t)w(t) + r_c(t)K_c(t) + r_d(t)K_d(t)$. The representative household maximizes its utility with respect to consumption, labor and capital investment,

$$\begin{aligned} U = \max_{L(t),c(t),i_c(t),i_d(t)} \quad & \int_0^{+\infty} \left(ln(c_t) - \chi ln(L_t)\right) e^{-\rho t} dt \\ s.t. \quad & Y(t) = L(t)w(t) + r_c(t)K_c(t) + r_d(t)K_d(t) \\ & c(t) = Y(t) - i_c(t) - i_d(t) \\ & \dot{K_c} = q(t)i_c(t) - \delta K_c(t) \\ & \dot{K_d} = i_d(t) - \delta K_d(t) \end{aligned} \tag{19}$$

First order conditions state:

$$\frac{\chi}{L(t)} = w(t)P(t) \qquad ; \qquad P_c(t)q(t) = P_d(t) = P(t)$$

Where $P(t), P_d, P_c$ are the multiplier of respectively the budget constraint, the “dirty" capital accumulation and the “clean" capital accumulation. FOC conditions in the decentralized equilibrium are similar to social planner.

Dynamic equations are also similar to what one can found in the centralized equilibrium,

$$\begin{aligned}\frac{\dot{P}_c}{P_c} &= \rho + \delta - q r_c \\ \frac{\dot{P}_d}{P_d} &= \rho + \delta - r_d\end{aligned}$$

Household do not take into account damages from pollution to the global production, the externality will need to be corrected by a tax rate as it will be shown below. The main difference here is for dynamic equation for the shadow price of the “dirty" capital because in the suboptimal equilibrium adverse pollution effects are not taken into account.

3.4.2 Final good

The final good firm maximizes its profit, it sells its production, buy work of the household and rent capital units,

$$\max_{L(t),K_c(t),K_d(t)} \pi = L^{1-\alpha}\left(K_c^\sigma + K_d^\sigma\right)^{\alpha/\sigma} - w(t)L(t) - r_c(t)K_c(t) - r_d K_d(t) \tag{20}$$

Deriving first order conditions leads to,

$$w(t) = (1-\alpha)\frac{Y(t)}{L(t)} \tag{21}$$

$$r_c(t) = \alpha\kappa(t)\frac{Y(t)}{K_c(t)} \tag{22}$$

$$r_d(t) = \alpha(1-\kappa(t))\frac{Y(t)}{K_d(t)} \tag{23}$$

$w(t)$, $r_c(t)$ and $r_d(t)$ represent, respectively, wages, rental price of “clean" capital and rental price of “dirty" capital. The next section will look at the optimal tax rate needed to coincide with the socail planner equilibrium and correct for the externality.

3.4.3 Optimal tax rate

Pollution accumulation due to use of “dirty" capital destroys a share, $d(t)$, of the production such that $d(t) = 1-e^{-\theta(S(t)-\bar{S})}$ is the damage function. In order to correct for this externality a government needs to introduce a tax on “dirty" capital for decentralized equilibrium to coincide with central planner scheme.

The tax will apply on the rental price of dirty capital, a slower rate of return for each “dirty" unit slow-down the investment in this kind of capital. The government modifies the household maximization (17) such that the budget constraint becomes

$$Y(t) = L(t)w(t) + r_c(t)K_c(t) + (r_d(t) - \tau(t))K_d(t)$$

Using (23) and solving the new maximization for dynamic equation it appears,

$$\frac{\dot{P}_d}{P_d} = \rho + \delta - \alpha(1-\kappa(t))\frac{Y(t)}{K_d(t)} + \tau(t)$$

Comparing to the central planner results for the dynamic equation of the shadow price of “dirty" capital it appears clearly that the tax rate is such that,

$$\tau(t) = \frac{-\varphi_2 Q(t)}{P_d(t)}$$

Proposition 1 states $Q(t)$ is constant and negative, and it still hold in the decentralized equilibrium making the tax rate dependent of the shadow price of “dirty" capital. The lower $P_d(t)$, the higher the tax rate. $\tau(t)$ is inversely proportional to $P_d(t)$, and so is proportional to $Y(t)$ because of (11) and (12). Using this and proposition 1 the expression for the tax rate can be rewritten,

$$\tau(t) = \frac{\varphi_2 \theta}{\rho + \varphi_1} Y(t) \tag{24}$$

To reach the optimal growth rate asks for an increasing tax rate on “dirty" capital, proportional to production in the economy, such a result is in line with recent literature on the topic of taxation of fossil energies, like Golosov et al. (2014), Lennox and Witajewski-Baltvilks (2017) , Li et al. (2016) or Adao et al. (2017). The taxation weight has to be bigger as the price of “dirty" input is decreasing (see theorem 1), due to capital deepening, to maintain the increasing attractivity of the “clean” backstop. The next section calibrates the model using US data to provide a numerical analysis of the model.

4 Numerical illustration

4.1 Calibration

Compare to a Ramsey model, this paper differs in its double capital market with investment-specific accumulation equations and the presence of a damage function, linked to the emissions of pollutants. The model is then characterized by the parameters summarized in table 1 and by 3 initial conditions, $Y(0)$, $K_c(0)$ and $K_d(0)$. Technological progress is computed using IEA technology R&D budget, delivering detailed budget for each type of source. γ is a proxy of technology differential in the energy sector, its value is extracted computing growth differential between fossil and renewable technology in public budget. On the considered period (1980-2015), R&D budget for “clean" energy grows 2.5% faster than for “dirty" energy sources, growth differential is then calibrated such as $\gamma = 0.025$. The value for α, the labor share is given directly by the bureau of labor and statistics, its average on the considered period is such as $\alpha = 0.4$. Depreciation rate of capital and discount factor, respectively δ and ρ are calibrated following Barro and Sala-i Martin (2004), the values $\delta = 0.05$ and $\rho = 0.02$ are widely use in macroeconomic literature and calibration of Ramsey models, this paper does not innovate in this regard. The value for the elasticity of substitution is chosen following Papageorgiou et al. (2017), they estimate its value in AABH framework which is close to the one in this paper, regarding to their computations the elasticity of substitution is calibrated as $\sigma = 0.44$. And lastly, the 3 parameters associated to environment are calibrated using the last IAM model used by Nordhaus, such that $\varphi_1 = 0.1$, $\varphi_2 = 0.0228$ and $\theta = 0.02$. The latter will be subject to 2 different values according to the region considered. EU, will be more pessimistic and the damage parameter will be set at $\theta = 0.03$ while US keep the value present in the table. This parameter difference characterizes the difference of priors exhibiting by the two regions of interest: the Eu and the US.

Parameter	Value	Data
γ	0.025	IEA technology R&D budget (2000-2018)
α	0.4	Bureau of Labor statistics
σ	0.33	Papageorgiou et al. (2017)
δ	0.05	Barro and Sala-i Martin (2004)
ρ	0.02	Barro and Sala-i Martin (2004)
φ_1	0.1	*Nordhaus (2014a)*
φ_2	0.0228	*Nordhaus (2014a)*
θ	0.02	*Nordhaus (2014a)*

Table 1: Parameters value

This part tries to calibrate initial values for GDP, “clean" and “dirty" capital in 2010. At this date, according to world bank data, GDP per capita was 48 kUS$, using this and equation (2) one can compute the level of “clean" and “dirty" capital. According to

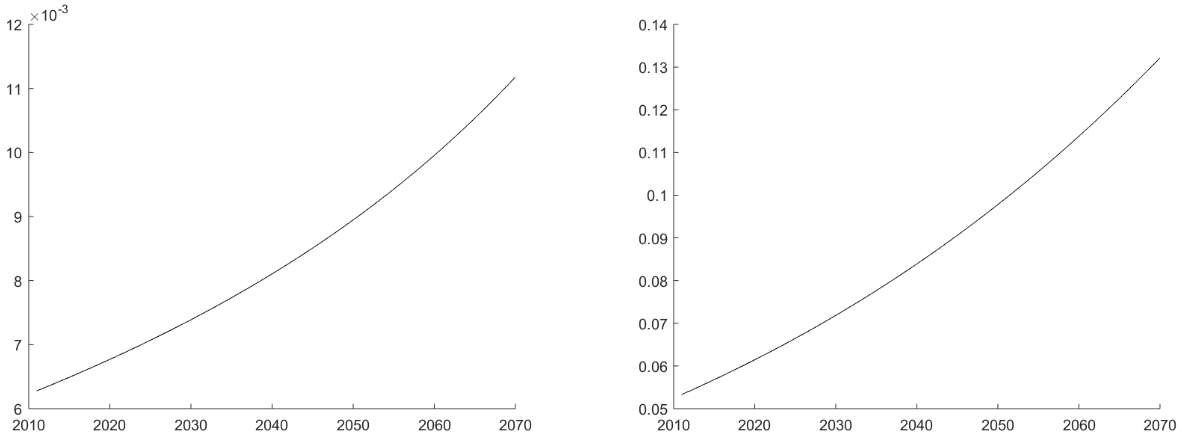
“US primary energy production by major sources", in 2010 renewable energies (without hydroelectricity, which is particular in the energy mix) represents 12.5% of the energy mix, we then solve $y(2010) = \left((\frac{0.125}{0.875}K_d(0))^\sigma + K_d(0)^\sigma\right)^{\alpha/\sigma}$. And then obtain $K_d(0) = 7.24$ and $K_c(0) = 1.03$. We can compute value of $\kappa(0)$ to obtain $\kappa(0) = 0.30$. Parameter values and initial conditions have been calibrated, the next part can now deal with the model simulations.

4.2 Simulations

This sections aims at providing some useful insight on the transitional pattern of this model, the asymptotic behavior of the model have already been described by theorem 1 but nothing was really clear about transition from initial conditions to Non-balanced growth path. Without any doubt the economy will switch from a fossil energy dominance to a renewable world, but in the presence of climate change the major problem is how long this switching will take. This section will first deal with the optimal tax rate of the economy, from whom the transitional timing will come from. And lastly, this model allows to derive the evolution of both “clean" and “dirty" capital level in the economy. A rising level of “dirty" capital would be problematic with respect to pollution emissions and threats of climate change.

Using equation (24), one is able to simulate the value of the tax rate, and it relative weight on the “dirty" capital rent. The latter is used to confront the real impact of the taxation instead of its absolute level. The tax rate diminishes the rent of “dirty" capital, simulating $\frac{\tau(t)}{r_d(t)}$ shows the real impact of taxation on “dirty" capital owners (figure 5).

The tax rate is increasing in absolute and relative terms. To be located on the



(a) Tax rate

(b) Weight of the tax rate

Figure 3: Optimal taxation

optimal path, the tax rate should have been set around 5% of the “dirty" capital rent in 2010, 6% in 2020 and reach 14% in 2070 if one wants to follow the optimal path. Such tax schedule is used if one wants to maximize growth in presence of environmental externality, other objectives might be considered, as minimizing the pollution without giving up to much on growth, it would end-up with a totality different tax schedule.

Next simulation (figure 4) displays the real share ($\frac{K_c}{K_c+K_d}$) of clean capital into the energy mix, for both EU and US. The evolution of the share of clean capital is close to the trend we observe in figure 1, for both countries. EU reaches 12% of clean energy around 2020 when US exhibits a 5% share. This result is only due to the difference of prior toward energy transition, explaining why the EU is increasing more rapidly its relative share of carbon-free energy. Following these paths, EU reaches 50% of “clean” technology before 2050, while it takes 25 more years for US to reach the same point.

The next plot (figure 5) is about absolute level of both “clean" and “dirty" capital and their evolution through time. An higher level of “dirty" capital should linked to an increase of pollutant emissions in the atmosphere

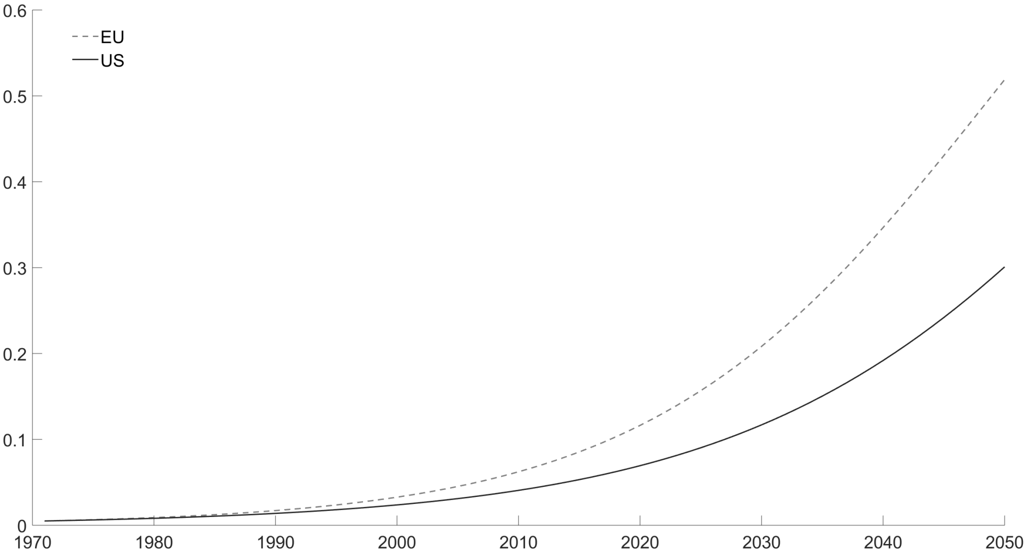


Figure 4: Share of clean technology across time, EU-US comparison

According to this simulated model, the level of dirty capital will continue to grow during 40 years for EU and 70 years for the US. It seems this model is able to replicate quite well what we observe in the data for both EU and the US. Looking at long term dynamic it seems that European continent can reduces heavily it dependence on fossil fuel before the end of the XXIth century while it seems more complicated for the US. Difference in views also affects the level of pollution emissions as figure 6 is showing.

According to the simulated values, the pollution stock vary quite heavily according to the point of view adopted about risks of climate change. The European stock starts to decrease in 2010 while US needs to wait until 2050. Adopting the optimistic view of the US means almost double the carbon stock present in the atmosphere between 1970 and 2050, while the European "green deal" asks to converge toward a carbon-neutral economy before 2050. However this model lacks of several features to reflect consistently the reality, in this framework I like the diversity of fossil sources which

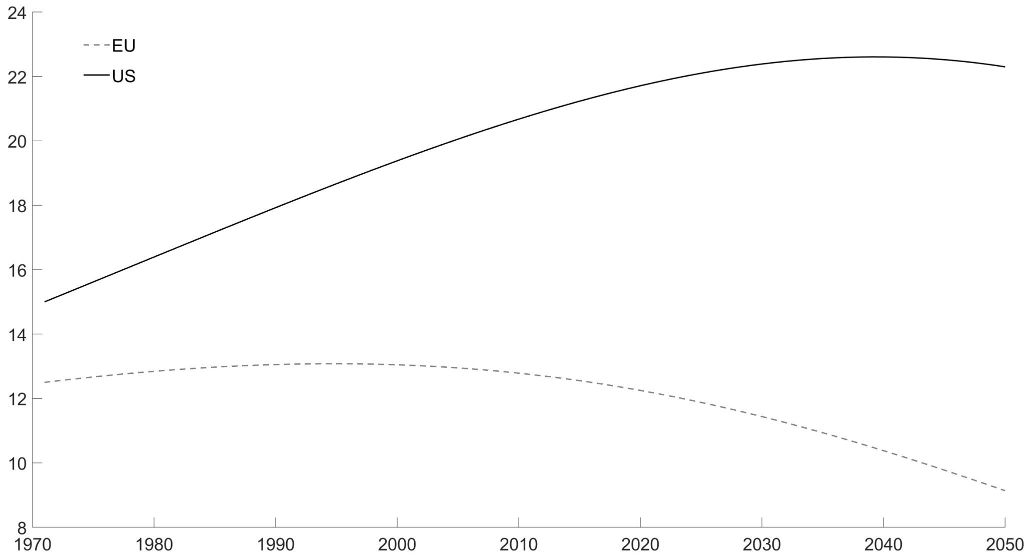


Figure 5: Use of “dirty” capital, EU-US comparison

does not emit the same amount of pollutant. Coal is known to emits more CO_2 than gas for the same amount of power, and US are actually relying mainly on oil and gas, instead of coal, which should reduce drastically the growth rate of pollution emissions.

The quantitative result that can be kept from these simulation is the very slow transitional process. Even in this simple framework with completely exogenous growth and expected outcome, there is an inertia effect from investment specific capital accumulation and initial conditions, calibrated on actual and historical data. Going back to the carbon lock-in argument of Unruh (2000), the pessimistic view would say that in practice we cannot escape carbon lock-in easily due to capital inertia. Looking at the actual discussion around carbon taxation and the work by Clements et al. (2013) and Coady et al. (2015), the economy is not so close for such a taxation scheme. However, the optimistic view would say this paper do not capture the complete inertia of the energy sector, nor political decisions, nor consumers behavior with respect to climate

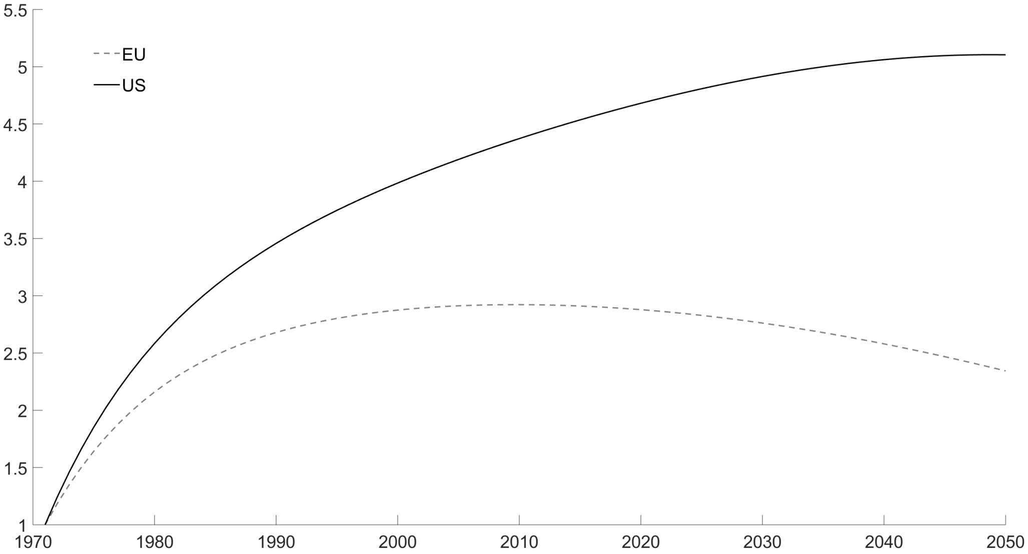


Figure 6: Pollution stock, EU-US comparison

change, nor uncertainties linked to the energy sector. Also, there are a couple of parameters of first importance in this model, like technology differential and damages to GDP, a government would be able to affect and change their value. Increasing public research, carbon capture storage or geoengineering technology may have significant consequences on the short and middle-run (on this topic, Acemoglu and Rafey (2018) shows that relying on geoengineering technology is suboptimal).

5 Conclusion

In this paper, I use an exogenous growth model with climate economics and embodied technical change to show that the existence of a lasting capital effect and difference in environmental prior can replicate patterns observe for EU and US in the energy data. The key mechanism here is a trade-off between efficient "dirty" capital and less efficient carbon-free alternative, which have a lasting effect due to the extended lifetime of capital units. While Eu seems able to reach its 2050 carbon-free objective, the optimistic view about climate change from US government tends to increase drastically the pollution level in the country. In this simple framework I've shown that in the context of the energy transition, we cannot rely only on technology even if it is biased in favor of the clean sector for a long period of time. As a further research I want to develop a Panel-VAR model to test the hypothesis of beliefs differential between regions on the use of dirty capital. For this purpose I will use the ratification of the Kyoto protocol as a measure of the belief in future risks of pollution emissions.

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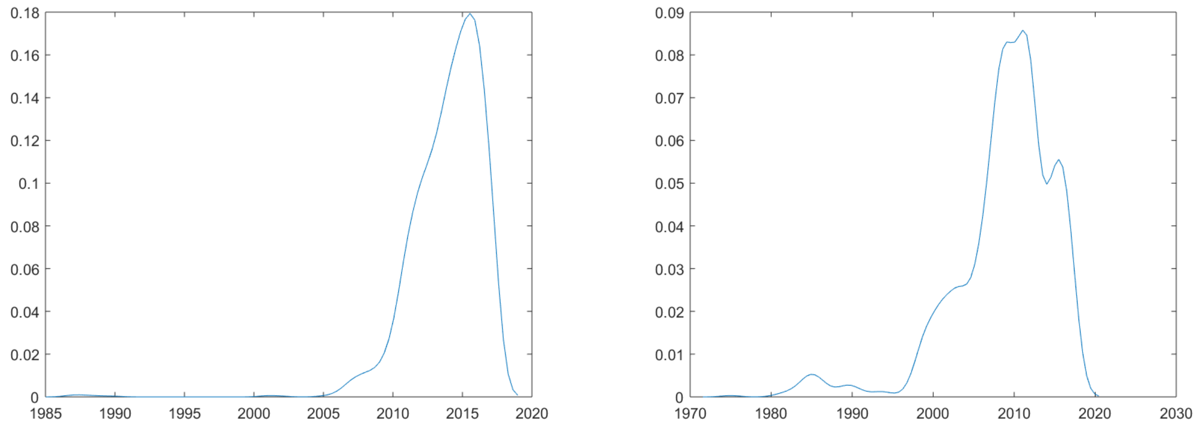
A Capital Inertia and lifetime of energy plants

This section aims to document more consistently capital inertia from energy power plants. The motivation of this paper is to look if the lifetime of energy power plants may be an issue for our transition from fossil fuel to renewable energies. Ii is argue that when capital units are long living, with embodied technology, there are frictions in the transition process.

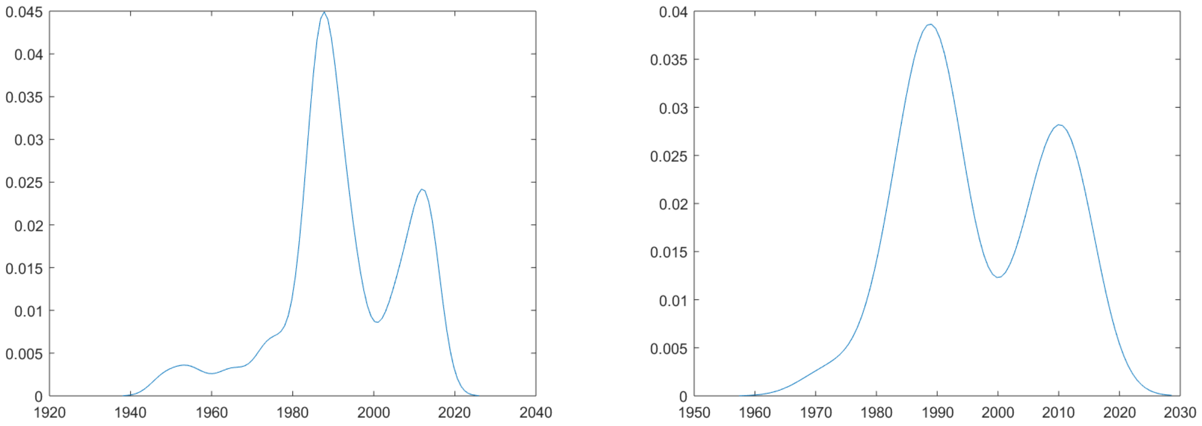
Power plants are using different type of energy sources with their own characteristics in term of production capacity, pollutant emission, geographical preferences,... Each energy source depends on one or several technologies, like solar energy, it can be transformed using photovoltaic (PV) or concentrating solar power (CSP) units. Technological progress is then embedded in each power plant, PV panels are formed of numerous photovoltaic modules which convert sun light into electricity using the photovoltaic effect. Performance of PV panels can be enhanced using more recent modules or coupling the system with an heat pump for example, but it is not possible to apply better modules or provide heat pump association on already engaged PV farms: technology is embedded in each generation (vintage) of panels, and is incompatible with other kind of solar energy like concentrating solar power (CSP). This special feature advocates for models with embodied technical change.

Using NEEDS data one can study, at least for US, lifetime of energy power plants. This dataset contains information about the commissioning year of actual power plants according to energy they use. Figures 2 and 3 show kernel density of On Line Year for renewable and fossil energy plants.



(a) Solar

(b) Wind

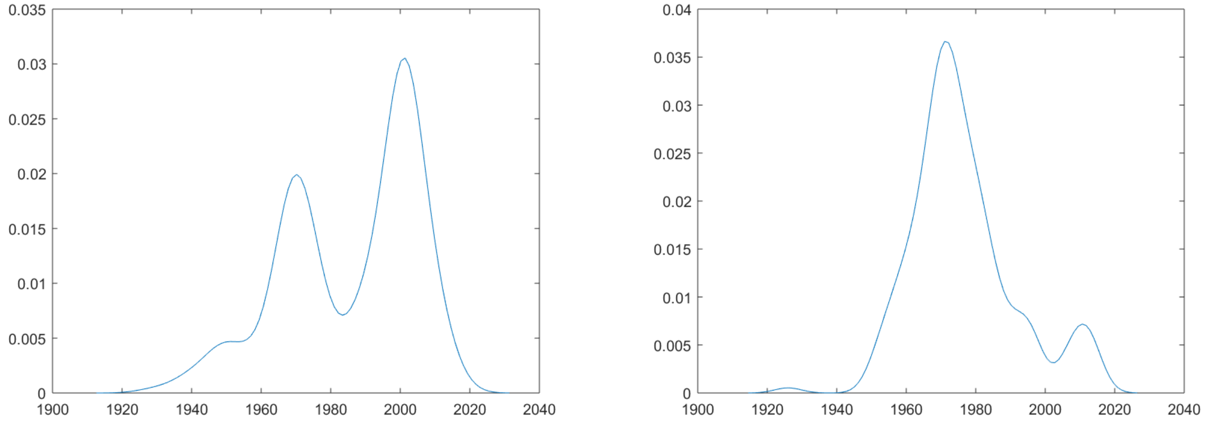


(c) Biomass

(d) Geothermal

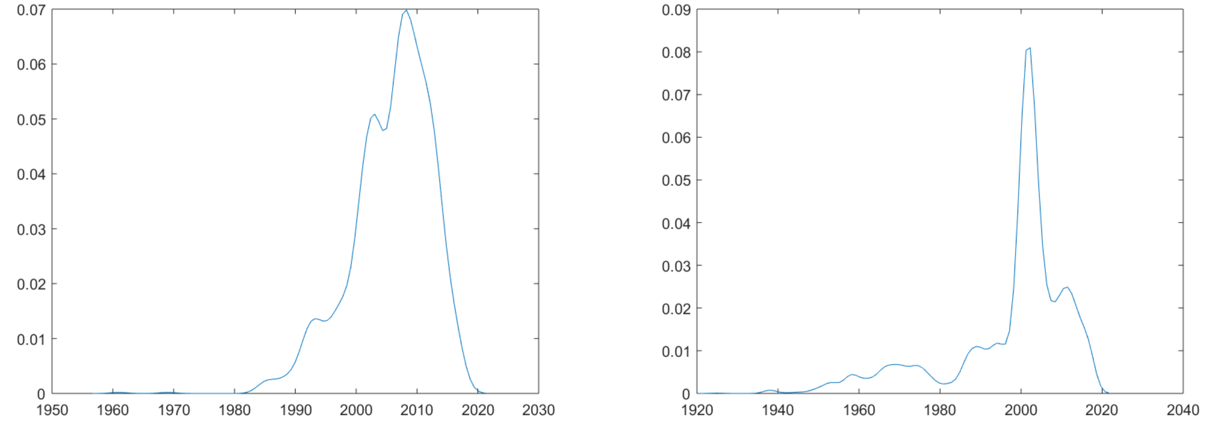
Figure 7: Renewable energy sources - On line year kernel density

We observe that fossil energy plants are older than renewable ones, but the later are still long living. especially for geothermal and biomass, there is a non-negligible share of them which between 40 and 60 years old. For gas power plants a majority of them were built around 2000 but some are a little bit older, for coal and oil power plants a big proportions of them are aged between 40 and 60 years. In conclusion, lifetime of power plants in US can be very long, almost 100 years for some specific units, slowing down the capacity to scrap old plants to build new ones.



(a) Oil

(b) Coal



(c) Landfill gas

(d) Natural gas

Figure 8: Fossil energy sources - On line year kernel density

B Embodied technical change in the data

Using The National Electric Energy Data System (NEEDS) v6 I am able to justify the functional forms used in the theoretical part of the paper. I am documenting here the validity of embodied technical change, that older power plants are less efficient and develop less power than new ones.

The NEEDS dataset contains information about all power plants operating in US and producing electricity. It details the capacity developed, the energy source used, an efficiency proxy (The net heat input required to generate 1 kilowatt hour of electricity), in which state the power plant is operating, when it started to operate and if the power plant is subject to pollution control (NOx and particulate matter). I then regress the efficiency measure and the power capacity on the other variables, using state and plant type fixed effect to show the negative relationship between lifetime of power plants and their capacity and efficiency. For this purpose I am using a simple OLS model, results can be found in tables 9, for efficiency, and 10 for capacity.

Variables used are the following,

• Efficiency: the neat heat input required to generate 1 kilowatt hour of electricity. This value is a proxy for efficiency of electricity generators, the higher this value is the lower the efficiency?. I have then inverse values, to have a positive trend for this variable. However this measure is not applicable for every power plants, then Photovoltaic panels and wind turbines are not concerned by this measure.

• Capacity (MW): the power developed by power plants, in megawatts

• renewable: dummy variable, it is equal to 1 if the plant is using renewable energy, 0 if it using fossil source. It is used to control for the capacity and efficiency delay of a power plant.

Then I use pollution control variable, for both NOx and particulate matter as other control variables. I am also controlling for a state fixed effect, because energy policies are different according to the state considered. California is using more intensively renewable energy and may develop learning by doing effect for renewable energies, while Texas have the same advantage with oil. And then I have a plant type fixed effect to control for each energy specificity.

Figure 9: Efficiency of Electricity power plants

	(1)	(2)	(3)	(4)
lifetime	-52.88***	-50.03***	-48.21***	-49.60***
	(-21.53)	(-19.97)	(-18.63)	(-18.98)
Capacity (MW)		2.825***	2.609***	2.756***
		(15.52)	(14.67)	(14.20)
renewable		1589.7**	1557.0**	1432.6*
		(3.01)	(3.02)	(1.96)
Constant	22237.7***	20284.0***	20305.6***	20787.4***
	(130.36)	(41.17)	(42.12)	(28.86)
Other control	No	No	Yes	Yes
Plant Type fixed effect	Yes	Yes	Yes	Yes
State fixed effect	No	No	No	Yes
Observations	10771	10771	10771	10771
R^2	0.41	0.42	0.42	0.44

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

More recent US power plants are more efficient than older one, this result is robust at the 0.1% threshold. it holds under many specifications and under state and plant type fixed effect. The assumption of embodied technical change fits quite well with energy power plants: older power plants are less efficient, it highlights the fact that technology do not spread over already built structures. The choice of an investment specific accumulation equation for both “clean” and “dirty” capital is confirmed by those results.

Additionally it seems 1) there is a scale effect. Capacity has a positive impact on efficiency, large power plants are then more efficient than smaller ones. 2) Renewable power plants are more efficient than fossil ones. It is logical as efficiency is constructed through a net heat input, we can think renewable power plants need less heat to produce electricity.

Figure 10: Capacity regression

	(1)	(2)	(3)	(4)
lifetime	-1.064***	-1.064***	-0.888***	-0.854***
	(-20.91)	(-20.91)	(-17.07)	(-15.53)
renewable		-48.84	-50.59	-28.43
		(-1.69)	(-1.72)	(-0.81)
Constant	129.9***	178.8***	177.1***	204.2***
	(8.01)	(7.44)	(7.26)	(6.38)
Other control	No	No	Yes	Yes
Plant Type fixed effect	Yes	Yes	Yes	Yes
State fixed effect	No	No	No	Yes
Observations	14434	14434	14434	14434
R^2	0.51	0.51	0.52	0.54

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

I find the same impact from lifetime on capacity generation than for efficiency measure. Older power plants are delivering less power than more recent ones, and this result holds under several specifications. It also validates the embodied technical change assumption for the theoretical model. We can also conclude that once we control for both power plant type and state fixed effect, there is no evidence that renewable plants are delivering a lowest amount of power. It justify the trade-off made in the final good production function, taking both “clean” and “dirty” capital as equal and imperfectly substitutable inputs.

C Additional simulations

The model also allows to simulate the level of GDP. For the following simulation I assume that the “true” value of damages lies between the optimistic and the pessimistic view. In this sens I can capture the positive effect of over estimating damages, and the negative effect of underestimating it.

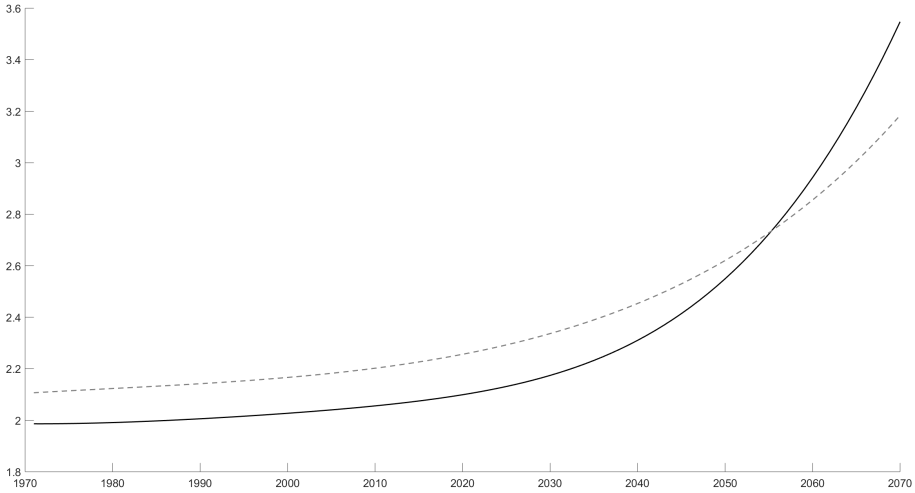


Figure 11: GDP level, comparison EU-US

I observe that the EU catches-up with US GDP level around 2055. Investing more in the beginning on clean energy allows to reduce the future damages without giving up on energy use, while US by underestimating the level of damages decided to rely more on fossil energy which harms more than expected the future economy. Then the US loses its economic leadership in favor of the EU in this simple simulated model, due to underestimation of the damages. I also observe there is an acceleration in the growth rate after clean energy catches-up with "dirty" technology efficiency.

D Embodied vs Disembodied technological change

The model uses embodied technical change for the accumulation of both "clean" and "dirty" capital, this feature is also present in Lennox and Witajewski-Baltvilks (2017). here I will make some adjustment to obtain a disembodied technical change model and study differences with the embodied version.

To do so I am reducing the dimension of the model by removing one state equation, there is one type of capital which can be divided between "clean" or "dirty" intermediate such that

$$K(t) = K_c(t) + K_d(t)$$

This unique capital stock stock is accumulating as follow

$$\dot{K} = Y(t) - \delta K(t) - c(t)$$

And lastly, technological progress is set on all type of capital through the production function

$$Y(t) = L(t)^{1-\alpha}\left((q(t)K_c(t))^{\sigma} + K_d(t)^{\sigma}\right)^{\frac{\alpha}{\sigma}} exp(-\theta(S(t) - \bar{S}))$$

I then solve this modified version of the model using an hamiltonian in current value, to obtain the dynamic system. I simulate the dynamic equations of the model to compare disembodied and embodied technical change. Using the US calibration I obtain the following graphs:

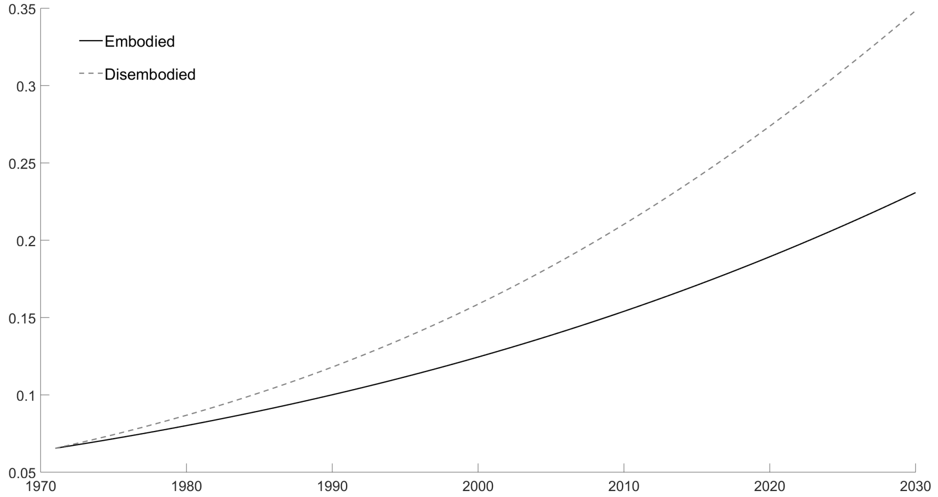


Figure 12: Share of clean energy proxy

Since for the disembodied version technological progress is assigned to the stock of “clean” capital, I cannot look at the real share of renewable energy because efficiency increases each period. I then use a proxy to compare the 2 patterns. It appears that the proxy with disembodied technical change is growing faster than the embodied version. It seems reasonable since in this version technological progress spread on each unit of “clean” capital, while in the embodied technical change it affects only newly installed units.

Share of renewable energy is increasing more rapidly but it might also be the case for the stock of “dirty” capital. It would coincides with an higher level of pollution.

When technology is disembodied, the level of dirty capital starts to decrease earlier

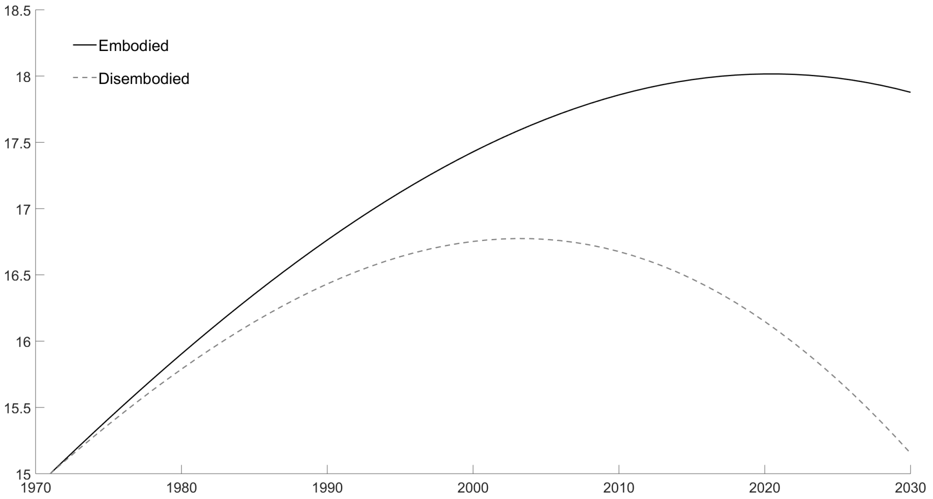


Figure 13: Level of “dirty” capital

than for embodied technology. It also reach a lower maximum with the same parameters with a 1970 level almost reached in 2030 according to these simulations.

These differences between embodied and disembodied technical change are highlighting the presence of the lasting capital effect described previously. By considering 2 distinct capital accumulation equation I impeach capital to move freely between the 2 sectors and each decision is lasting over approximately 40 years. This mechanism is absent from the disembodied version because the only capital good can be freely dedicated to “clean” or “dirty” sector.